CS 173, Fall 2015
Examlet 10, Part B

NETID:

FIRST:	**LAST:**

Discussion: **Thursday 2 3 4 5 Friday 9 10 11 12 1 2**

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

   $T(4) = 7 \qquad T(n) = 5T\left(\frac{n}{2}\right) + n$

   (a) The height:

   (b) The number of leaves (please simplify):

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $42n!$ $\quad 7^n$ $\quad 100\log n$ $\quad n\log(n^7)$ $\quad 2^{3n}$ $\quad \log(2^n)$ $\quad (n^3)^7$

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Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

1. (9 points) Fill in key facts about the recursion tree for T, assuming that T is even.

$$T(0) = 5 \qquad T(n) = 3T(n-2) + n^2$$

(a) The height:

(b) The number of leaves (please simplify):

(c) Value in each node at level k:

Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

$(3^n)^2$ 10 $0.001n^3$ $30 \log n$ $n \log(n^7)$ $8n! + 18$ $3n^2$

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Discussion: **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

1. (7 points) Prof. Flitwick claims that for any functions f and g from the reals to the reals, if $f(x) \ll g(x)$ then $\log(f(x)) \ll \log(g(x))$. Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n/2) + c$

$\Theta(\log n)$ ☐ $\Theta(n)$ ☐
$\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐

$T(1) = d$
$T(n) = T(n-1) + n$

$\Theta(n)$ ☐ $\Theta(n^2)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(2^n)$ ☐

$n!$

$O(2^n)$ ☐ $\Theta(2^n)$ ☐ neither of these ☐

3^n is

$\Theta(5^n)$ ☐ $O(5^n)$ ☐ neither of these ☐

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 7.

   $T(1) = 5$ $\qquad$ $T(n) = 3T\left(\frac{n}{7}\right) + n^2$

   (a) The height:

   (b) The number of leaves (please simplify):

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $2^n + 3^n$ $\qquad$ n^3 $\qquad$ $100 \log n$ $\qquad$ 3^{31} $\qquad$ $3n \log(n^3)$ $\qquad$ $7n! + 2$ $\qquad$ $173n - 173$

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Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is odd.

   $T(1) = 7 \qquad T(n) = nT(n-2) + n$

   (a) The height:

   (b) The number of leaves:

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $3n^2 \qquad \frac{n \log n}{7} \qquad (10^{10^{10}})n \qquad 0.001n^3 \qquad 30\log(n^{17}) \qquad 8n! + 18 \qquad 3^n + 11^n$

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1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that f is $O(g)$ and g is $O(h)$. Must f be $O(h)$? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

$k < m$ ☐ $k = m$ ☐

$k > m$ ☐ $km = 1$ ☐

$T(1) = c$
$T(n) = 3T(n/3) + n$

$\Theta(n)$ ☐ $\Theta(n^2)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(2^n)$ ☐

Suppose $f(n) \ll g(n)$.
Is $g(n) \ll f(n)$?

no ☐ perhaps ☐ yes ☐

$n^{log_2 3}$ grows

faster than n^2 ☐ slower than n^2 ☐

at the same rate as n^2 ☐