CS 173, Fall 2014
Examlet 4, Part A

NETID:

FIRST: | **LAST:**

Discussion: Thursday 2 3 4 5 Friday 9 10 11 12 1 2

Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, i.e. pairs of positive integers. Consider the relation T on A defined by

$(a, b)T(p, q)$ if and only if $ab \mid p$

Working directly from the definition of divides, prove that T is transitive.

Solution: Let (a, b), (p, q), and (m, n) be elements of A. Suppose that $(a, b)T(p, q)$ and $(p, q)T(m, n)$. By the definition of T, this means that $ab \mid p$ and $pq \mid m$.

By the definition of divides, we then have $abx = p$ and $pqy = m$, for some integers x and y. Substituting the first equation into the second, we get $(abx)qy = m$. That is $(ab)(xqy) = m$. Since x, y, and q are all integers, so is xqy. So this implies that $ab \mid m$. So $(a, b)T(m, n)$, which is what we needed to show.

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Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, i.e. pairs of positive integers. Consider the relation T on A defined by

$(x, y)T(p, q)$ if and only if $(xy)(p + q) = (pq)(x + y)$

Prove that T is transitive.

Solution: Let (a, b), (p, q), and (m, n) be elements of A. Suppose that $(a, b)T(p, q)$ and $(p, q)T(m, n)$. By the definition of T, this means that $(xy)(p + q) = (pq)(x + y)$ and $(pq)(m + n) = (mn)(p + q)$

Since $m+n$ is positive, we can divide both sides by it, to get $(pq) = (mn)(p+q)/(m+n)$. Substituting this into the first equation, we get

$$(xy)(p + q) = (mn)(p + q)/(m + n) \times (x + y)$$

Multiplying both sides by $(m + n)$, we get

$$(xy)(p + q)(m + n) = (mn)(p + q)(x + y)$$

Since $(p + q)$ is positive, we can cancel it from both sides to get

$$(xy)(m + n) = (mn)(x + y)$$

By the definition of T, this means that $(a, b)T(m, n)$, which is what we needed to show.

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Recall how to multiply a real number α by a 2D point $(x, y) \in \mathbb{R}^2$: $\alpha(x, y) = (\alpha x, \alpha y)$.

Let $A = \mathbb{R}^+ \times \mathbb{R}^+$, i.e. pairs of positive real numbers.

Define a relation $\gg$ on A as follows:

$(x, y) \gg (p, q)$ if and only if there exists a real number $\alpha \geq 1$ such that $(x, y) = \alpha(p, q)$.

Prove that $\gg$ is antisymmetric.

Solution: Let (x, y) and (p, q) be elements of A. Suppose that $(x, y) \gg (p, q)$ and $(p, q) \gg (x, y)$.

By the definition of $\gg$, there are real numbers $\alpha \geq 1$ and $\beta \geq 1$ such that $(x, y) = \alpha(p, q)$ and $(p, q) = \beta(a, b)$.

Substituting the second equation into the first, we get $(x, y) = \alpha\beta(x, y)$. This means that $\alpha\beta = 1$. Since $\alpha \geq 1$ and $\beta \geq 1$, this implies that $\alpha = \beta = 1$. So therefore $(x, y) = (p, q)$, which is what we needed to show.

CS 173, Fall 2014
Examlet 3, Part A

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Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, i.e. pairs of positive integers. Consider the relation T on A defined by

$(x, y)T(p, q)$ if and only if $x \leq p$ and $xy \leq pq$

Prove that T is antisymmetric.

Solution: Let (x, y) and (p, q) be elements of A. Suppose that $(x, y)T(p, q)$ and $(p, q)T(x, y)$.

By the definition of T, $(x, y)T(p, q)$ implies that $x \leq p$ and $xy \leq pq$.

Similarly $(p, q)T(x, y)$ implies that that $p \leq x$ and $pq \leq xy$.

Since $x \leq p$ and $p \leq x$, $x = p$. Since $xy \leq pq$ and $pq \leq xy$, $xy = pq$.

Notice that x and o are positive, by the definition of A. So $x = p$ and $xy = pq$ implies that $y = q$.

We now know that $x = p$ and $y = q$. So therefore $(x, y) = (p, q)$, which is what we needed to show.