CS 173, Fall 2015
Examlet 4, Part A

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Discussion: **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

Let T be the relation defined on $\mathbb{N}^2$ by

$(x, y)T(p, q)$ if and only if $x \leq p$ or ($x = p$ and $y \leq q$)

Prove that T is transitive.

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The closed interval $[a, b]$ is defined by $[a, b] = \{x \in \mathbb{R} \; : \; a \leq x \leq b\}$. Let J be the set containing all closed intervals $[a, b]$. Let's define the relation F on J as follows:

$[s, t]F[p, q]$ if and only if $q \leq s$

Prove that F is antisymmetric.

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Let $A = \mathbb{N} \times \mathbb{N}$, i.e. pairs of natural numbers.

Define a relation $\gg$ on A as follows:

$(x, y) \gg (p, q)$ if and only if there exists an integer $n \geq 1$ such that $(x, y) = (np, nq)$.

Prove that $\gg$ is antisymmetric.

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Let T be the relation defined on $\mathbb{Z}^2$ by

$(x, y)T(p, q)$ if and only if $x < p$ or ($x = p$ and $y \leq q$)

Prove that T is antisymmetric.

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Suppose that n is some positive integer. Let's define the relation R_n on the integers such that aR_nb if and only if $a \equiv b + 1 \pmod{n}$. Prove the following claim

Claim: For any integers x, y, and z, if xR_ny and yR_nz and xR_nz, then $n = 1$.

You must work directly from the definition of congruence mod k, using the following version of the definition: $x \equiv y \pmod{k}$ iff $x - y = mk$ for some integer m. You may use the following fact about divisibility: for any non-zero integers p and q, if $p \mid q$, then $|p| \leq |q|$.

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A closed interval of the real line can be represented as a pair (c, r), where c is the center of the interval and r is its radius. Let $X = \{(c, r) \mid c, r \in \mathbb{R}, r \geq 0\}$ be the set of closed intervals represented this way.

Now, let's define the interval containment $\preceq$ on X as follows

$(c, r) \preceq (d, q)$ if and only if $r \leq q$ and $|c - d| + r \leq q$.

Prove that $\preceq$ is antisymmetric.