CS 173, Fall 2014
Examlet 8, Part B

NETID:

FIRST: | **LAST:**

Discussion: **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

(10 points) Suppose we have a function f defined by

$$\begin{aligned} f(1) &= 5 \\ f(n) &= 3f(n-1) + n^2 \text{ for } n \geq 2 \end{aligned}$$

Express $f(n)$ in terms of $f(n-3)$ (where $n \geq 4$). Show your work and simplify your answer.

Solution:

$$\begin{aligned} f(n) &= 3f(n-1) + n^2 \\ &= 3(3f(n-2) + (n-1)^2) + n^2 \\ &= 3(3(3f(n-3) + (n-2)^2) + (n-1)^2) + n^2 \\ &= 27f(n-3) + 9(n-2)^2 + 3(n-1)^2 + n^2 \end{aligned}$$

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(10 points) Suppose we have a function g defined (for n a power of 3) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 3g(n/3) + n \text{ for } n \geq 3 \end{aligned}$$

Express $g(n)$ in terms of $g(n/3^3)$ (where $n \geq 27$). Show your work and simplify your answer.

Solution:

$$\begin{aligned} g(n) &= 3g(n/3) + n \\ &= 3(3g(n/9) + n/3) + n \\ &= 3(3(3g(n/27) + n/9) + n/3) + n \\ &= 27g(n/27) + n + n + n \\ &= 27g(n/27) + 3n \end{aligned}$$

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(10 points) Suppose we have a function T defined (for n a power of 2) by

$$\begin{aligned} T(4) &= c \\ T(n) &= T(n/2) + n \text{ for } n \geq 8 \end{aligned}$$

Your partner has already figured out that

$$T(n) = T(n/2^k) + \sum_{i=0}^{k-1} n\frac{1}{2^i}$$

Finish finding the closed form for T. Show your work and simplify your answer.

Solution:

To find the value of k at the base case, we need to set $n/2^k = 4$. This means that $n = 4 \cdot 2^k$. So $n = 2^{k+2}$. So $k + 2 = \log n$. So $k = \log n - 2$. Substituting this value into the above equation, we get

$$\begin{aligned} T(n) &= T(n/2^k) + \sum_{i=0}^{k-1} n\frac{1}{2^i} = T(4) + \sum_{i=0}^{\log n - 3} n\frac{1}{2^i} \\ &= c + n\sum_{i=0}^{\log n - 3} \frac{1}{2^i} = c + n(2 - \frac{1}{2^{\log n - 3}}) \\ &= c + n(2 - \frac{1}{2^{\log n} \cdot 2^{-3}}) = c + n(2 - \frac{8}{2^{\log n}}) \\ &= c + n(2 - \frac{8}{n}) = c + 2n - 8 \end{aligned}$$