CS 173, Fall 2015
Examlet 3, Part B

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Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, $(A \cup B) - C = A \cup (B - C)$.

   Solution:

   Let $A = \{1, 2\}$, $B = \{3\}$, and $C = \{2\}$.

   Then $(A \cup B) - C = \{1, 2, 3\} - C = \{1, 3\}$.

   But $A \cup (B - C) = \{1, 2\} \cup \{3\} = \{1, 2, 3\}$

2. (4 points) Check the (single) box that best characterizes each item.

   If $x \in A - B$, then $x \in B$.

true for all sets A and B	☐	true for some sets A and B ☐
false for all sets A and B	☑	

   Sets A and B are disjoint

$A - B = B - A$	☐	$A = \overline{B}$	☐
$A \cap B = \{\emptyset\}$	☐	$A \cap B = \emptyset$	☑

3. (7 points) In $\mathbb{Z}_9$, find the value of $[5]^{38}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 8$.

   Solution: $[5]^2 = [25] = [7]$

   $[5]^4 = [7]^2 = [49] = [4]$

   $[5]^8 = [4]^2 = [16] = [7]$

   $[5]^{16} = [7]^2 = [49] = [4]$

   $[5]^{32} = [4]^2 = [16] = [7]$

   $[5]^{38} = [5]^{32} \cdot [5]^4 \cdot [5]^2 = [7] \cdot [4] \cdot [7] = [28] \cdot [7] = 1 \cdot [7] = [7]$

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1. (4 points) $A = \{\text{fox, tiger, wolf, eagle, cat}\}$ $B = \{3, 4\}$ $C = \{6, 7\}$

   Solution:

   $A \times (B \cap C) = A \times \emptyset = \emptyset$

   $| A \times (B \cup C) | = 4 \times 5 = 20$

2. (4 points) Check the (single) box that best characterizes each item.

   $A \times B = B \times A$ — true for all sets A and B [ ] — false for all sets A and B [ ] — true for some sets A and B [√]

   $\emptyset \subseteq A$ — true for all sets A [√] — true for some sets A [ ] — false for all sets A [ ]

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{38}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[7]^2 = [49] = [5]$

   $[7]^4 = ([7]^2)^2 = [5]^2 = [25] = [3]$

   $[7]^8 = ([7]^4)^2 = [3]^2 = [9] = [-2]$

   $[7]^16 = ([7]^8)^2 = [-2]^2 = [4]$

   $[7]^32 = ([7]^{16})^2 = [4]^2 = [16] = [5]$

   $[7]^{38} = [7]^{32} \cdot [7]^4 \cdot [7]^2 = [5] \cdot [3] \cdot [5] = [15] \cdot [5] = [4] \cdot [5] = [20] = [9]$

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, if $A \subseteq B$ then $A \times C \subseteq B \times C$.

   Solution: This is true. Suppose we pick (x, y) from $A \times C$. Then x is in A and y is in C. Since x is in A and $A \subseteq B$, x is in B. So (x, y) is in $B \times C$.

2. (4 points) Check the (single) box that best characterizes each item.

   $A \cup B = A$

true for all sets A and B		false for all sets A and B	
true for some sets A and B	√		

   $\emptyset \times \emptyset =$

$\emptyset$	√	$\{\emptyset\}$		$\{\emptyset, \emptyset\}$		$\{(\emptyset, \emptyset)\}$	

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{40}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[7]^2 = [49] = [5]$

   $[7]^4 = ([7]^2)^2 = [5]^2 = [25] = [3]$

   $[7]^8 = ([7]^4)^2 = [3]^2 = [9] = [-2]$

   $[7]^16 = ([7]^8)^2 = [-2]^2 = [4]$

   $[7]^32 = ([7]^{16})^2 = [4]^2 = [16] = [5]$

   $[7]^{40} = [7]^{32} \cdot [7]^8 = [5] \cdot [-2] = [-10] = [1]$

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1. (4 points) $A = \{4, 5, 9\}$ $B = \{\text{arya}, \text{bran}\}$ $C = \{\ 2, 4, 10\ \}$

   Solution:

   $(A \cap C) \times B = \{4\} \times B = \{(4, \text{arya}), (4, \text{bran})\}$

   $|A \times B \times C| = 3 \times 2 \times 3 = 18$

2. (4 points) Check the (single) box that best characterizes each item.

   $\{13, 14, 15\} \times \emptyset =$ $\emptyset$ [√] $\{\emptyset\}$ [ ] $\{13, 14, 15\}$ [ ]

   $\emptyset \in A$ true for all sets A [ ] true for some sets A [√]
   false for all sets A [ ]

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{19}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10]$

   $[7]^4 = [100] = [9]$

   $[7]^8 = [9]^2 = [81] = [3]$

   $[7]^{16} = [3]^2 = [9]$

   $[7]^{19} = [7]^{16} \cdot [7]^{[}2] \cdot [7] = [9] \cdot [10] \cdot [7]$

   $[9] \cdot [10] \cdot [7] = [90] \cdot [7] = [-1] \cdot [7] = [-7] = [6]$

   So $[7]^{19} = [6]$

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1. (4 points) $A = \{\text{apple}, \text{lemon}\}$ $B = \{4, 5, 9\}$ $C = \{\ (\text{apple}, 4),\ (5, \text{lemon})\ \}$

   Solution:

   $\emptyset \times B = \emptyset$

   $(A \times B) \cap C = \{(\text{apple}, 4)\}$

2. (4 points) Check the (single) box that best characterizes each item.

   $|A \cup B| = |A| + |B|$ true for all sets A ☐ true for some sets A ☑
   false for all sets A ☐

   $A \times B = A$ true for all sets A and B ☐ false for all sets A and B ☐
   true for some sets A and B ☑

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[8]^{37}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[8]^2 = [64] = 9$

   $[8]^4 = [9]^2 = [81] = [4]$

   $[8]^8 = [4]^2 = [16] = [5]$

   $[8]^{16} = [5]^2 = [3]$

   $[8]^{32} = [3]^2 = [9]$

   $[8]^{37} = [8]^{32} \cdot [8]^4 \cdot [8] = [9] \cdot [4] \cdot [8] = [36] \cdot [8] = [3] \cdot [8] = [24] = [2]$

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1. (4 points) $A = \{\text{water}, \text{beer}, \text{wine}\}$ $B = \{\text{ cup}, \text{mug }\}$ $C = \{\text{wine}, (\text{water}, \text{beer})\}$

   Solution:

   $A \times B = \{\ (\text{water}, \text{cup}), (\text{beer}, \text{cup}), (\text{wine}, \text{cup}), (\text{water}, \text{mug}), (\text{beer}, \text{mug}), (\text{wine}, \text{mug})\ \}$

   $A \cap C = \{\text{wine}\}$

2. (4 points) Check the (single) box that best characterizes each item.

   $\emptyset$ is — an element of $\mathbb{Z}$ ☐ both ☐ a subset of $\mathbb{Z}$ ☑ neither ☐

   $\{1,2\} \cup \emptyset =$ — $\emptyset$ ☐ $\{(1,\emptyset),(2,\emptyset)\}$ ☐ $\{1,2,\emptyset\}$ ☐ $\{\emptyset\}$ ☐ $\{1,2\}$ ☑ undefined ☐

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{21}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10] = [-3]$

   $[7]^4 = ([7]^2)^2 = [-3]^2 = [9]$

   $[7]^8 = ([7]^4)^2 = [9]^2 = [81] = [3]$

   $[7]^{16} = ([7]^8)^2 = [3]^2 = [9]$

   $[7]^{21} = [7]^{16} \cdot [7]^4 \cdot [7] = [9] \cdot [9] \cdot [7] = [81] \cdot [7] = [3] \cdot [7] = [21] = [8]$