CS 173, Fall 2015
Examlet 12, Part B

NETID:

FIRST: **LAST:**

Discussion: **Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

(9 points) Suppose that R is a relation on $\mathbb{Z}$ which is reflexive and symmetric, but not transitive. Let's define $T(n) = \{a \in \mathbb{Z} \mid aRn\}$. Notice that $n \in T(n)$ for any integer n. The collection of all sets $T(n)$ does not form a partition of $\mathbb{Z}$. Explain (informally but clearly) why the fact that R is not transitive can cause one of the partition properties to fail.

(6 points) Check the (single) box that best characterizes each item.

How many ways can I choose 5 bagels from among 10 varieties, if I can have any number of bagels from any type?

$\frac{10!}{5!5!}$ ☐ $\frac{14!}{10!4!}$ ☐ $\frac{14!}{9!5!}$ ☐

$\frac{15!}{10!5!}$ ☐ 10^5 ☐ 5^{10} ☐

If $n \geq k \geq 0$, then $\binom{n}{k} = \binom{n}{n-k}$

True ☐ True for some n and k ☐ False ☐

If $f : \mathbb{N} \to \mathbb{P}(\mathbb{Q})$ then $f(3)$ is

a rational ☐ a power set of rationals ☐

a set of rationals ☐ undefined ☐

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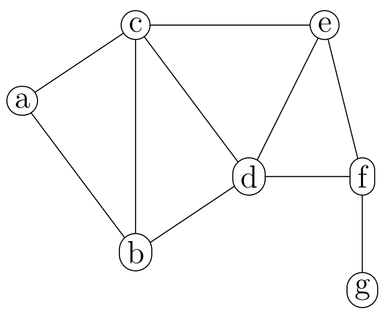
Graph G with set of nodes V is shown below. Recall that $deg(n)$ is the degree of node n. Let's define $f : \mathbb{N} \rightarrow \mathbb{P}(V)$ by $f(k) = \{n \in V : \deg(n) = k\}$. Also let $T = \{f(k) \mid k \in \mathbb{N}\}$.



(6 points) Fill in the following values:

$f(4) =$

$f(1) =$

$|T| =$

(7 points) Is T a partition of V? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

(2 points) State the definition of $\binom{n}{k}$, i.e. express $\binom{n}{k}$ in terms of more basic arithmetic operations.

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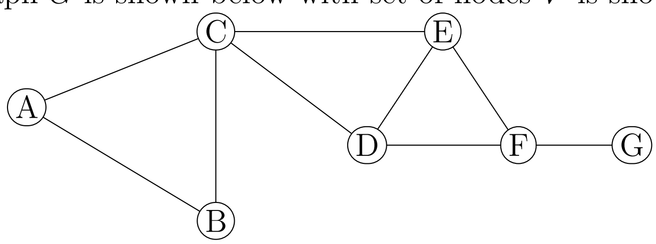
Graph G is shown below with set of nodes V is shown below.



Suppose that deg(n) is the degree of node n. Now let's define a function $f : V \to \mathbb{P}(V)$ by $f(p) = \{n \in V \ : \ \deg(n) = \deg(p)\}$. Then let $P = \{f(p) \mid p \in V\}$.

(6 points) Fill in the following values:

f(A) =

f(C) =

P =

(7 points) Is P a partition of V? For each of the conditions required to be a partition, briefly explain why P does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

$|\mathbb{P}(\mathbb{P}(\emptyset))|$ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ undefined ☐

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(9 points) Suppose that A is a set and P is a collection of subsets of A. Using precise language and/or notation, state the conditions P must satisfy to be a partition of A.

(6 points) Check the (single) box that best characterizes each item.

V is the vertex set of a tree with n edges. $|\mathbb{P}(V)| =$

2^{n-1} ☐ 2^n ☐ not determined ☐

2^{n+1} ☐ n ☐

$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$

true for all sets ☐ true for some sets ☐

false for all sets ☐

Set B is a partition of a finite set A. Then $|B|$

$\leq 2^{|A|}$ ☐ $\leq |A|$ ☐

$= 2^{|A|}$ ☐ $\leq |A+1|$ ☐

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Let $f : \mathbb{R}^2 \to \mathbb{P}(\mathbb{R}^2)$ be defined by $f(x, y) = \{(p, q) \in \mathbb{R}^2 \mid \exists \alpha \in \mathbb{R}, (p, q) = \alpha(x, y)\}$.
Let $T = \{f(x, y) \mid (x, y) \in \mathbb{R}^2\}$.

(6 points) Answer the following questions:

$f(0, 0) =$

Describe (at a high level) the elements of $f(0, 36)$:

Give an element of $\mathbb{P}(\mathbb{R}^2) - T$:

(7 points) Is T a partition of $\mathbb{R}^2$? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

[Buggy question: answer depends on whether you assumed A could be empty.] Let A be a set, $\{A\}$ is a partition of A.	always ☐	sometimes ☐	never ☐

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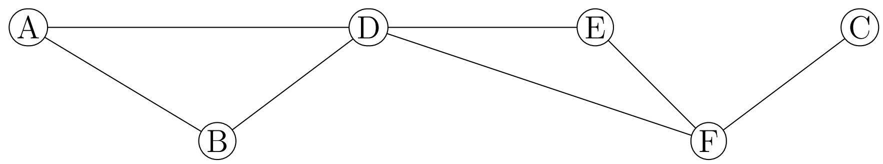
Graph G is at right.
V is the set of nodes in G.
$M = \{0, 1, 2, 3\}$



Define $f : M \rightarrow \mathbb{P}(V)$ by $f(n) = \{p \in V \ : \ d(p, B) < n\}$, where $d(a, b)$ is the (shortest-path) distance between a and b. Let $P = \{f(n) \mid n \in M\}$.

(6 points) Fill in the following values:

$f(0) =$

$f(1) =$

$P =$

(7 points) Is P a partition of V? For each of the conditions required to be a partition, briefly explain why P does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

$\binom{0}{0}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ undefined ☐