CS 173, Fall 2015
Examlet 3, Part A

NETID:

FIRST: **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

$A = \{(p, q) \in \mathbb{Z}^2 \ \mid 3pq + 15p - 5q - 25 \geq 0\}$

$B = \{(s, t) \in \mathbb{Z}^2 \ \mid t \geq 0\}$

$C = \{(x, y) \in \mathbb{Z}^2 \ \mid x \geq 0\}$

Prove that $(A \cap B) \subseteq C$.

Solution: Proof: Let (p, q) be an element of $\mathbb{Z}^2$ and suppose that (p, q) is an element of $A \cap B$.

Then (p, q) is an element of A, so $3pq + 15p - 5q - 25 \geq 0$. But (p, q) is also an element of B, so $q \geq 0$.

Notice that $3pq + 15p - 5q - 25 = (q + 5)(3p - 5)$. So we know that $(q + 5)(3p - 5) \geq 0$. So q+5 and 3p-5 are either both non-negative or both non-positive. Since $q \geq 0$, $q + 5 \geq 0$. So we must have $3p - 5 \geq 0$.

Now, if $3p - 5 \geq 0$, then $3p \geq 5$. So $p \geq \frac{5}{3}$, and therefore $p \geq 0$.

Since (p, q) be an element of $\mathbb{Z}^2$ and $p \geq 0$, (p, q) must be an element of C, which is what we needed to show.

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$A = \{\lambda(0,3) + (1-\lambda)(2,4) \ \mid \ \lambda \in [0,1]\}$

$B = \{(x,y) \in \mathbb{R}^2 \ \mid \ x \leq y\}$

Prove that $A \subseteq B$.

Solution:

Let $(x,y) \in A$. Then $(x,y) = \lambda(0,3) + (1-\lambda)(2,4)$ for some $\lambda \in [0.1]$. So $x = 2 - 2\lambda$ and $y = 3\lambda + 4(1-\lambda) = 4 - \lambda$. So $y = x + 2 + \lambda$

Since $\lambda \in [0.1]$, $\lambda \geq 0$.

So $y = x + 2 + \lambda \geq x$.

Since $x \leq y$, $(x,y) \in B$, which is what we needed to show.

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$A = \{(x, y) \in \mathbb{R}^2 \ | \ y = x^2 - 3x + 2\}$

$B = \{(p, q) \in \mathbb{R}^2 \ | \ p \geq 0\}$

$C = \{(m, n) \in \mathbb{R}^2 \ | \ n \geq 1\}$

Prove that $A \subseteq B \cup C$.

Solution:

Suppose that (x, y) is an element of A. Then $y = x^2 - 3x + 2 = (x - 1)(x - 2)$. There are two cases:

Case 1: $x \geq 0$. Then $(x, y) \in B$ so $(x, y) \in B \cup C$.

Case 2: $x \leq 0$. Then $x - 1 \leq -1$ and $x - 2 \leq -2$. So $y = (x - 1)(x - 2) \geq 2 \geq 1$. So $(x, y) \in C$. And therefore $(x, y) \in B \cup C$.

In both cases $(x, y) \in B \cup C$, which is what we needed to show.

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$A = \{(x, y) \in \mathbb{R}^2 \ \mid \ x = \lfloor 3y + 5 \rfloor \ \}$

$B = \{(p, q) \in \mathbb{Z}^2 \ \mid \ 2p + q \equiv 3 \pmod 7 \ \}$

Prove that $A \cap \mathbb{Z}^2 \subseteq B$.

Use the following definition of congruence mod k: if s, t, k are integers, k positive, then $s \equiv t \pmod k$ if and only if $s = t + nk$ for some integer n.

Solution: Let (x, y) be an element of $A \cap \mathbb{Z}^2$. Then (x, y) is an element of A and, also, both x and y are integers.

By the definition of Z, $x = \lfloor 3y + 5 \rfloor$. Since y is an integer, $3y + 5$ must also be an integer. So $\lfloor 3y + 5 \rfloor = 3y + 5$. Therefore, $x = 3y + 5$.

Now, consider $2x + y$.

$$2x + y = 2(3y + 5) + y = 7y + 10 = 7(y + 1) + 3$$

$y + 1$ is an integer, since y is an integer. So this means that $2x + y \equiv 3 \pmod 7$. Therefore, (x, y) is an element of B, which is what we needed to show.

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$A = \{(p, q) \in \mathbb{R} \mid p = 0\}$

$B = \{(x, y) \in \mathbb{R} \mid (x-1)^2 + y^2 = 4\}$

$C = \{(s, t) \in \mathbb{R} \mid (s+1)^2 + t^2 = 4\}$

Prove that $B \cap C \subseteq$ A.

Solution: Let $(x, y) \in \mathbb{R}$ and suppose that $(x, y) \in B \cap C$. Then $(x, y) \in B$ and $(x, y) \in C$.

By the definitions of B and C, this means that $(x-1)^2 + y^2 = 4$ and $(x+1)^2 + y^2 = 4$. So $(x-1)^2 + y^2 = (x+1)^2 + y^2$, which means that $(x-1)^2 = (x+1)^2$. Multiplying out the two sides, we get $x^2 - 2x + 2 = x^2 + 2x + 2$ So $-2x = 2x$, so $4x = 0$, so $x = 0$.

Since $x = 0$, $(x, y) \in A$, which is what we needed to show.

[This shows more algebra steps than I'd expect for full credit.]

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$A = \{\lambda(3,2) + (1-\lambda)(5,0) \ \mid\ \lambda \in [0,1]\}$

$B = \{(x,y) \in \mathbb{R}^2 \ \mid\ x \geq y\}$

Prove that $A \subseteq B$.

Solution:

Let $(x,y) \in A$. Then $(x,y) = \lambda(3,2) + (1-\lambda)(5,0)$ for some $\lambda \in [0.1]$. So $x = 3\lambda + 5(1-\lambda) = 5 - 2\lambda$ and $y = 2\lambda$.

Since $\lambda \in [0.1]$, $\lambda \leq 1$. So $4\lambda \leq 5\lambda \leq 5$. So $5 - 4\lambda \geq 0$. And therefore $x = 5 - 2\lambda \geq 2\lambda = y$.

Since $x \geq y$, $(x,y) \in B$, which is what we needed to show.