Problem 1: Sets (14 points) Prove the following claim about sets. You must use the method of choosing an element from the smaller set and showing that it belongs to the larger set.

Claim: For any sets $A, B, C, D$, $(A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D)$

Problem 2: Relations Let $R$ be the following relation over $\mathbb{N}^2$:

$(a, b) \ R (c, d)$ iff $(a < c$ or $(a = c$ and $b \leq d))$

(a) Is $R$ reflexive? Prove it or disprove it. (6 points)

(b) Is $R$ antisymmetric? Prove it or disprove it. (15 points)

(c) Is $R$ transitive? Prove it or disprove it. (15 points)

(d) Let $f : \mathbb{N}^2 \rightarrow \mathbb{N}^2$ be the following function:

$$f(a, b) = \begin{cases} 
(a, b - 100) & \text{if } b \geq 100 \\
(a - 1, 10000) & \text{if } b < 100 \text{ and } a > 0 \\
(0, 0) & \text{otherwise}
\end{cases}$$

Prove that, for every $(p, q) \in \mathbb{N}^2$, $f(p, q)R(p, q)$ (14 points)