CS 173, Fall 2015
Examlet 10, Part B

NETID:

FIRST:	**LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

   $T(4) = 7 \qquad T(n) = 5T\left(\frac{n}{2}\right) + n$

   (a) The height: $\log_2(n) - 2$

   (b) The number of leaves (please simplify): $5^{\log_2(n)-2} = \frac{1}{25}5^{log_2(n)} = \frac{1}{25}5^{log_5 n \log_2 5} = \frac{1}{25}n^{\log_2 5}$

   (c) Value in each node at level k: $\frac{n}{2^k}$

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $42n!$ $\quad 7^n$ $\quad 100\log n$ $\quad n\log(n^7)$ $\quad 2^{3n}$ $\quad \log(2^n)$ $\quad (n^3)^7$

$100\log n$	$\log(2^n)$	$n\log(n^7)$	$(n^3)^7$	7^n	2^{3n}	$42n!$

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Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

1. (9 points) Fill in key facts about the recursion tree for T, assuming that T is even.

   $T(0) = 5 \qquad T(n) = 3T(n-2) + n^2$

   (a) The height: $\frac{n}{2}$

   (b) The number of leaves (please simplify): $3^{\frac{n}{2}} = (\sqrt{3})^n$

   (c) Value in each node at level k: $(n-2k)^2$

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $(3^n)^2 \qquad 10 \qquad 0.001n^3 \qquad 30\log n \qquad n\log(n^7) \qquad 8n! + 18 \qquad 3n^2$

10	$30\log n$	$n\log(n^7)$	$3n^2$	$0.001n^3$	$(3^n)^2$	$8n! + 18$

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Discussion: **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

1. (7 points) Prof. Flitwick claims that for any functions f and g from the reals to the reals, if $f(x) \ll g(x)$ then $\log(f(x)) \ll \log(g(x))$. Is this true? Briefly justify your answer.

   Solution: This is not true. Consider $f(x) = x$ and $g(x) = x^2$. Then $\log(g(x)) = 2\log(f(x))$ So it can't be the case that $\log(f(x)) \ll \log(g(x))$.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n/2) + c$

$\Theta(\log n)$ ☑ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐

$T(1) = d$
$T(n) = T(n-1) + n$

$\Theta(n)$ ☐ $\Theta(n^2)$ ☑ $\Theta(n \log n)$ ☐ $\Theta(2^n)$ ☐

$n!$

$O(2^n)$ ☐ $\Theta(2^n)$ ☐ neither of these ☑

3^n is

$\Theta(5^n)$ ☐ $O(5^n)$ ☑ neither of these ☐

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 7.

$T(1) = 5 \qquad T(n) = 3T\left(\frac{n}{7}\right) + n^2$

(a) The height: $\log_7 n$.

(b) The number of leaves (please simplify): $3^{\log_7 n} = 3^{\log_3 n \log_7 3} = n^{\log_7 3}$

(c) Value in each node at level k: $(\frac{n}{7^k})^2$

Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

$2^n + 3^n \qquad n^3 \qquad 100 \log n \qquad 3^{31} \qquad 3n\log(n^3) \qquad 7n! + 2 \qquad 173n - 173$

3^{31}	$100 \log n$	$173n - 173$	$3n\log(n^3)$	n^3	$2^n + 3^n$	$7n! + 2$

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Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is odd.

   $T(1) = 7 \qquad T(n) = nT(n-2) + n$

   (a) The height: $\frac{n-1}{2}$

   (b) The number of leaves: $n(n-2)(n-4)\ldots 5 \cdot 3 \cdot 1$

   (c) Value in each node at level k: $n - 2k$

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $3n^2 \qquad \frac{n \log n}{7} \qquad (10^{10^{10}})n \qquad 0.001n^3 \qquad 30\log(n^{17}) \qquad 8n! + 18 \qquad 3^n + 11^n$

$30\log(n^{17})$	$(10^{10^{10}})n$	$\frac{n \log n}{7}$	$3n^2$	$0.001n^3$	$3^n + 11^n$	$8n! + 18$

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1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that f is $O(g)$ and g is $O(h)$. Must f be $O(h)$? Briefly justify your answer.

   Solution: This is true. Since f is $O(g)$ and g is $O(h)$, there are positive reals c, k, C and K such that

   $0 \leq f(x) \leq cg(x)$ and $0 \leq g(y) \leq Ch(y)$

   for every $x \geq k$ and $y \geq K$.

   But then if we let $p = cC$, we have

   $O \leq f(x) \leq ph(x)$

   for every $x \geq \max(k, K)$.

2. (8 points) Check the (single) box that best characterizes each item.

   Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

   $k < m$ [√] $k = m$ [ ] $k > m$ [ ] $km = 1$ [ ]

   $T(1) = c$
   $T(n) = 3T(n/3) + n$

   $\Theta(n)$ [ ] $\Theta(n^2)$ [ ] $\Theta(n \log n)$ [√] $\Theta(2^n)$ [ ]

   Suppose $f(n) \ll g(n)$.
   Is $g(n) \ll f(n)$?

   no [√] perhaps [ ] yes [ ]

   $n^{log_2 3}$ grows

   faster than n^2 [ ] slower than n^2 [√] at the same rate as n^2 [ ]