CS 173, Fall 2015
Examlet 5, Part B

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Discussion: **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

1. (5 points) The departmental proctor needs to arrange 9 students in a row of 9 chairs for a makeup exam. 4 of these students are from CS 173 and cannot sit next to one another. How many options does the proctor have?

2. (10 points) Check the (single) box that best characterizes each item.

If a function from $\mathbb{R}$ to $\mathbb{R}$ is increasing, it must be one-to-one. true ☐ false ☐

$g : \mathbb{N} \to \mathbb{N}$, $g(x) = x$ onto ☐ not onto ☐ not a function ☐

$g : \mathbb{R}^2 \to \mathbb{R}^2$, $g(x, y) = (y, 3x)$ one-to-one ☐ not one-to-one ☐ not a function ☐

$g : \mathbb{N} \to \mathbb{Z}$, $g(x) = x^2$ one-to-one ☐ not one-to-one ☐ not a function ☐

$\exists y \in \mathbb{R},\ \forall x \in \mathbb{R},\ xy = 1$ true ☐ false ☐

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1. (5 points) How many different 13-letter strings ending with s can be made be rearranging the characters in the word ``massachusetts''? Show your work.

2. (10 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the set of all even integers is the _____ of f. domain ☐ co-domain ☐ image ☐

$g : \mathbb{N} \to \mathbb{Z}$, $g(x) = x$ onto ☐ not onto ☐ not a function ☐

$g : \mathbb{Z} \to \mathbb{R}$, $g(x) = x + 2.137$ one-to-one ☐ not one-to-one ☐ not a function ☐

$g : \mathbb{R} \to \mathbb{Z}$, $g(x) = |x|$ one-to-one ☐ not one-to-one ☐ not a function ☐

$\forall p \in \mathbb{Z}^+,\ \exists t \in \mathbb{Z}^+,\ \gcd(p, t) = 1$ true ☐ false ☐

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1. (5 points) Hermione Granger has 7000 socks in her magically expanding drawer. The socks are colored purple, magenta, shocking pink, and neon green. How many socks must she pull out of the drawer before she is guaranteed to have two socks of the same color. Briefly justify your answer.

2. (10 points) Check the (single) box that best characterizes each item.

A function is onto if and only if its image is the same as its co-domain.	true	☐	false	☐		
$g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = 7 - \lfloor \frac{x}{3} \rfloor$	one-to-one	☐	not one-to-one	☐	not a function	☐
$g : \mathbb{Z} \to \mathbb{N}$, $g(x) = x$	onto	☐	not onto	☐	not a function	☐
$f : \mathbb{N} \to \mathbb{R}$, $f(x) = x^2 + 2$	one-to-one	☐	not one-to-one	☐	not a function	☐
$\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ xy = 1$	true	☐	false	☐		

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1. (5 points) Suppose that $|A| = p$, $|B| = q$, $|C| = n$. How many different functions are there from $A \times B$ to C?

2. (10 points) Check the (single) box that best characterizes each item.

If $f : A \to B$ is onto, then	$\lvert A\rvert \geq \lvert B\rvert$ ☐	$\lvert A\rvert \leq \lvert B\rvert$ ☐	$\lvert A\rvert = \lvert B\rvert$ ☐
$f : \mathbb{N}^2 \to \mathbb{Z}$, $f(p,q) = 2^p 3^q$	onto ☐	not onto ☐	not a function ☐
$f : \mathbb{R} \to \mathbb{Z}$, $f(x) = x$	one-to-one ☐	not one-to-one ☐	not a function ☐
$g : \mathbb{N} \to \mathbb{Z}$, $g(x) = x^2$	one-to-one ☐	not one-to-one ☐	not a function ☐
$\exists t \in \mathbb{N},\ \forall p \in \mathbb{Z}^+,\ \gcd(p,t) = p$	true ☐	false ☐	

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1. (5 points) Suppose that $|A| = 3$ and $|B| = 2$. How many onto functions are there from A to B? Briefly justify or show work.

2. (10 points) Check the (single) box that best characterizes each item.

$f : \mathbb{Z} \to \mathbb{Z}$,
$f(x) = x + 3$ (x even),
$f(x) = x - 21$ (x odd)
one-to-one ☐ not one-to-one ☐ not a function ☐

Suppose a graph with 12 vertices is colored with exactly 5 colors. By the pigeonhole principle, there is a color that appear on at least two vertices.
true ☐ false ☐

$g : \mathbb{Z} \to \mathbb{Z}$,
$g(x) = |x|$
onto ☐ not onto ☐ not a function ☐

$f : \mathbb{R} \to \mathbb{Z}$,
$f(x) = x$
one-to-one ☐ not one-to-one ☐ not a function ☐

$\exists t \in \mathbb{Z}^+,\ \forall p \in \mathbb{Z}^+,\ \gcd(p, t) = 1$
true ☐ false ☐

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1. (5 points) Suppose that A is a set containing $k+1$ (distinct) integers. Use the Pigeonhole Principle to show that there are x and y in A ($x \neq y$) such that and $x - y$ is a multiple of k.

2. (10 points) Check the (single) box that best characterizes each item.

If $f : A \to B$ is onto, then	$\|A\| \geq \|B\|$ ☐	$\|A\| \leq \|B\|$ ☐	$\|A\| = \|B\|$ ☐
$g : \mathbb{N} \to \mathbb{Z}$, $g(x) = \|x\|$	one-to-one ☐	not one-to-one ☐	not a function ☐
$f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x + 3$ (x even), $f(x) = x - 21$ (x odd)	onto ☐	not onto ☐	not a function ☐
$g : \mathbb{Z} \to \mathbb{N}$, $g(x) = x$	one-to-one ☐	not one-to-one ☐	not a function ☐
$\forall x \in \mathbb{Z},\ \exists y \in \mathbb{N},\ x^2 = y$	true ☐	false ☐	