CS 173, Fall 2014
Examlet 7, Part A

NETID:

FIRST: | **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

Use (strong) induction to prove the following claim:

Claim: $\sum_{p=0}^{n}(p \cdot p!) = (n+1)! - 1$, for all natural numbers n.

Recall that 0! is defined to be 1.

Proof by induction on n.

Base case(s): $n = 0$. Then $\sum_{p=0}^{n}(p \cdot p!) = 0 \cdot 0! = 0 \cdot 1 = 0$ and also $(n+1)! - 1 = 0! - 1 = 1 - 1 = 0$. So the claim holds at $n = 0$.

Inductive hypothesis [Be specific, don't just refer to "the claim"]:

$\sum_{p=0}^{n}(p \cdot p!) = (n+1)! - 1$, for $n = 0, 1, \ldots, k$ for some integer $k \geq 0$.

Rest of the inductive step: By removing the top term of the summation and then applying the inductive hypothesis, we get:

$$\sum_{p=0}^{k+1}(p \cdot p!) = (k+1) \cdot (k+1)! + \sum_{p=0}^{k}(p \cdot p!) = (k+1) \cdot (k+1)! + ((k+1)! - 1)$$

Simplifying the algebra:

$$(k+1) \cdot (k+1)! + ((k+1)! - 1) = [(k+1) + 1] \cdot (k+1)! - 1 = (k+2) \cdot (k+1)! - 1$$

So $\sum_{p=0}^{k+1}(p \cdot p!) = (k+2)! - 1$, which is what we needed to show.

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Use (strong) induction to prove the following claim:

Claim: $3^{2n+1} + 1$ is divisible by 4, for all natural numbers n

Proof by induction on n.

Base case(s): $n = 0$. At $n = 0$, $3^{2n+1} + 1 = 3^1 + 1 = 4$ which is clearly divisible by 4.

Inductive hypothesis [Be specific, don't just refer to "the claim"]:

$3^{2n+1} + 1$ is divisible by 4 for $n = 0, 1, \ldots, k$ for some integer $k \geq 0$.

Rest of the inductive step: In particular, by the inductive hypothesis, we know that $3^{2k+1} + 1$ is divisible by 4. So $3^{2k+1} + 1 = 4p$ for some integer p. So $3^{2k+1} = 4p - 1$.

Consider $3^{2(k+1)+1} + 1$. $3^{2(k+1)+1} + 1 = 3^{2k+3} + 1 = 9 \cdot 3^{2k+1} + 1$

Substituting in $3^{2k+1} = 4p - 1$, we get $9 \cdot 3^{2k+1} + 1 = 9 \cdot (4p - 1) + 1 = 36p - 9 + 1 = 36p - 8 = 4(9p - 2)$.

So $3^{2(k+1)+1} + 1 = 4(9p - 2)$. $9p - 2$ is an integer since p is an integer. So this means that $3^{2(k+1)+1} + 1$ is divisible by 4, which is what we needed to show.

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Use (strong) induction to prove the following claim:

Claim: $\sum_{j=1}^{n} j(j+1) = \frac{n(n+1)(n+2)}{3}$, for all positive integers n.

Proof by induction on n.

Base case(s): $n = 1$. At $n = 1$, $\sum_{j=1}^{n} j(j+1) = 1(1+1) = 2$ Also, $\frac{n(n+1)(n+2)}{3} = \frac{1 \cdot 2 \cdot 3}{3} = 2$. So the two sides of the equation are equal at $n = 1$.

Inductive hypothesis [Be specific, don't just refer to "the claim"]:

Suppose that $\sum_{j=1}^{n} j(j+1) = \frac{n(n+1)(n+2)}{3}$, for $n = 1, \ldots, k$, for some integer $k \geq 1$.

Rest of the inductive step:

Consider $\sum_{j=1}^{k+1} j(j+1)$. By removing the top term of the summation and applying the inductive hypothesis, we get

$$\sum_{j=1}^{k+1} j(j+1) = (k+1)(k+2) + \sum_{j=1}^{k} j(j+1) = (k+1)(k+2) + \frac{k(k+1)(k+2)}{3}$$

Simplifying the algebra:

$$(k+1)(k+2) + \frac{k(k+1)(k+2)}{3} = \frac{3(k+1)(k+2)}{3} + \frac{k(k+1)(k+2)}{3} = \frac{3(k+1)(k+2)\ +\ k(k+1)(k+2)}{3} = \frac{(k+1)(k+2)(k+3)}{3}$$

So $\sum_{j=1}^{k+1} j(j+1) = \frac{(k+1)(k+2)(k+3)}{3}$, which is what we needed to show.

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Use (strong) induction to prove the following claim:

Claim: $\sum_{j=1}^{n} \frac{1}{j(j+1)} = \frac{n}{n+1}$ for all positive integers n.

Proof by induction on n.

Base case(s): $n = 1$. At $n = 1$, $\sum_{j=1}^{n} \frac{1}{j(j+1)} = \frac{1}{1(1+1)} = \frac{1}{2}$. Also $\frac{n}{n+1} = \frac{1}{2}$. So the two sides of the equation are equal.

Inductive hypothesis [Be specific, don't just refer to "the claim"]: Suppose that $\sum_{j=1}^{n} \frac{1}{j(j+1)} = \frac{n}{n+1}$ for $n = 1, \ldots, k$ for some integer $k \geq 1$.

Rest of the inductive step:

Consider $\sum_{j=1}^{k+1} \frac{1}{j(j+1)}$. By removing the top term of the summation and then applying the inductive hypothesis, we get

$$\sum_{j=1}^{k+1} \frac{1}{j(j+1)} = \frac{1}{(k+1)(k+2)} + \sum_{j=1}^{k} \frac{1}{j(j+1)} = \frac{1}{(k+1)(k+2)} + \frac{k}{k+1}.$$

Adding the two fractions together:

$$\frac{1}{(k+1)(k+2)} + \frac{k}{k+1} = \frac{1}{(k+1)(k+2)} + \frac{k(k+2)}{(k+1)(k+2)} = \frac{1+k(k+2)}{(k+1)(k+2)} = \frac{k^2+2k+1}{(k+1)(k+2)} = \frac{(k+1)^2}{(k+1)(k+2)} = \frac{k+1}{k+2}$$

So $\sum_{j=1}^{k+1} \frac{1}{j(j+1)} = \frac{k+1}{k+2}$, which is what we needed to show.