CS 173, Fall 2015
Examlet 5, Part B

NETID:

FIRST: | **LAST:**

Discussion: **Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

1. (5 points) The departmental proctor needs to arrange 9 students in a row of 9 chairs for a makeup exam. 4 of these students are from CS 173 and cannot sit next to one another. How many options does the proctor have?

   Solution: There are 5! ways to arrange the non-173 students.

   There are 6 potential positions for a CS 173 student. So there are 6 choices for where to put the first one, 5 for the second, 4 for the third, and 3 for the fourth.

   So the total number of choices is $5! \cdot 6 \cdot 5 \cdot 4 \cdot 3$.

2. (10 points) Check the (single) box that best characterizes each item.

If a function from $\mathbb{R}$ to $\mathbb{R}$ is increasing, it must be one-to-one.	true ☐	false $\boxed{\surd}$	
$g : \mathbb{N} \to \mathbb{N}$, $g(x) = x$	onto $\boxed{\surd}$	not onto ☐	not a function ☐
$g : \mathbb{R}^2 \to \mathbb{R}^2$, $g(x, y) = (y, 3x)$	one-to-one $\boxed{\surd}$	not one-to-one ☐	not a function ☐
$g : \mathbb{N} \to \mathbb{Z}$, $g(x) = x^2$	one-to-one $\boxed{\surd}$	not one-to-one ☐	not a function ☐
$\exists y \in \mathbb{R},\ \forall x \in \mathbb{R},\ xy = 1$	true ☐	false $\boxed{\surd}$	

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1. (5 points) How many different 13-letter strings ending with s can be made be rearranging the characters in the word ``massachusetts''? Show your work.

   Solution: Notice that the final character (s) is fixed. There are 12 letters total to rearrange, with 3 copies of s, two t's, and 2 a's. So the number of possibilities is

$$\frac{12!}{3!2!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the set of all even integers is the _____ of f. domain ☐ co-domain ☐ image ☑

$g : \mathbb{N} \to \mathbb{Z}$, $g(x) = x$ onto ☐ not onto ☑ not a function ☐

$g : \mathbb{Z} \to \mathbb{R}$, $g(x) = x + 2.137$ one-to-one ☑ not one-to-one ☐ not a function ☐

$g : \mathbb{R} \to \mathbb{Z}$, $g(x) = |x|$ one-to-one ☐ not one-to-one ☐ not a function ☑

$\forall p \in \mathbb{Z}^+,\ \exists t \in \mathbb{Z}^+,\ \gcd(p, t) = 1$ true ☑ false ☐

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1. (5 points) Hermione Granger has 7000 socks in her magically expanding drawer. The socks are colored purple, magenta, shocking pink, and neon green. How many socks must she pull out of the drawer before she is guaranteed to have two socks of the same color. Briefly justify your answer.

   Solution: She needs to pull out five socks. By the pigeonhole principle, five socks and only four colors means that two must have the same color.

2. (10 points) Check the (single) box that best characterizes each item.

A function is onto if and only if its image is the same as its co-domain.	true	$\surd$	false			
$g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = 7 - \left\lfloor \frac{x}{3} \right\rfloor$	one-to-one		not one-to-one	$\surd$	not a function	
$g : \mathbb{Z} \to \mathbb{N}$, $g(x) = x$	onto		not onto		not a function	$\surd$
$f : \mathbb{N} \to \mathbb{R}$, $f(x) = x^2 + 2$	one-to-one	$\surd$	not one-to-one		not a function	
$\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ xy = 1$	true		false	$\surd$		

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1. (5 points) Suppose that $|A| = p$, $|B| = q$, $|C| = n$. How many different functions are there from $A \times B$ to C?

   Solution: There are pq elements in $A \times B$. So there are $(n)^{pq}$ ways to build a function from $A \times B$ to C.

2. (10 points) Check the (single) box that best characterizes each item.

If $f : A \to B$ is onto, then	$\|A\| \geq \|B\|$ ☑	$\|A\| \leq \|B\|$ ☐	$\|A\| = \|B\|$ ☐
$f : \mathbb{N}^2 \to \mathbb{Z}$, $f(p, q) = 2^p 3^q$	onto ☐	not onto ☑	not a function ☐
$f : \mathbb{R} \to \mathbb{Z}$, $f(x) = x$	one-to-one ☐	not one-to-one ☐	not a function ☑
$g : \mathbb{N} \to \mathbb{Z}$, $g(x) = x^2$	one-to-one ☑	not one-to-one ☐	not a function ☐
$\exists t \in \mathbb{N},\ \forall p \in \mathbb{Z}^+,\ \gcd(p, t) = p$	true ☑	false ☐	

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1. (5 points) Suppose that $|A| = 3$ and $|B| = 2$. How many onto functions are there from A to B? Briefly justify or show work.

   Solution: It doesn't matter what the elements of A and B are, so let's suppose that $A = \{1, 2, 3\}$ and $B = \{4, 5\}$. Two elements of A must map to the same output value, with the third element x mapping to the other output value. There are three choices for which element x is. And then there are two choices for which output value corresponds to x. So 6 onto functions total.

2. (10 points) Check the (single) box that best characterizes each item.

$f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x + 3$ (x even), $f(x) = x - 21$ (x odd)	one-to-one ☑ $\surd$	not one-to-one ☐	not a function ☐
Suppose a graph with 12 vertices is colored with exactly 5 colors. By the pigeonhole principle, there is a color that appear on at least two vertices.	true ☑ $\surd$	false ☐	
$g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = \|x\|$	onto ☐	not onto ☑ $\surd$	not a function ☐
$f : \mathbb{R} \to \mathbb{Z}$, $f(x) = x$	one-to-one ☐	not one-to-one ☐	not a function ☑ $\surd$
$\exists t \in \mathbb{Z}^+,\ \forall p \in \mathbb{Z}^+,\ \gcd(p, t) = 1$	true ☑ $\surd$	false ☐	

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1. (5 points) Suppose that A is a set containing $k+1$ (distinct) integers. Use the Pigeonhole Principle to show that there are x and y in A ($x \neq y$) such that and $x - y$ is a multiple of k.

   Solution: (You don't need to be this formal for full credit.) Let $A = \{x_1, x_2, \ldots, x_{k+1}\}$. We can represent each integer x_i in terms of its quotient and remainder mod k, i.e. $x_i = kq_i + r_i$, where $0 \leq r_i < k$. There are k possible remainders, but $k+1$ numbers in A. So two numbers must share the same remainder, which implies that they differ by a multiple of k.

2. (10 points) Check the (single) box that best characterizes each item.

If $f : A \to B$ is onto, then	$\lvert A\rvert \geq \lvert B\rvert$ ☑	$\lvert A\rvert \leq \lvert B\rvert$ ☐	$\lvert A\rvert = \lvert B\rvert$ ☐
$g : \mathbb{N} \to \mathbb{Z}$, $g(x) = \lvert x\rvert$	one-to-one ☑	not one-to-one ☐	not a function ☐
$f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x + 3$ (x even), $f(x) = x - 21$ (x odd)	onto ☑	not onto ☐	not a function ☐
$g : \mathbb{Z} \to \mathbb{N}$, $g(x) = x$	one-to-one ☐	not one-to-one ☐	not a function ☑
$\forall x \in \mathbb{Z},\ \exists y \in \mathbb{N},\ x^2 = y$	true ☑	false ☐	