

CS 173, Fall 2015  
Examlet 7, Part B

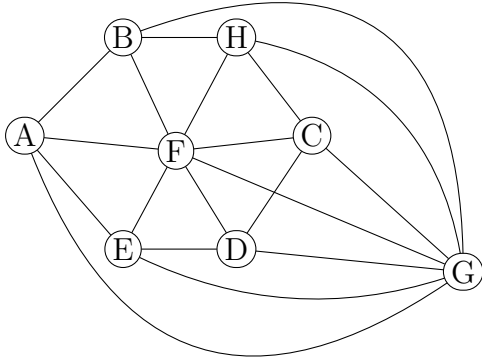
NETID:

FIRST:

LAST:

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

1. (9 points) What is the chromatic number of graph  $G$  (below)? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Exactly 11 Xboxes fit in my suitcase by volume, but I haven't checked their total weight. 11 is \_\_\_\_\_ how many Xboxes the suitcase can hold.

an upper bound on  exactly   
a lower bound on  not a bound on

All elements of  $M$  are also elements of  $X$ .

$M = X$    $M \subseteq X$    $X \subseteq M$

$$\sum_{i=1}^{p-1} i =$$

$$\frac{p(p-1)}{2} \quad \input{checkbox}$$

$$\frac{(p-1)^2}{2} \quad \input{checkbox}$$

$$\frac{p(p+1)}{2} \quad \input{checkbox}$$

$$\frac{(p-1)(p+1)}{2} \quad \input{checkbox}$$

CS 173, Fall 2015  
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NETID:

FIRST:

LAST:

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1. (11 points) Let's define two sets as follows:

$$A = \{x \in \mathbb{R} : |x + 1| \leq 2\}$$

$$B = \{w \in \mathbb{R} : w^2 + 2w - 3 \leq 0\}$$

Prove that  $A = B$  by proving two subset inclusions.

2. (4 points) Check the (single) box that best characterizes each item.

I found 143 identical marbles in my saucepan last Saturday. 143 is \_\_\_\_\_ how many marbles this size will fit in my saucepan.

an upper bound on

exactly

a lower bound on

not a bound on

Chromatic number of a bipartite graph with at least two vertices.

1

2

3

can't tell

CS 173, Fall 2015  
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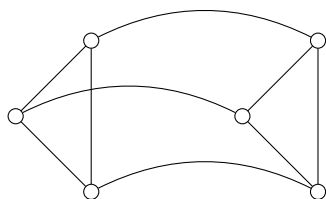
NETID:

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LAST:

Discussion:    Thursday    2    3    4    5    Friday    9    10    11    12    1    2

1. (11 points) Recall that if  $G$  is a graph, then  $\chi(G)$  is its chromatic number. Let's define the "doubled" version of a graph  $G$  as follows: make two copies of  $G$  and add an edge joining each pair of corresponding nodes. For example, the doubled version of  $C_3$  looks like:



Suppose that  $T$  is the doubled version of a graph  $G$ . Describe how  $\chi(T)$  is related to  $\chi(G)$ , justifying your answer. Your answer should handle any choice for  $G$ , not just  $C_3$ .

2. (4 points) Check the (single) box that best characterizes each item.

All elements of  $X$  are also elements of  $M$ .

$$M = X \quad \square$$

$$M \subseteq X \quad \square$$

$$X \subseteq M \quad \square$$

$$\sum_{k=1}^n \frac{1}{2^k}$$

$$1 - \left(\frac{1}{2}\right)^{n-1} \quad \square$$

$$2 - \left(\frac{1}{2}\right)^n \quad \square$$

$$1 - \left(\frac{1}{2}\right)^n \quad \square$$

$$2 - \left(\frac{1}{2}\right)^{n-1} \quad \square$$

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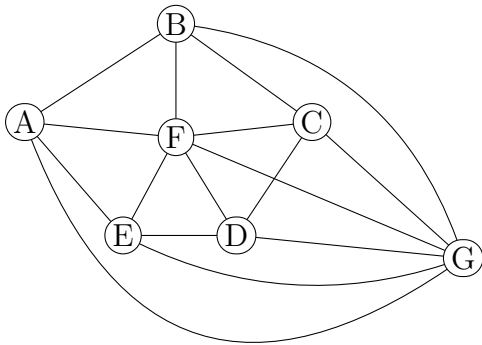
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1. (9 points) What is the chromatic number of graph  $G$  (below)? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{i=0}^{k-1} (k \cdot i + 2) = \frac{k^2(k-1)}{2} + 2k \quad \boxed{\phantom{x}} \quad \frac{k(k+1)}{2} + 2(k-1) \quad \boxed{\phantom{x}}$$

$$\frac{k^2(k+1)}{2} + 2k \quad \boxed{\phantom{x}} \quad \frac{k(k-1)}{2} + 2(k-1) \quad \boxed{\phantom{x}}$$

Putting 10 people in the canoe caused it to sink. 10 is \_\_\_\_\_ how many people the canoe can carry.

an upper bound on

exactly

a lower bound on

not a bound on

The chromatic number of a graph with maximum vertex degree  $D$

$= D$    
 $\leq D + 1$

$= D + 1$    
 $\geq D + 1$

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1. (11 points) Let's define two sets as follows:

$$A = \{(p + 1, p) : p \in \mathbb{R}\}$$

$$B = \{\lambda(1, 0) + (1 - \lambda)(2, 1) : \lambda \in \mathbb{R}\}$$

Prove that  $A = B$  by proving two subset inclusions.

2. (4 points) Check the (single) box that best characterizes each item.

Suppose I want to estimate  $\frac{103}{20}$ .  
3 is \_\_\_\_\_

an upper bound

an exact answer

a lower bound

not a bound on

The chromatic number of  $C_n$ .

2

3

$\leq 3$

$\leq 4$

CS 173, Fall 2015  
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1. (11 points) Recall that if  $G$  is a graph, then  $\chi(G)$  is its chromatic number. Suppose that  $G$  is a graph and  $H$  is another graph not connected to  $G$ . Suppose  $G$  and  $H$  each have at least two nodes and at least one edge. Dr. Evil picks two adjacent nodes  $a$  and  $b$  from  $G$ , and also two adjacent nodes  $c$  and  $d$  from  $H$ . He merges  $G$  and  $H$  into a single graph  $T$  by merging  $b$  and  $d$  into a single node, and adding an edge connecting  $a$  and  $c$ . So, if  $G$  and  $H$  are as shown on the left, then  $T$  might look as shown on the right.



Describe how  $\chi(T)$  is related to  $\chi(G)$  and  $\chi(H)$ , justifying your answer.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=3}^n k^7 = \sum_{p=1}^{n-2} p^9 \quad \square \quad \sum_{p=1}^{n-2} k^7 \quad \square \quad \sum_{p=1}^{n-2} k^9 \quad \square \quad \sum_{p=1}^{n-2} (p+2)^7 \quad \square$$

$W_7$  is a subgraph of graph  $H$ . 4 is  
\_\_\_\_\_ the chromatic number of  $H$ .

an upper bound on   
a lower bound on

exactly   
not a bound on