CS 173, Fall 2014
Examlet 10, Part B

NETID:

FIRST:	**LAST:**

Discussion: **Thursday** **2** **3** **4** **5** **Friday** **9** **10** **11** **12** **1** **2**

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2:

   $T(4) = 7$ $\qquad$ $T(n) = 5T\left(\frac{n}{2}\right) + n$

   (a) The height: $\log_2(n) - 2$

   (b) The number of leaves (please simplify):
   $5^{\log_2(n)-2} = \frac{1}{25}5^{log_2(n)} = \frac{1}{25}5^{log_5 n \log_2 5} = \frac{1}{25}n^{\log_2 5}$

   (c) Value in each node at level k: $\frac{n}{2^k}$

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $2^n + 3^n$ $\qquad$ n^3 $\qquad$ $100 \log n$ $\qquad$ 3^{31} $\qquad$ $3n \log(n^3)$ $\qquad$ $7n! + 2$ $\qquad$ $173n - 173$

3^{31}	$100 \log n$	$173n - 173$	$3n \log(n^3)$	n^3	$2^n + 3^n$	$7n! + 2$

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1. (7 points) Prof. Flitwick claims that for any functions f and g from the reals to the reals, if $f(x) \ll g(x)$ then $\log(f(x)) \ll \log(g(x))$. Is this true? Briefly justify your answer.

   This is not true. Consider $f(x) = x$ and $g(x) = x^2$. Then $\log(g(x)) = 2\log(f(x))$ So it can't be the case that $\log(f(x)) \ll \log(g(x))$.

2. (8 points) Check the (single) box that best characterizes each item.

Suppose $f(n)$ is $O(g(n))$. Will $g(n)$ be $O(f(n))$?	no ☐	perhaps ☑	yes ☐	
$T(1) = d$ $T(n) = T(n-1) + n$	$\Theta(n)$ ☐	$\Theta(n^2)$ ☑	$\Theta(n \log n)$ ☐	$\Theta(2^n)$ ☐
$T(1) = c$ $T(n) = 3T(n/3) + n$	$\Theta(n)$ ☐	$\Theta(n^2)$ ☐	$\Theta(n \log n)$ ☑	$\Theta(2^n)$ ☐
$n!$	$O(2^n)$ ☐	$\Theta(2^n)$ ☐	neither of these ☑	

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 7:

$$T(1) = 5 \qquad T(n) = 3T\left(\frac{n}{7}\right) + n^2$$

   (a) The height: $\log_7 n$.

   (b) The number of leaves (please simplify):
   $3^{\log_7 n} = 3^{\log_3 n \log_7 3} = n^{\log_7 3}$

   (c) Value in each node at level k:
   $(\frac{n}{7^k})^2$

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

$3n^2 \quad \frac{n \log n}{7} \quad (10^{10^{10}})n \quad 0.001n^3 \quad 30\log(n^{17}) \quad 8n! + 18 \quad 3^n + 11^n$

$30\log(n^{17})$	$(10^{10^{10}})n$	$\frac{n \log n}{7}$	$3n^2$	$0.001n^3$	$3^n + 11^n$	$8n! + 18$

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1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for f to be $O(g)$.

   There are positive reals c and k such that

   $0 \leq f(x) \leq cg(x)$

   for every $x \geq k$.

2. (8 points) Check the (single) box that best characterizes each item.

Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

$k < m$ [√] $k = m$ [ ]

$k > m$ [ ] $km = 1$ [ ]

$T(1) = d$
$T(n) = T(n/2) + c$

$\Theta(\log n)$ [√] $\Theta(n)$ [ ]

$\Theta(n \log n)$ [ ] $\Theta(n^2)$ [ ]

$n^{1.5}$ is

$\Theta(n^{1.414})$ [ ] $O(n^{1.414})$ [ ] neither of these [√]

Suppose $f(n) \ll g(n)$.
Will $f(n)$ be $O(g(n))$?

no [ ] perhaps [ ] yes [√]