CS 173, Fall 2015
Examlet 12, Part B

NETID:

FIRST: | **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

(9 points) Suppose that R is a relation on $\mathbb{Z}$ which is reflexive and symmetric, but not transitive. Let's define $T(n) = \{a \in \mathbb{Z} \mid aRn\}$. Notice that $n \in T(n)$ for any integer n. The collection of all sets $T(n)$ does not form a partition of $\mathbb{Z}$. Explain (informally but clearly) why the fact that R is not transitive can cause one of the partition properties to fail.

Solution: For full credit, it's enough to give a well-explained specific example (e.g. using the relation $|a - b| \leq 10$) showing how partial overlap can occur.

Here's a more general (but still informal) argument. Since R is not transitive, there's three integers a, b, and c such that aRb and bRc but not aRc. Since aRb, $T(a)$ must contain a and b. Since bRc, $T(c)$ must contain c and b. So $T(a)$ and $T(c)$ overlap. But they can't be the same set because it's not true that aRc. So there is partial overlap.

Making the above argument full formal would require using the reflexive and symmetric properties of R.

(6 points) Check the (single) box that best characterizes each item.

How many ways can I choose 5 bagels from among 10 varieties, if I can have any number of bagels from any type?

$\frac{10!}{5!5!}$ ☐ $\frac{14!}{10!4!}$ ☑ $\frac{14!}{9!5!}$ ☐

$\frac{15!}{10!5!}$ ☐ 10^5 ☐ 5^{10} ☐

If $n \geq k \geq 0$, then $\binom{n}{k} = \binom{n}{n-k}$

True ☑ True for some n and k ☐ False ☐

If $f : \mathbb{N} \to \mathbb{P}(\mathbb{Q})$ then $f(3)$ is

a rational ☐ a power set of rationals ☐

a set of rationals ☑ undefined ☐

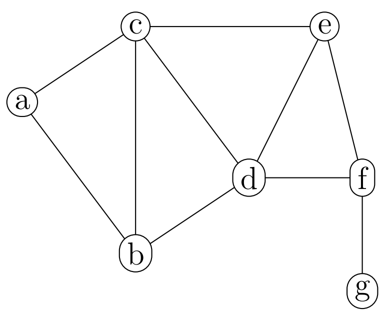
CS 173, Fall 2015
Examlet 12, Part B

NETID:

FIRST: **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

Graph G with set of nodes V is shown below. Recall that $deg(n)$ is the degree of node n. Let's define $f : \mathbb{N} \to \mathbb{P}(V)$ by $f(k) = \{n \in V : \deg(n) = k\}$. Also let $T = \{f(k) \mid k \in \mathbb{N}\}$.



(6 points) Fill in the following values:

$f(4) =$
Solution: $\{c, d\}$
$f(1) =$
Solution: $\{g\}$
$|T| =$
Solution: 5. (The distinct members are f(0), f(1), f(2), f(3), and f(4).)

(7 points) Is T a partition of V? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: No, it is not a partition. There is no partial overlap between the sets in T and they cover all nodes in V. However, T contains the empty set (e.g. as the value of $f(17)$).

(2 points) State the definition of $\binom{n}{k}$, i.e. express $\binom{n}{k}$ in terms of more basic arithmetic operations.

Solution: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

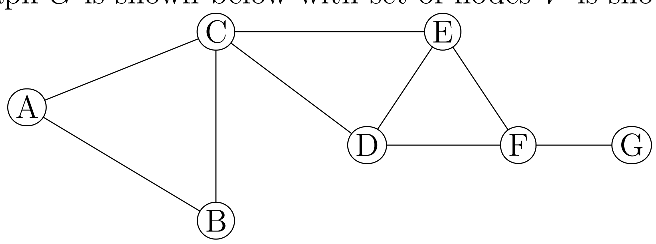
CS 173, Fall 2015
Examlet 12, Part B

NETID:

FIRST:	**LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

Graph G is shown below with set of nodes V is shown below.



Suppose that deg(n) is the degree of node n. Now let's define a function $f : V \to \mathbb{P}(V)$ by $f(p) = \{n \in V \ : \ \deg(n) = \deg(p)\}$. Then let $P = \{f(p) \mid p \in V\}$.

(6 points) Fill in the following values:

f(A) =

Solution: $\{A, B\}$

f(C) =

Solution: $\{C\}$

P =

Solution: $\{\{A, B\}, \{C\}, \{D, E, F\}, \{G\}, \}$

(7 points) Is P a partition of V? For each of the conditions required to be a partition, briefly explain why P does or doesn't satisfy that condition.

Solution: Yes, it is a partition of V. Every element of V is in exactly one subset in P. And P does not contain the empty set.

(2 points) Check the (single) box that best characterizes each item.

$|\mathbb{P}(\mathbb{P}(\emptyset))|$ 0 ☐ 1 ☐ 2 ☑ 3 ☐ 4 ☐ undefined ☐

CS 173, Fall 2015
Examlet 12, Part B

NETID:

FIRST: | **LAST:**

Discussion: **Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

(9 points) Suppose that A is a set and P is a collection of subsets of A. Using precise language and/or notation, state the conditions P must satisfy to be a partition of A.

Solution: P cannot contain the empty set. Every element of A must belong to exactly one element of P.

The second condition is frequently split into two sepaate conditions. That is, every element of A must belong some to element of P, and two distinct elements of P cannot overlap.

(6 points) Check the (single) box that best characterizes each item.

V is the vertex set of a tree with n edges. $|\mathbb{P}(V)| =$

- 2^{n-1} ☐
- 2^n ☐
- not determined ☐
- 2^{n+1} ☑ (√)
- n ☐

$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$

- true for all sets ☑ (√)
- true for some sets ☐
- false for all sets ☐

Set B is a partition of a finite set A. Then $|B|$

- $\leq 2^{|A|}$ ☐
- $\leq |A|$ ☑ (√)
- $= 2^{|A|}$ ☐
- $\leq |A+1|$ ☐

CS 173, Fall 2015
Examlet 12, Part B

NETID:

FIRST: **LAST:**

Discussion: **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

Let $f : \mathbb{R}^2 \to \mathbb{P}(\mathbb{R}^2)$ be defined by $f(x, y) = \{(p, q) \in \mathbb{R}^2 \mid \exists \alpha \in \mathbb{R}, (p, q) = \alpha(x, y)\}$.
Let $T = \{f(x, y) \mid (x, y) \in \mathbb{R}^2\}$.

(6 points) Answer the following questions:

$f(0, 0) =$

Solution: $\{(0, 0)\}$

Describe (at a high level) the elements of $f(0, 36)$:

Solution: $f(0, 36)$ is the line passing through the origin and $(0, 36)$.

Give an element of $\mathbb{P}(\mathbb{R}^2) - T$:

Solution: Many possible answers here. For example, $\emptyset$, or any finite set or any circle.

(7 points) Is T a partition of $\mathbb{R}^2$? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: This is not a partition of $\mathbb{R}^2$. It doesn't contain the empty set (good). And the elements of T do cover all of the plane (good). However, all the lines contain the origin, so there is partial overlap (bad).

(2 points) Check the (single) box that best characterizes each item.

[Buggy question: answer depends on whether you assumed A could be empty.] Let A be a set, $\{A\}$ is a partition of A.	always ☑ $\surd$	sometimes ☑ $\surd$	never ☐

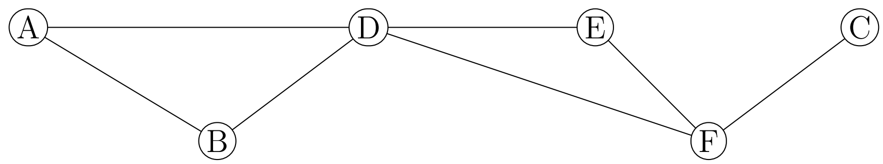
CS 173, Fall 2015
Examlet 12, Part B

NETID:

FIRST: **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

Graph G is at right.
V is the set of nodes in G.
$M = \{0, 1, 2, 3\}$



Define $f : M \to \mathbb{P}(V)$ by $f(n) = \{p \in V \ : \ d(p, B) < n\}$, where $d(a, b)$ is the (shortest-path) distance between a and b. Let $P = \{f(n) \mid n \in M\}$.

(6 points) Fill in the following values:

$f(0) =$

Solution: $\emptyset$

$f(1) =$

Solution: $\{B\}$

$P =$

Solution: $\{\emptyset, \{B\}, \{A, B, D\}, \{A, B, D, E, F\}\}$

(7 points) Is P a partition of V? For each of the conditions required to be a partition, briefly explain why P does or doesn't satisfy that condition.

Solution: No, P is not a partition of V. The subsets do not cover all of V, because the node C is missing. There is partial overlap among the subsets. And P contains the empty set.

(2 points) Check the (single) box that best characterizes each item.

$\binom{0}{0}$ -1 ☐ 0 ☐ 1 ☑ 2 ☐ undefined ☐