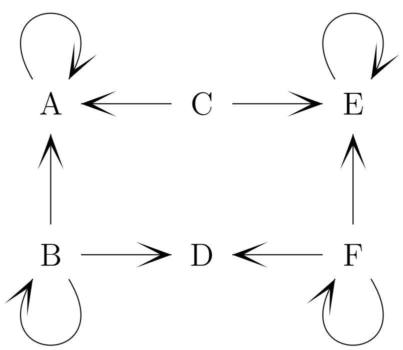
CS 173, Fall 2015
Examlet 4, Part B

NETID:

FIRST:	**LAST:**

Discussion: **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	☐
Symmetric:	☐	Antisymmetric:	$\surd$
Transitive:	$\surd$		

2. (5 points) Suppose that R is a partial order on a set A. What additional property is required for R to be a linear order (aka total order)? Give specific details of the property, not just its name.

   Solution: All pairs of elements must be comparable. That is, for any elements x and y in A, either xRy or yRx.

3. (5 points) Recall that $\mathbb{N}^2$ is the set of all pairs of natural numbers. Let's define the equivalence relation $\sim$ on $\mathbb{N}^2$ as follows: $(x, y) \sim (p, q)$ if and only $|x - y| = |p - q|$. List three members of $[(2, 3)]$.

   Solution: (2,3), (3,4), (14,13)

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1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.

A	C	E
B	D	F

(that is, 6 nodes and no arrows at all)

Reflexive:	$\square$	Irreflexive:	$\boxed{\surd}$
Symmetric:	$\boxed{\surd}$	Antisymmetric:	$\boxed{\surd}$
Transitive:	$\boxed{\surd}$		

2. (5 points) Suppose that R is a relation on a set A. Using precise mathematical words and notation, define what it means for R to be transitive.

   Solution: For any $x, y, z \in A$, if xRy and yRz, then xRz.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x \leq p$ or $y \leq q$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is not transitive. We have $(0, 0)T(-10, 10)$ (look at the second coordinate). We also have $(-10, 10)T(-5, -5)$ (look at the first coordinate). But it's not the case that $(0, 0)T(-5, -5)$.

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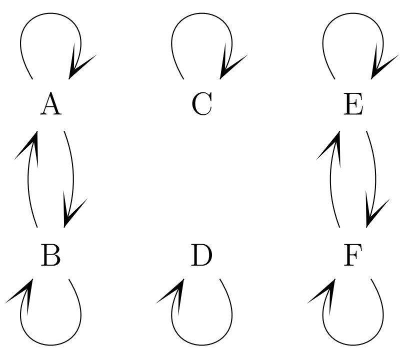
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	$\boxed{\surd}$	Irreflexive:	$\boxed{}$
Symmetric:	$\boxed{\surd}$	Antisymmetric:	$\boxed{}$
Transitive:	$\boxed{\surd}$		

2. (5 points) A relation is an equivalence relation if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

   Solution: reflexive, symmetric, transitive

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $xp + yq = 0$. Is T irreflexive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is not irreflexive, because $(0, 0)$ is related to itself.

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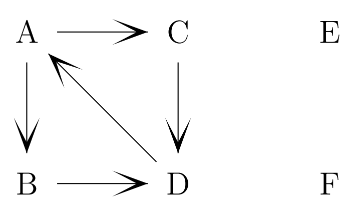
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	√
Symmetric:	☐	Antisymmetric:	√
Transitive:	☐		

2. (5 points) A relation is a strict partial order if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

   Solution: irreflexive, antisymmetric, transitive

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x - p \leq y - q$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is transitive. Suppose that $(x, y)T(p, q)$ and $(p, q)T(m, n)$. Then $x - p \leq y - q$ and also $p - m \leq q - n$. Adding these two equations together, we get $(x - p) + (p - m) \leq (y - q) + (q - n)$. This simplifies to $x - m \leq y - n$. So $(x, y)T(m, n)$.

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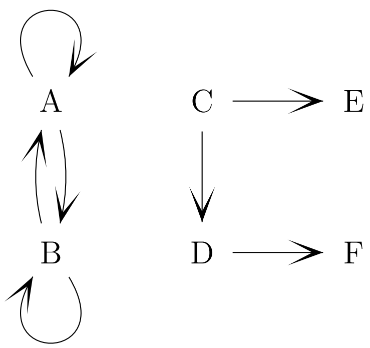
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☐

2. (5 points) Suppose that R is an equivalence relation on a set A. Using precise set notation, define $[x]_R$, i.e. the equivalence class of x under the relation R.

   Solution: $[x]_R = \{y \in A \mid xRy\}$

3. (5 points) Suppose that T is the relation on the set of integers such that aTb if and only if $\gcd(a, b) = 3$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is not transitive. Suppose $a = 6$, $b = 15$, and $c = 12$. Then $\gcd(a, b) = 3$ and $\gcd(b, c) = 3$, but $\gcd(a, c) = 6$.

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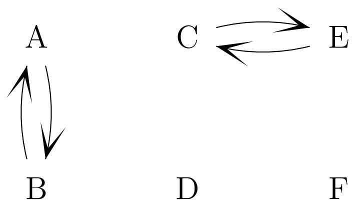
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	☑ √
Symmetric:	☑ √	Antisymmetric:	☐
Transitive:	☐		

2. (5 points) Can a relation be irreflexive, symmetric, and also transitive? Either give such a relation or briefly explain why it's not possible to construct one.

   Solution: Yes, this is possible. However, this must be the empty relation in which no elements are related. (No edges in the graph representation.) If we have aRb, then we must have bRa (by symmetry) and so aRa (by transitivity), which is inconsistent with the relation being irreflexive.

3. (5 points) Let $\sim$ be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y) \sim (p, q)$ if and only if $x^2 + y^2 = p^2 + q^2$. Find three elements in the equivalence class $[(0, 1)]$

   Solution:

   $(0, 1)$, $(1, 0))$, $(-1, 0)$ (for example)