CS 173, Fall 2015
Examlet 2, Part B

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Discussion: **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers s, t, p, q, if $s \equiv t \pmod p$ and $q \mid p$, then $s \equiv t \pmod q$.

   Solution: This is true.

   Informally, since q is smaller, congruence mod q makes coarser distinctions than congruence mod q. So this is in the right direction and the relationship $q \mid p$ ensures that the details work out.

   More formally, from $s \equiv t \pmod p$ and $q \mid p$, we get that $s = t + pk$ and $p = qj$, where k and j are integers. So $s = t + q(jk)$, which means that $s \equiv t \pmod q$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1183, 351)$. Show your work.

   Solution:

   $1183 - 3 \times 351 = 1183 - 1053 = 130$

   $351 - 2 \times 130 = 351 - 260 = 91$

   $130 - 91 = 39$

   $91 - 3 \times 39 = 91 - 78 = 13$

   $39 - 3 \times 13 = 0$

   So the GCD is 13.

3. (4 points) Check the (single) box that best characterizes each item.

   $-7 \equiv 13 \pmod 6$ — true ☐ false ☑ (√)

   For any positive integers p and q, if $\text{lcm}(p, q) = pq$, then p and q are relatively prime. — true ☑ (√) false ☐

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all natural numbers a and b, if $a \mid b$ and $b \mid a$, then $a = b$.

   Solution: This is true. If $a \mid b$ and $b \mid a$, then $b = pa$ and $a = qb$, where p and q are integers. Since a and b are natural numbers and therefore not negative, p and q cannot be negative. So $b = pqb$, so $pq = 1$. So $p = q = 1$ and therefore $a = b$. [This is more detail than you'd need for full credit.]

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1609, 563)$. Show your work.

   Solution:

   $1609 - 2 \times 563 = 1609 - 1126 = 483$

   $563 - 483 = 80$

   $483 - 6 \times 80 = 3$

   $80 - 26 \times 3 = 80 - 78 = 2$

   $3 - 2 = 1$

   $2 - 2 \times 1 = 0$

   So the GCD is 1.

3. (4 points) Check the (single) box that best characterizes each item.

   For any positive integers p, q, and k, if $p \equiv q \pmod{k}$, then $p^2 \equiv q^2 \pmod{k}$ — true ☑ false ☐

   Zero is a multiple of 7. — true ☑ false ☐

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a,b) = 1$ and $\gcd(b,c) = 1$, then $\gcd(a,c) = 1$.

   Solution: This is false. Consider $a = c = 3$ and $b = 2$. Then a and b have no common factors, i.e. $\gcd(a,b) = 1$. Also b and c have no common factors, i.e. $\gcd(b,c) = 1$. But $\gcd(a,c) = 3$.

2. (6 points) Use the Euclidean algorithm to compute gcd(1012, 299). Show your work.

   Solution: $1012 - 3 \times 299 = 1012 - 897 = 115$

   $299 - 2 \times 115 = 299 - 230 = 69$

   $115 - 69 = 46$

   $69 - 46 = 23$

   $46 - 2 \times 23 = 0$

   So $\gcd(1012, 299) = 23$

3. (4 points) Check the (single) box that best characterizes each item.

$k \equiv -k \pmod{k}$	true for all k [√]	true for some k [ ]	false for all k [ ]
$\gcd(0,0)$	0 [ ]	k [ ]	undefined [√]

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1. (5 points) Is the following claim true? Informally explain why it is false, or give a concrete example showing that it is true.

   There is an integer n such that $n \equiv 5 \pmod 6$ and $n \equiv 2 \pmod{10}$?

   Solution: There is no such n. If $n \equiv 5 \pmod 6$ and $n \equiv 2 \pmod{10}$, then $n = 5 + 6k$ and $n = 2 + 10j$, where k and j are integers. So $5 + 6k = 2 + 10j$. This implies that $3 = 10j - 6k$ which is impossible because the right side is divisible by 2 and the left side isn't.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1568, 546)$. Show your work.

   Solution:

   $1568 - 546 \times 2 = 1568 - 1092 = 476$

   $546 - 476 = 70$

   $476 - 70 \times 6 = 476 - 420 = 56$

   $70 - 56 = 14$

   $56 - 14 \times 3 = 0$

   So the GCD is 14.

3. (4 points) Check the (single) box that best characterizes each item.

   For any integers p and q, if $p \mid q$ then $p \leq q$. true ☐ false ☑ ($\surd$)

   Two positive integers p and q are relatively prime if and only if $\gcd(p, q) > 1$. true ☐ false ☑ ($\surd$)

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For any positive integers p and q, $p \equiv q \pmod 1$.

   Solution: This is true. $p \equiv q \pmod 1$ is equivalent to $p - q = n \times 1 = n$ for some integer n. But we can always find an integer that will make this equation balance!

2. (6 points) Use the Euclidean algorithm to compute $\gcd(7839, 1474)$. Show your work.

   Solution:

   $7839 - 5 \times 1474 = 7839 - 7370 = 469$

   $1474 - 3 \times 469 = 1474 - 1407 = 67$

   $469 - 7 \times 67 = 0$

   So the GCD is 67.

3. (4 points) Check the (single) box that best characterizes each item.

   Two positive integers p and q are relatively prime if and only if $\gcd(p, q) = 1$. true [√] false [ ]

   $\gcd(p, q) = \dfrac{pq}{\mathrm{lcm}(p, q)}$ where p and q are positive integers

   true for all p, q [√] true for some p, q [ ]

   true for p, q prime [ ]

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1. (5 points) Let a and b be integers, $b > 0$. We used two formulas to define the quotient q and the remainder r of a divided by b. One of these is $a = bq + r$. What is the other?

   Solution: $0 \leq r < b$

2. (6 points) Use the Euclidean algorithm to compute gcd(221, 1224). Show your work.

   Solution:

   $1224 - 5 \times 221 = 1224 - 1105 = 119$

   $221 - 119 = 102$

   $119 - 102 = 17$

   $102 - 17 \times 6 = 0$

   So the GCD is 17.

3. (4 points) Check the (single) box that best characterizes each item.

   For any positive integers p and q, if $\text{lcm}(p, q) = pq$, then p and q are relatively prime. true [√] false [ ]

   $25 \equiv 4 \pmod 7$ true [√] false [ ]