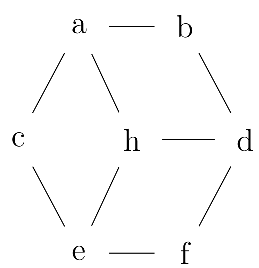
# CS 173, Fall 2015
# Examlet 6, Part A

**NETID:**

**FIRST:** **LAST:**

**Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

1. (10 points) How many isomorphisms are there from $G$ (below) to itself? Justify your answer and/or show your work clearly .




**Solution:** 6. Node b can map to c, f, or itself. Then node a can map to either of the two adjacent nodes. After that, the rest of the mapping is forced.

2. (5 points) Complete this statement of the Handshaking Theorem.

   For any graph G with set of nodes V and set of edges E, ...

   **Solution:** The sum of the degrees of all the nodes is equal to twice the number of edges.

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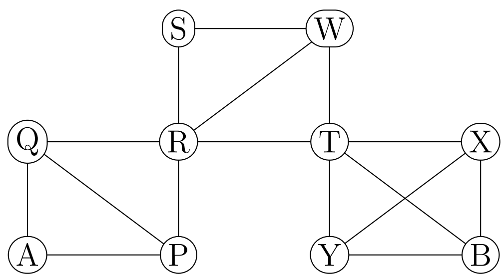


**Solution:** Nodes R and T must map to themselves. They are the only degree-5 nodes and only R is adjacent to a degree-2 node. So S, W, and A must also map to themselves. Q and P can be swapped. X, Y, and B can be permuted, creating 3! choices. So there are $2 \cdot 3!$ different isomorphisms of the whole graph.

2. (5 points) The complete graph $K_7$ contains 7 vertices. How many edges does it have?

**Solution:** It has $\frac{7 \cdot 6}{2} = 21$ edges.

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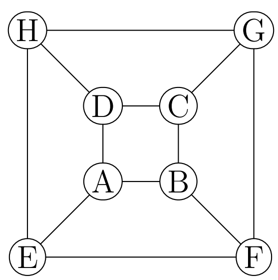
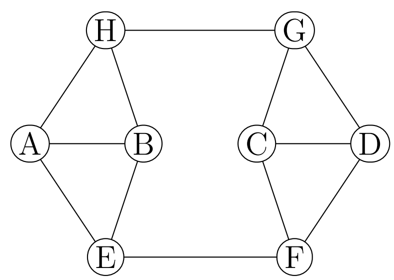
1. (10 points) Are the graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



**Solution:** No, they are not isomorphic. Graph Y contains several 3-cycles (e.g. A,B,H) where Graph X contains no 3-cycles.

2. (5 points) Is the cycle graph $C_4$ a subgraph of graph $K_{3,3}$? Briefly justify your answer.

**Solution:** Yes, it is. Pick two nodes on each side of $K_{3,3}$ and follow a path back-and-forth between the two sides.

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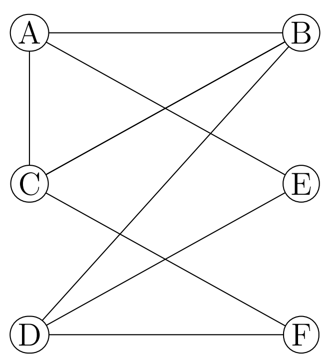
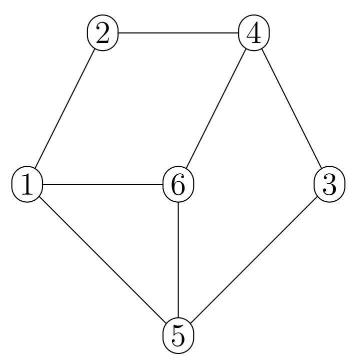
   Graph X

   Graph Y

   **Solution:** Yes, they are isomorphic. Consider the map f such that f(A) = 5, f(B) = 6, f(C) = 1, f(D) = 4, f(E) = 3, and f(F) = 2.

2. (5 points) What is the difference between a cycle and a closed walk?

   **Solution:** A cycle uses each node only once, except that the first and last nodes are the same. Also, a cycle must contain at least three nodes.

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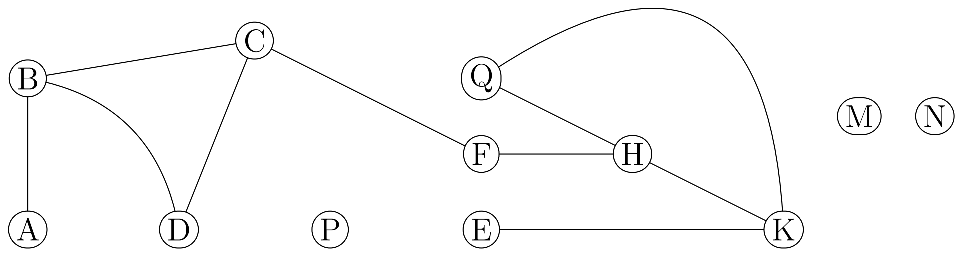


**Solution:** There are 3! ways to permute the degree-0 nodes (M, N, P). We can also swap C and H. After that, we have no more choices. So there are $2 \cdot 3! = 12$ isomorphisms.

2. (5 points) Is the graph $C_7$ bipartite? Briefly justify your answer.

**Solution:** No, it isn't bipartite. As you walk around the cycle, you must assign nodes to the two subsets in an alternating manner. But there's no way to assign the last (odd) node.

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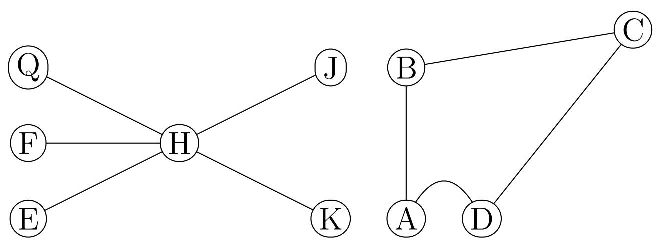


**Solution:** The left part needs to map to itself, similarly for the right part. There are 5! ways to map the left part to itself, because any of the degree-1 nodes can be interchanged. There are eight isomorphisms of the right part to itself (four choices for where to map A, then two choices for B). So there are $8 \cdot 5!$ total isomorphisms.

2. (5 points) Does the complete graph $K_7$ have an Euler circuit?

**Solution:** Yes. Each node has degree 6, which is even.