Graphs
(And Review)

Lecture 11

Register your i>Clicker!

<u>Exam</u>

Bring your ID!

More preparation material:
to be posted

Graph Isomorphism

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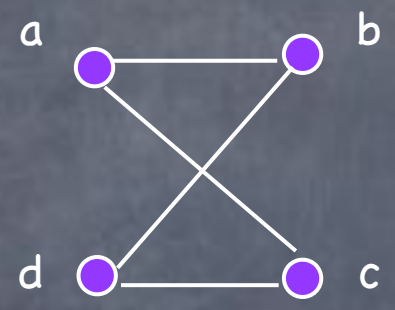

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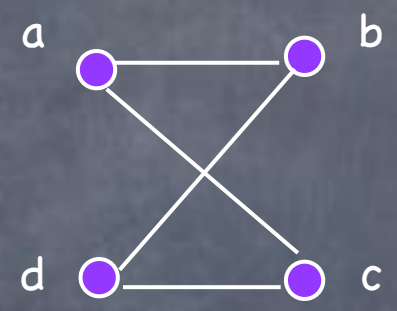

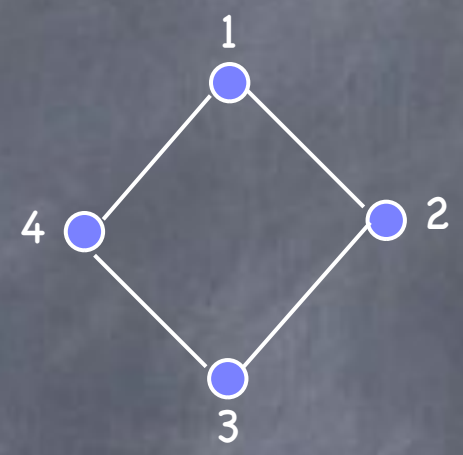

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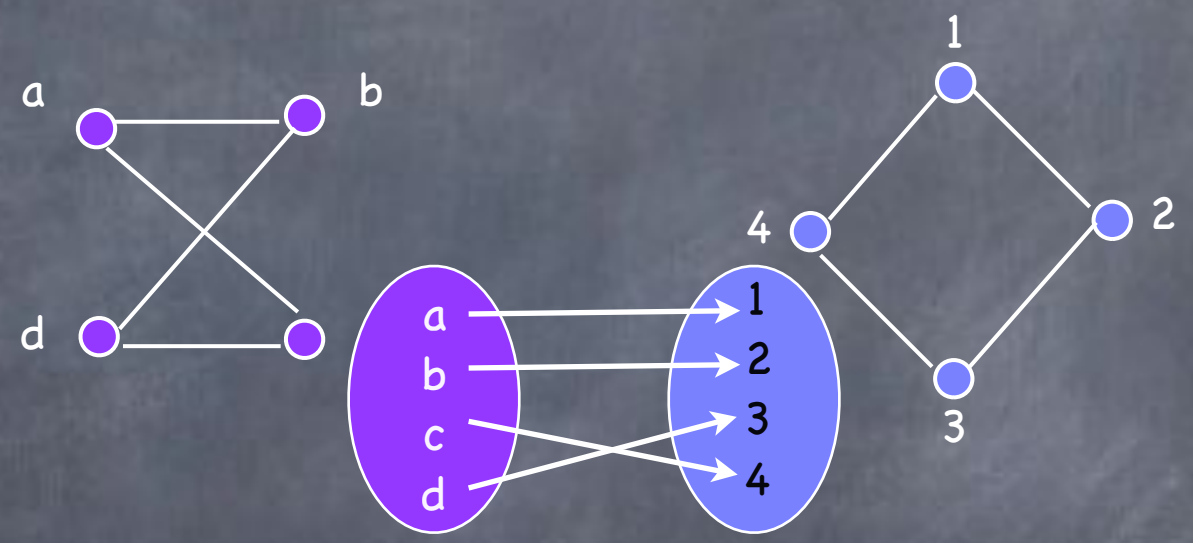

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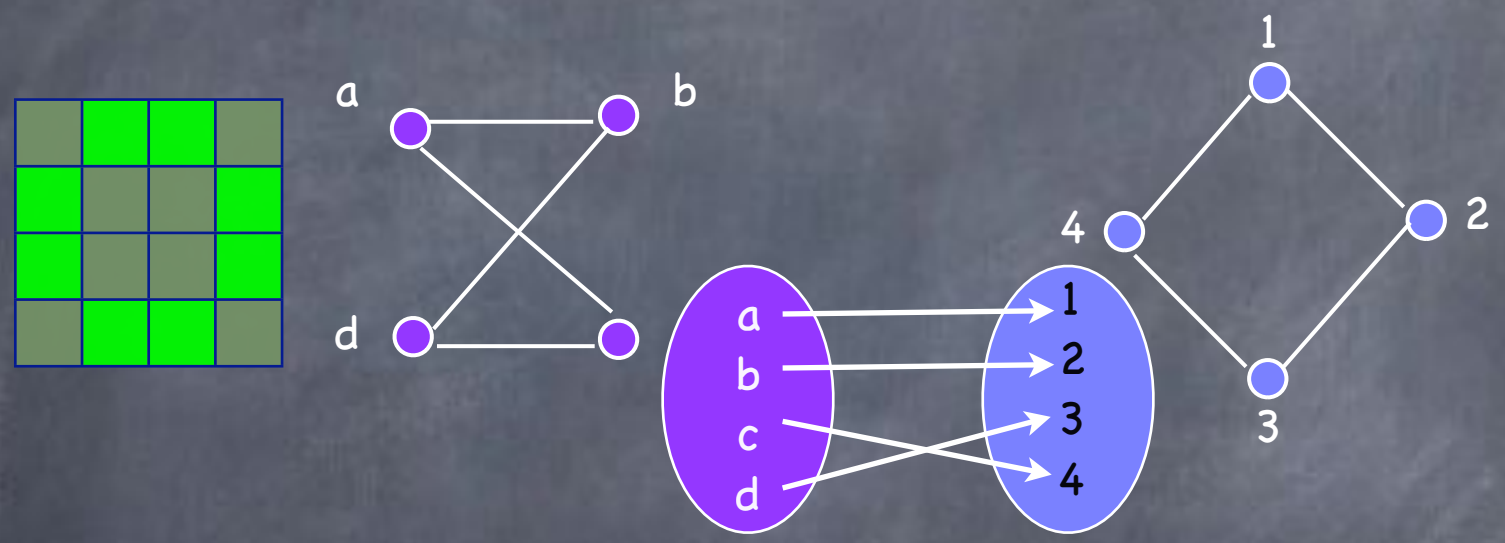

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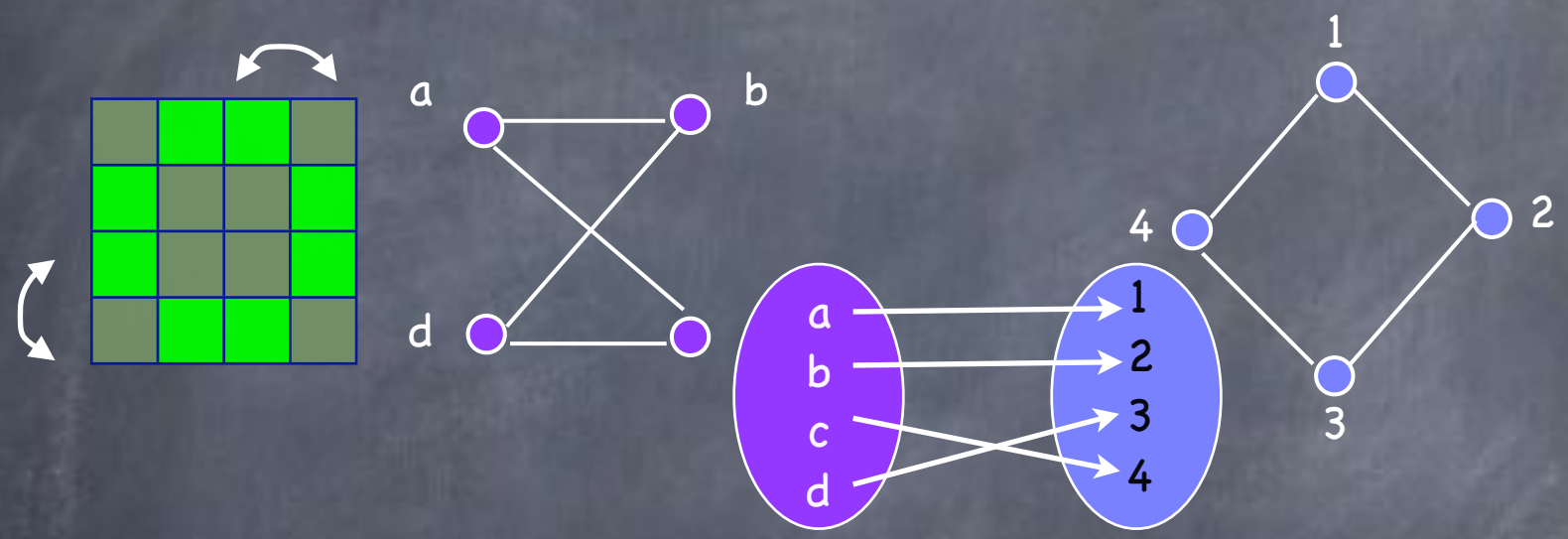

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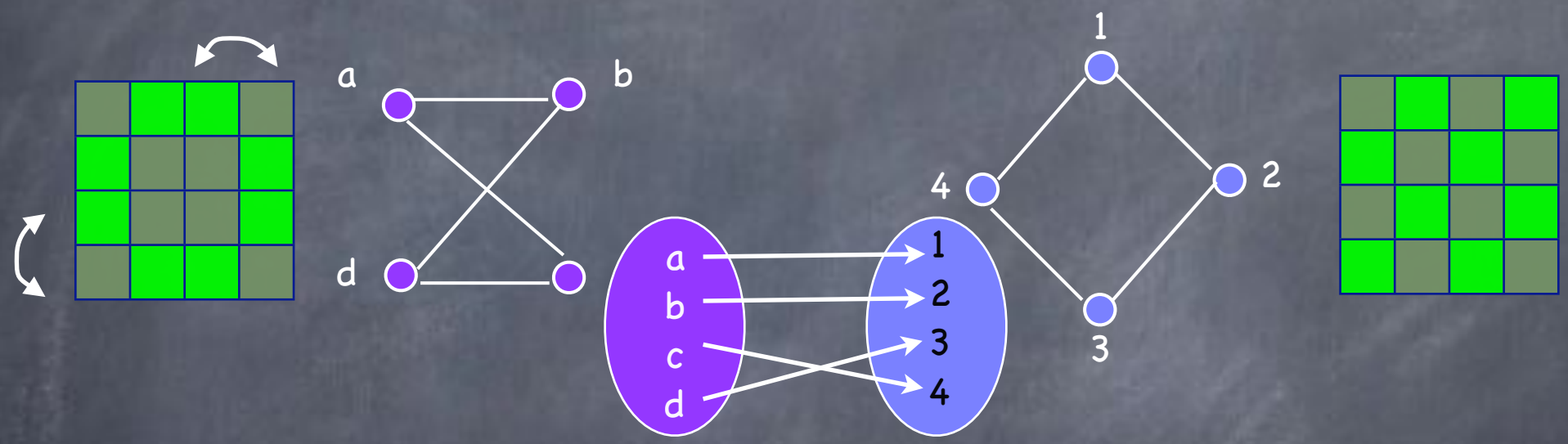

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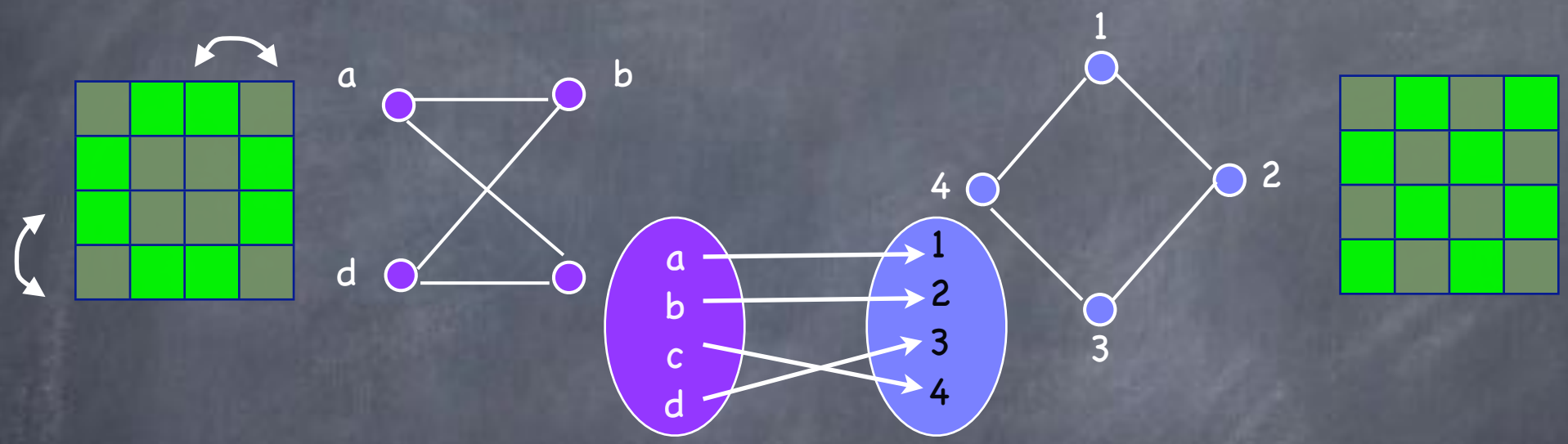

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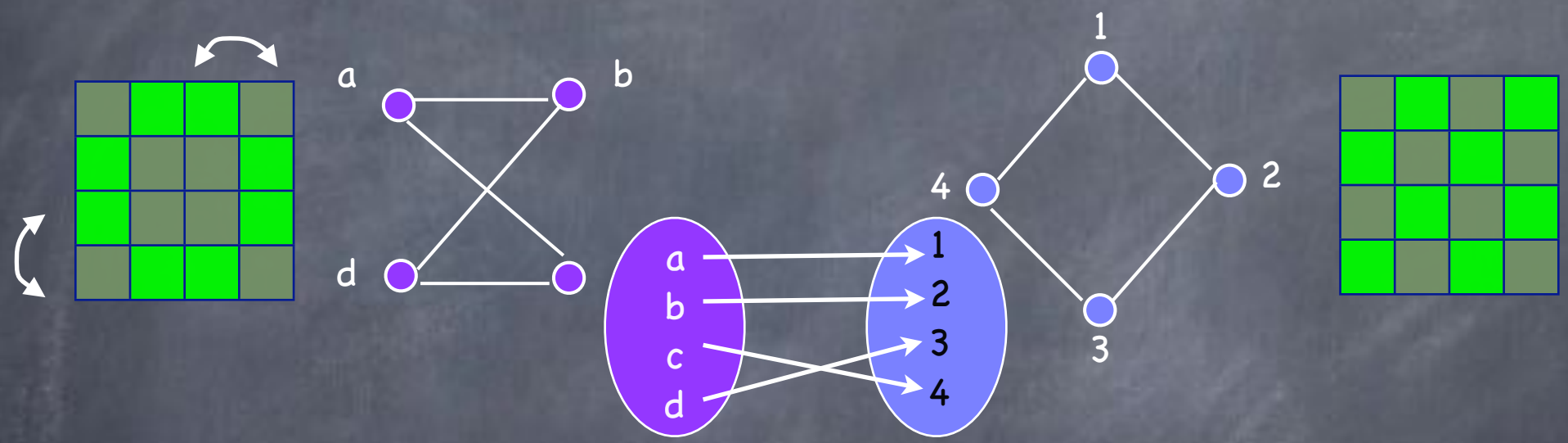

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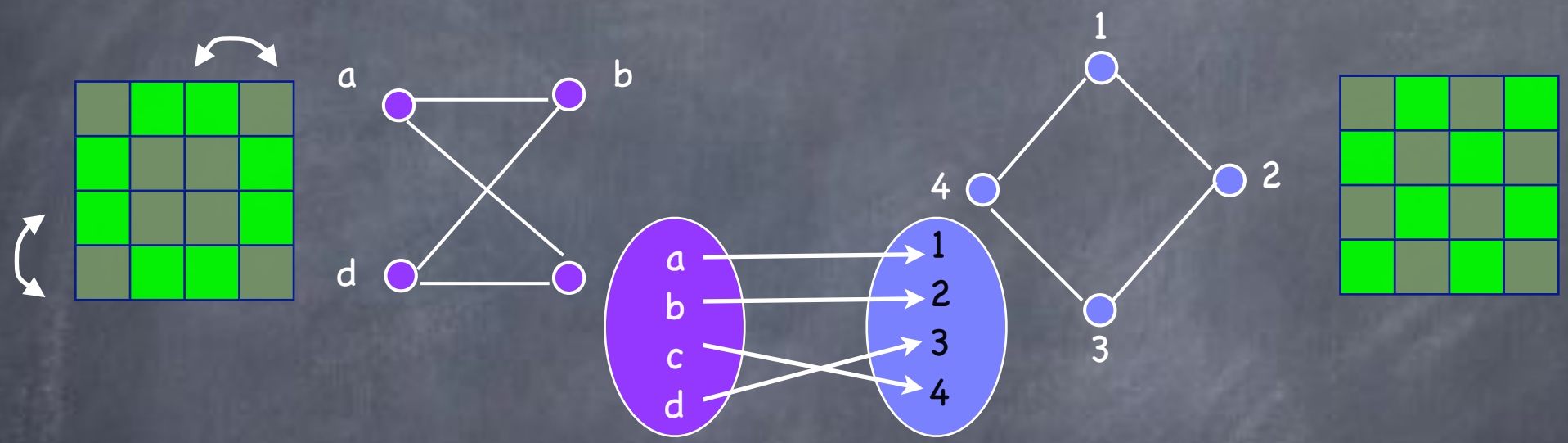

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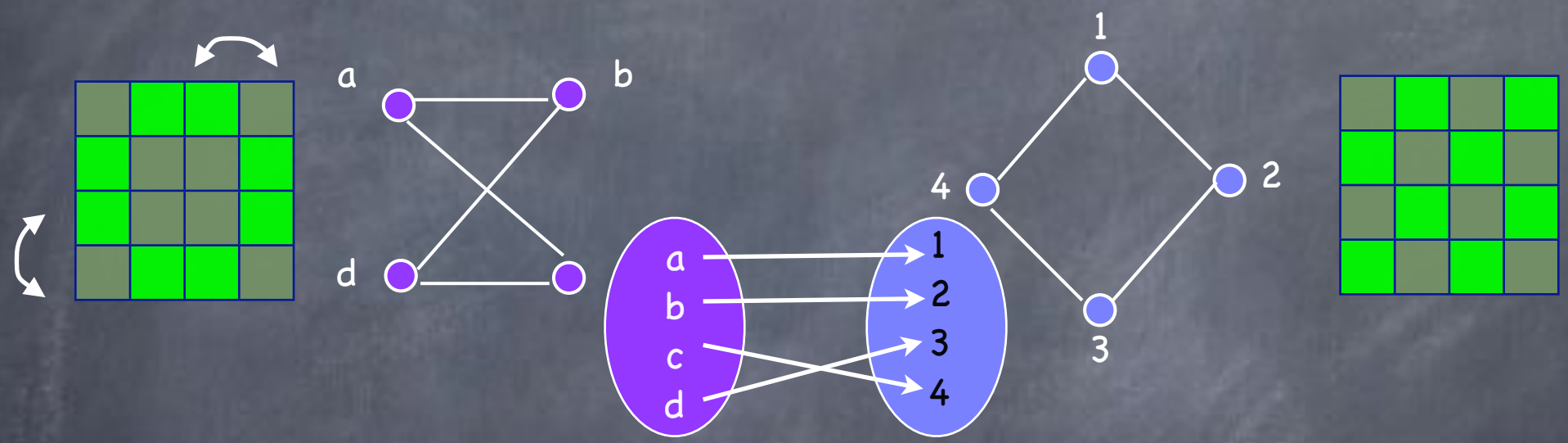

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  - Some believe no efficient algorithm "exists"

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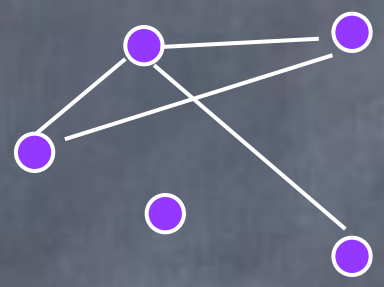

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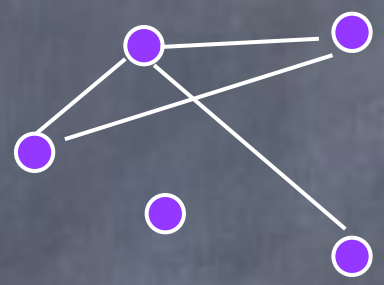

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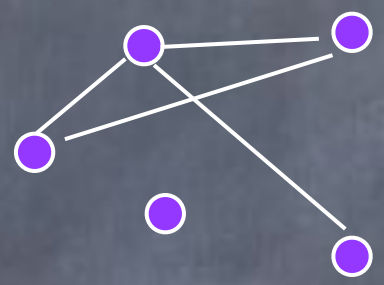

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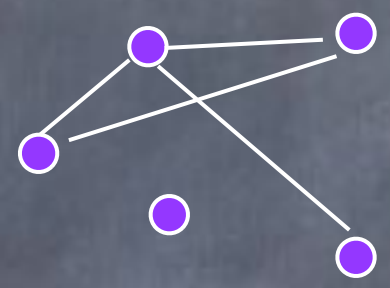

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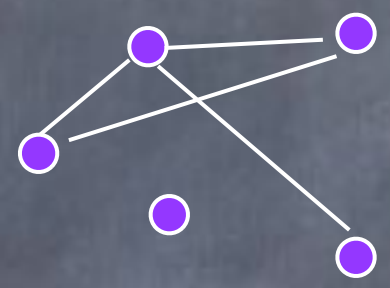

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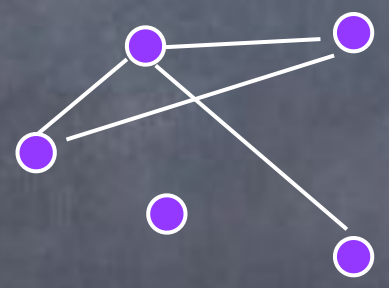

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- If G_1 and G_2 are isomorphic, then every subgraph of G_1 is isomorphic to some subgraph of G_2

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- A graph is acyclic if it has no cycles (i.e., no C_k is a subgraph of G)

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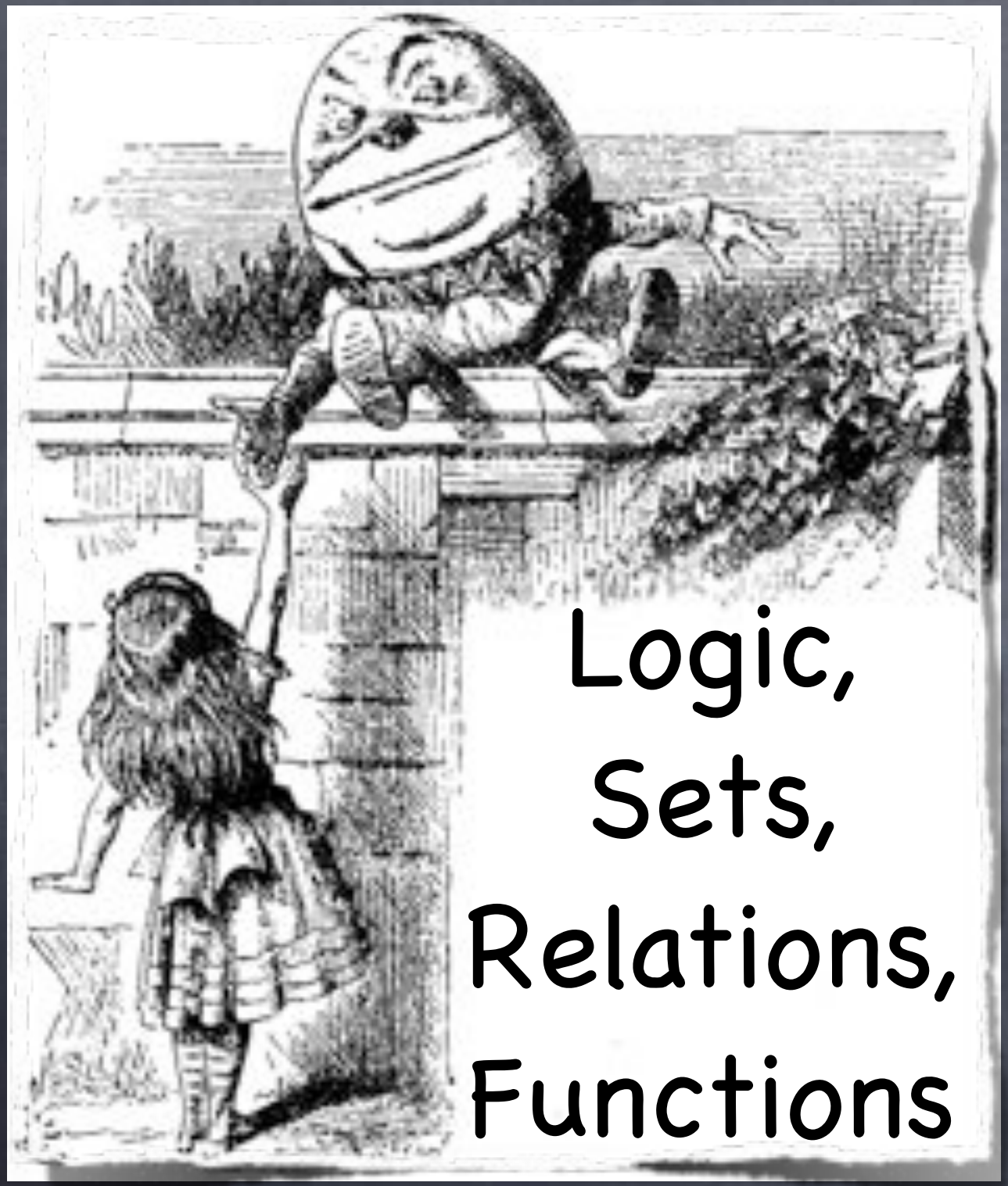
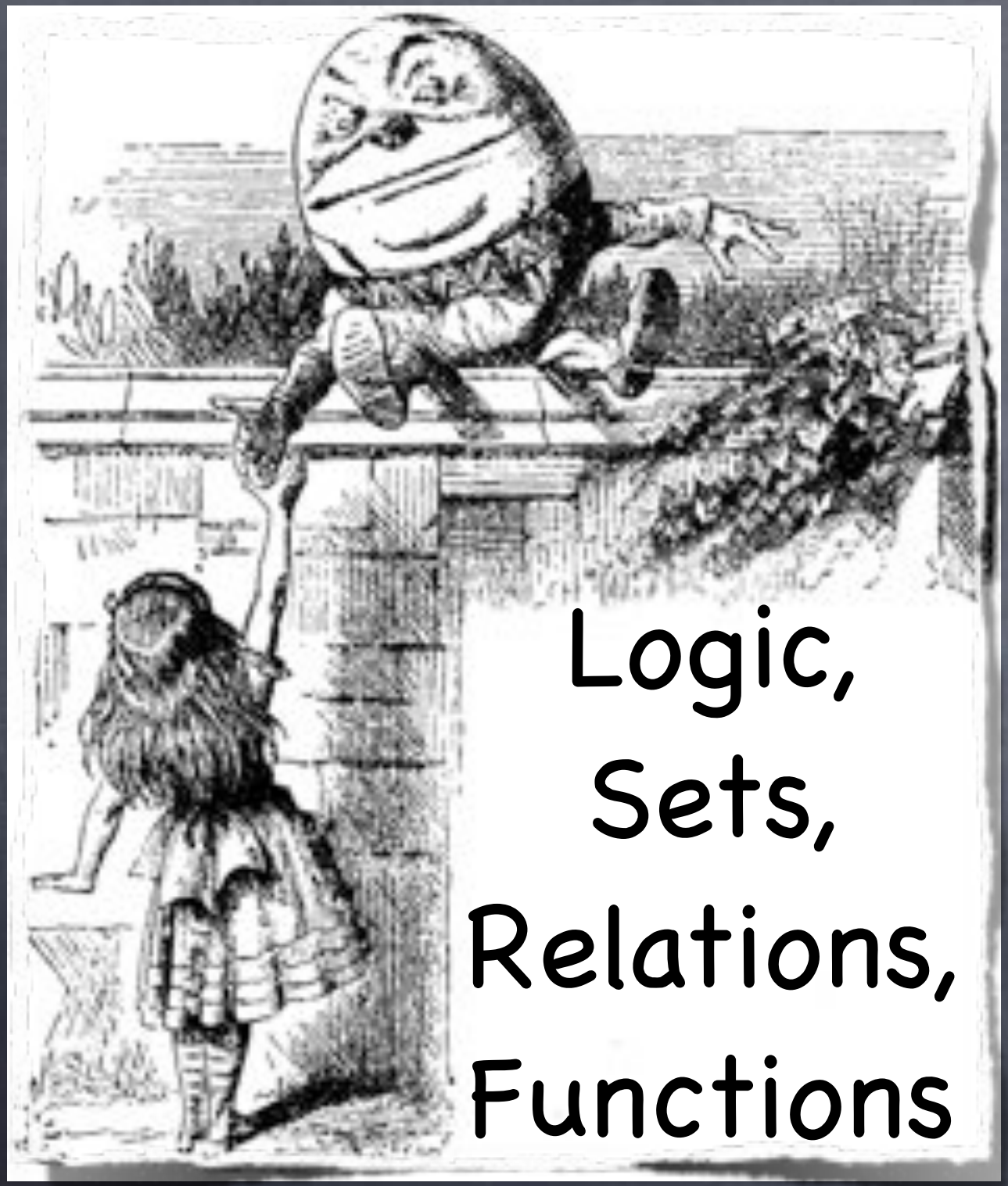
- Number of edges in the shortest path between u and v is called the distance between u and v



Logic, Sets, Relations, Functions



Logic, Sets, Relations, Functions

Review
(continued)

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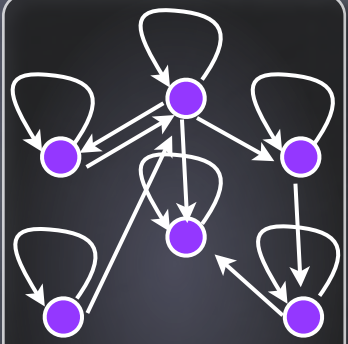
- For two sets S,T, which is a valid proof approach for showing that S = T

  A. Suppose $x \in S$. Then argue $x \in T$.
  B. Suppose $x \notin S$. Then argue $x \notin T$.
  C. Either A or B
  D. A & B together
  E. None of the above



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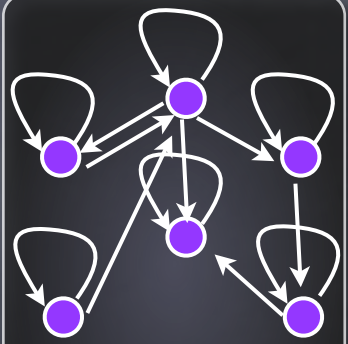

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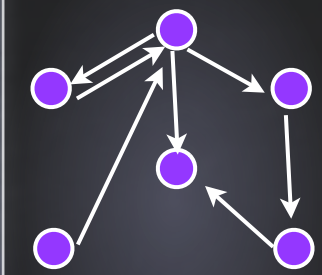

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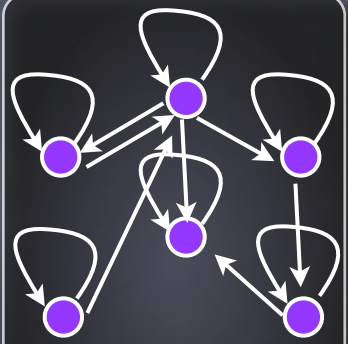

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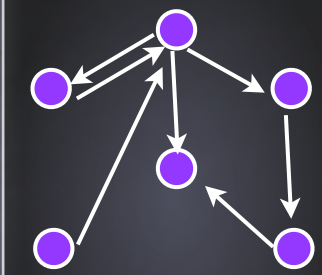

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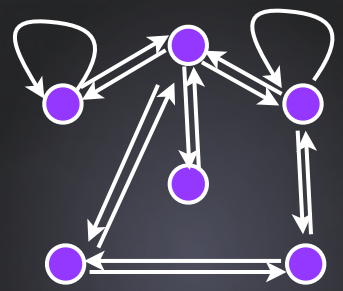

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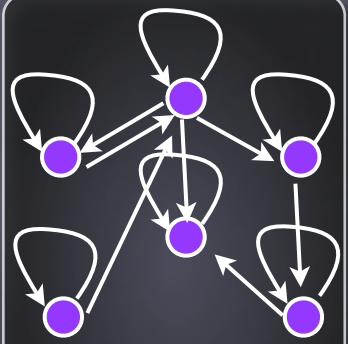

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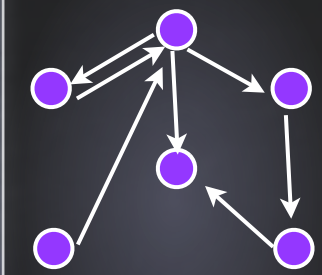

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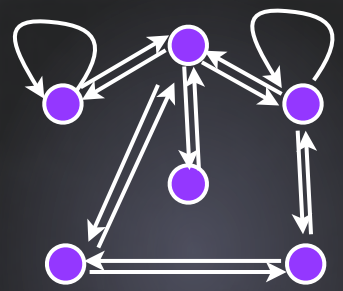

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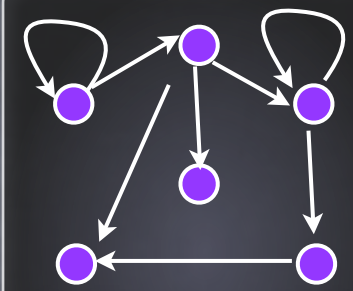

Anti-symmetric:
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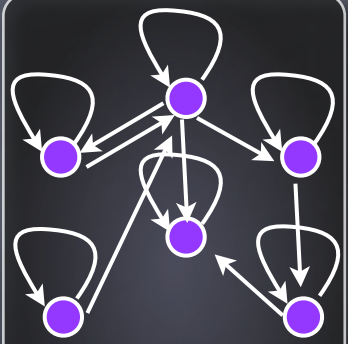

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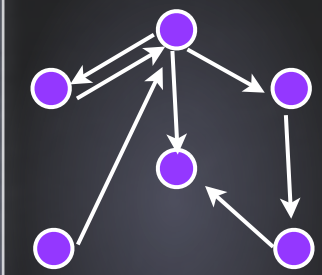
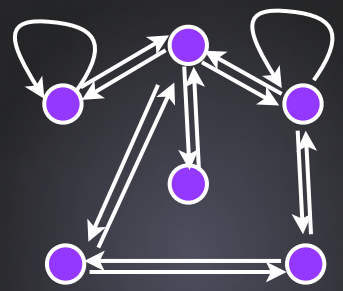
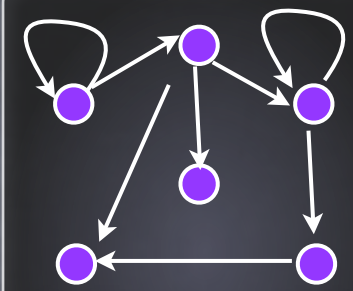


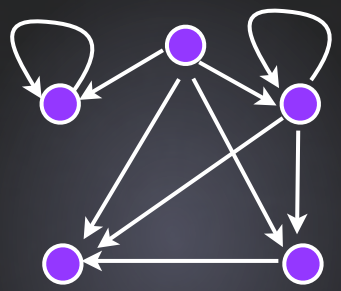
Irreflexive:
No self-loops



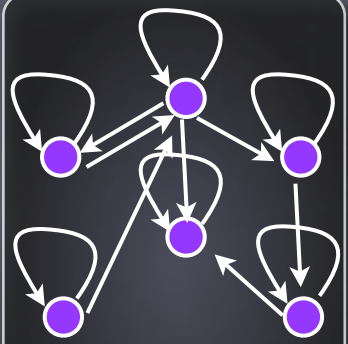
Symmetric:
Only self-loops &
bidirectional edges



Anti-symmetric:
No bidirectional
edges



Transitive:
Path from a to b
implies edge (a,b)



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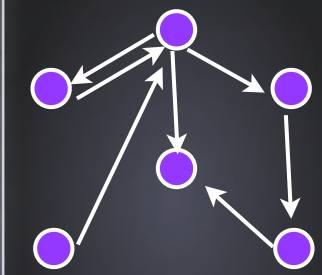

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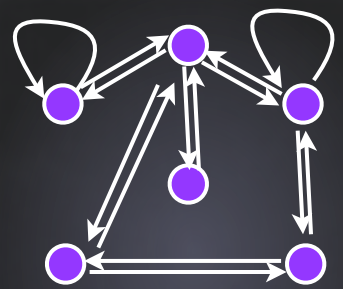

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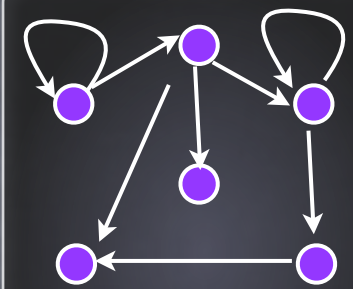

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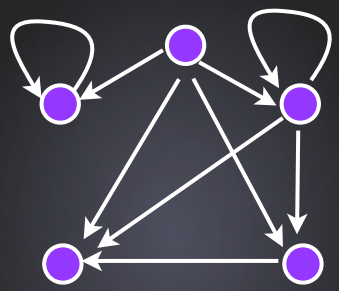
No bidirectional edges



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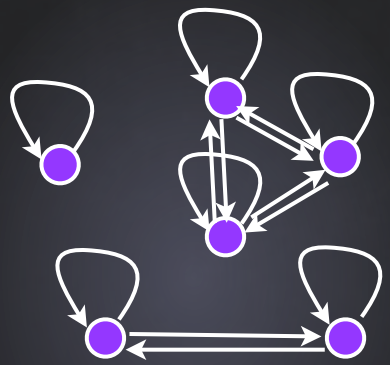
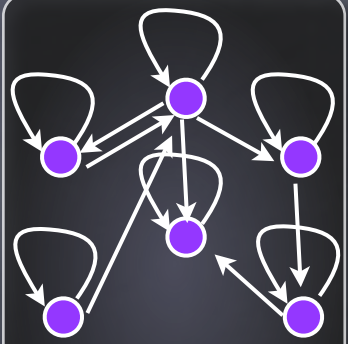


Equivalence:
Cliques, disconnected from each other



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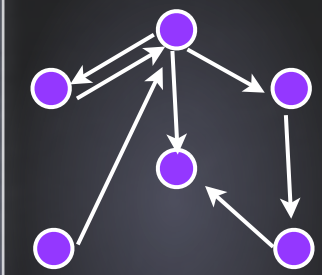

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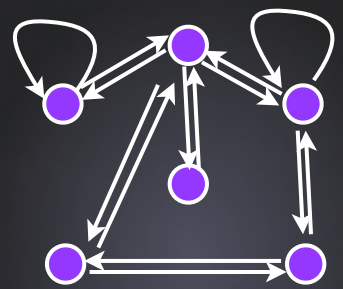

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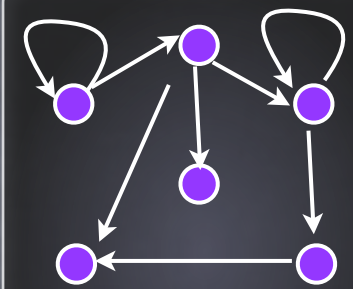

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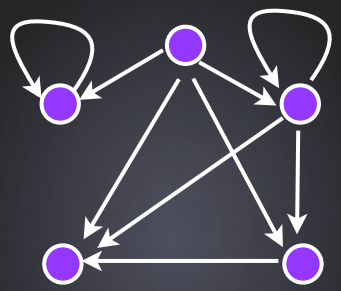

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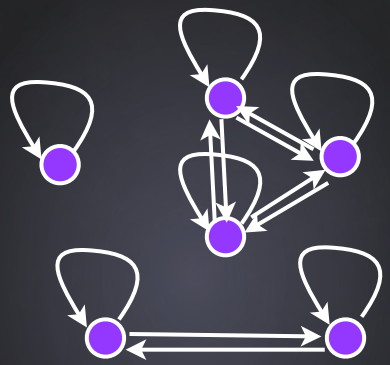

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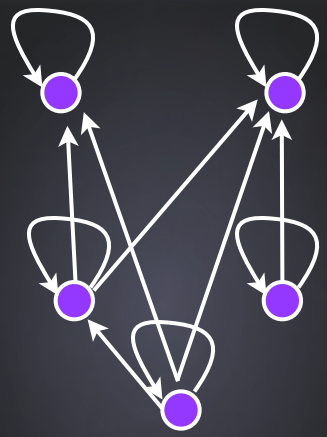
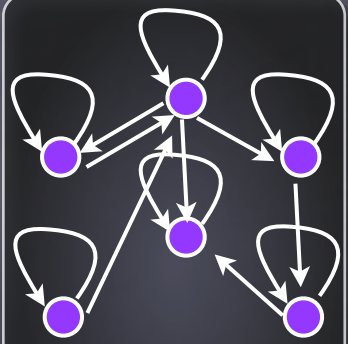


Partial Order:
Reflexive, with no cycles



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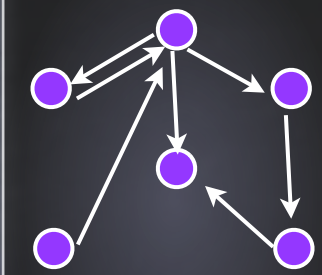

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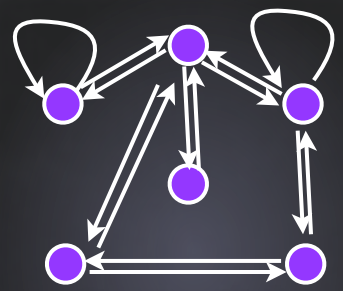

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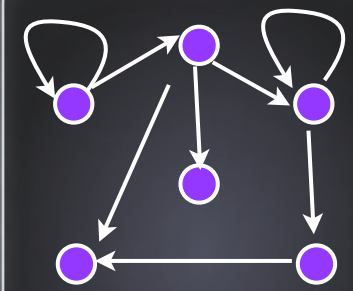

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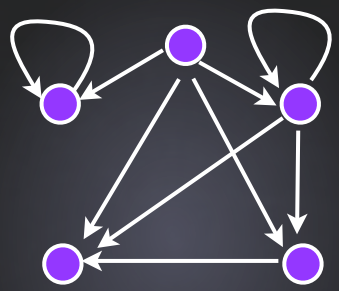

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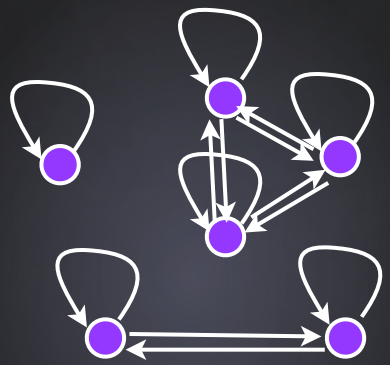

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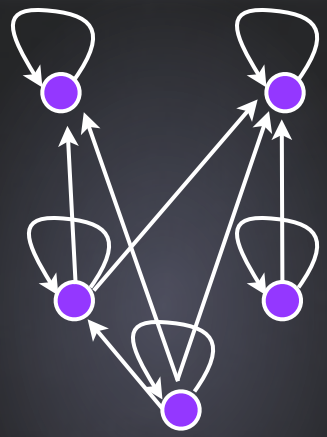

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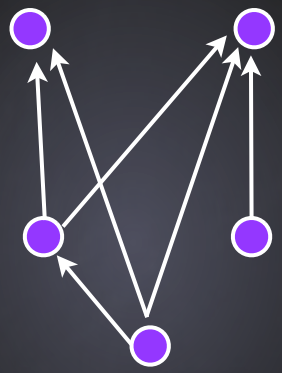
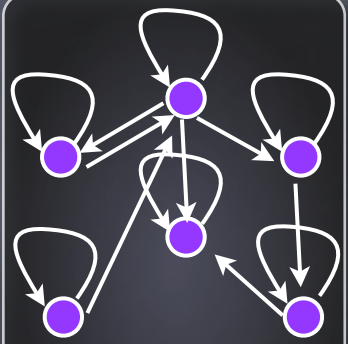


Strict partial Order:
Irreflexive, with no cycles



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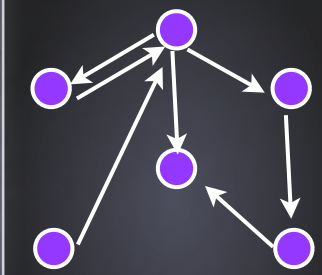

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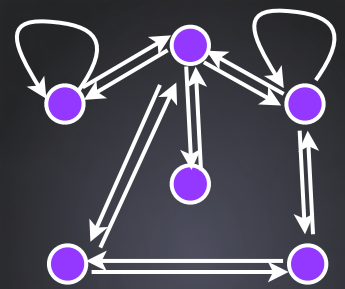

Symmetric:
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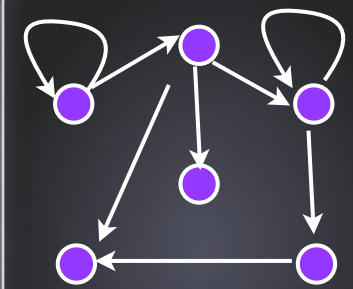

Anti-symmetric:
No bidirectional
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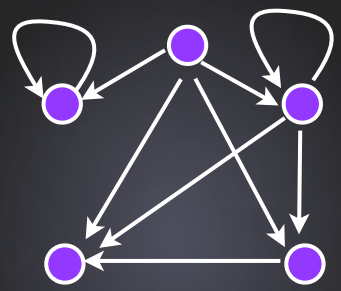

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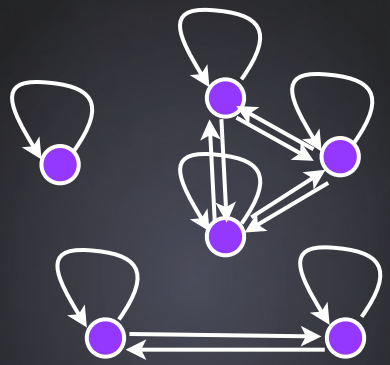

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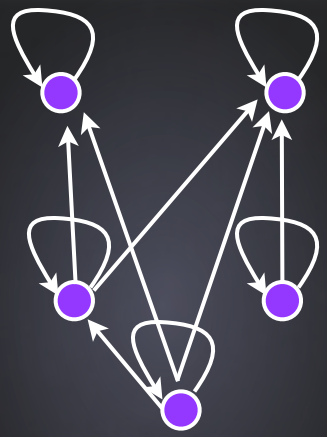

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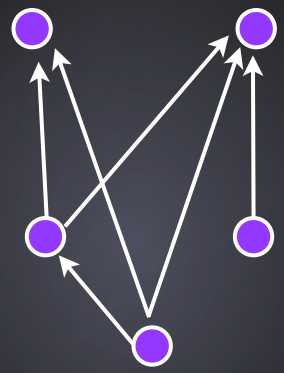
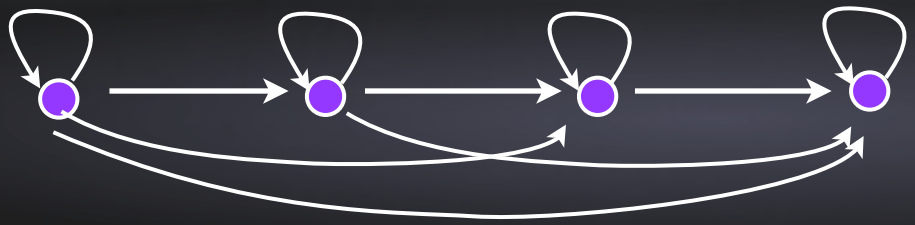


Strict partial Order:
Irreflexive, with no
cycles



Total (or Linear) Order:
Partial order, with
an edge between every pair

Question

- Find the wrong statement

  A. $(\forall a,b \quad a \sqsubset b) \rightarrow \sqsubset$ is an equivalence relation
  B. $(\forall a,b \quad \neg(a \sqsubset b)) \rightarrow \sqsubset$ is a partial order
  C. if $\sqsubset$ is a strict partial order, it is not a partial order
  D. if $\sqsubset$ is a linear order, it is a partial order
  E. None of the above

Types of Functions

Types of Functions

- Function $f:A \to B$. Every $x \in A$ has exactly one image $f(x)$
- Onto Function: Every $y \in B$ has <u>at least</u> one pre-image (x s.t. $y=f(x)$)
  - $|Im(f)| = |B| \leq |A|$
- One-to-One function: Every $y \in B$ has <u>at most</u> one pre-image
  - $|Im(f)| = |A| \leq |B|$
- Bijection: Every $y \in B$ has <u>exactly</u> one pre-image
  - $|Im(f)| = |A| = |B|$
- Strictly increasing/strictly decreasing functions are one-to-one
- Function Composition: $g \circ f : Dom_f \to CoDom_g$, $g \circ f(x) = g(f(x))$
- A one-to-one function f is invertible: $\exists g$ s.t. $g \circ f \equiv Id$
- A bijection f has a unique inverse f^{-1}. f and f^{-1} are inverses of each other

Question

- Below $f:\mathbb{Z}\rightarrow\mathbb{Z}$ and $g:\mathbb{Z}\rightarrow\mathbb{Z}$ defined as follows:

  $f(x) = 5x$, and $g(x)=\lfloor x/5 \rfloor$. Then:

  A. $f\circ g$ is identity over $\mathbb{Z}$
  B. $g\circ f$ is identity over $\mathbb{Z}$
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Numb3rs

Review

Division

- For any two integers a and b, $a \neq 0$, there is a <u>unique</u> quotient q and remainder r, such that
$$b = q \cdot a + r, \quad \text{and } 0 \leq r < |a|$$

-2	-14	-13	-12	-11	-10	-9	-8
-1	-7	-6	-5	-4	-3	-2	-1
	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	7	8	9	10	11	12	13
2	14	15	16	17	18	19	20

r

q

a=7

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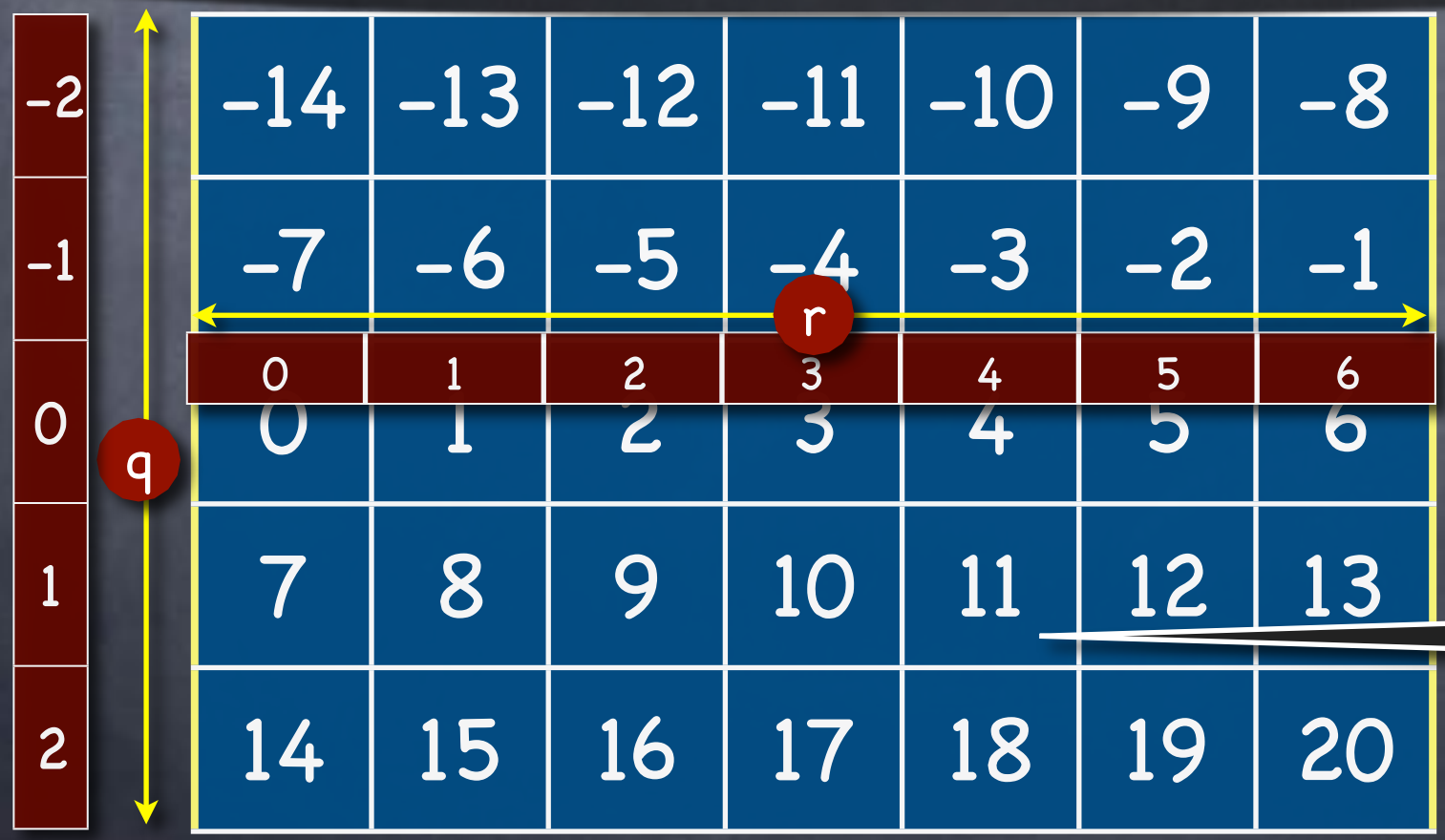

a=7

e.g.
b=11
q=1, r=4

Division

$q = \lfloor b/a \rfloor$

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	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	7	8	9	10	11	12	13
2	14	15	16	17	18	19	20

r

q

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Common Divisors & Multiples

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  Smallest positive number d s.t. $\exists u,v\in\mathbb{Z}$ $d = ua+vb$

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- $\gcd(a,b) \cdot \mathrm{lcm}(a,b) = |a\cdot b|$ [Why?]

Congruence

-14	-13	-12	-11	-10	-9	-8
-7	-6	-5	-4	-3	-2	-1
0	1	2	3	4	5	6
7	8	9	10	11	12	13
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modulus=7

Congruence

- For a "modulus" m and two integers p and q, we say $p \equiv q \pmod{m}$ if $m|(p-q)$

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p&q on same column: <u>distance</u> between p&q is a multiple of m

	3	-12	-11	-10	-9	-8
	6	-5	-4	-3	-2	-1
0	1	2	3	4	5	6
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0	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20

Congruence

- For a "modulus" m and two integers p and q, we say $p \equiv q \pmod{m}$ if $m|(p-q)$

p&q on same column: <u>distance</u> between p&q is a multiple of m

modulus=7

	3	-12	-11	-10	-9	-8
	6	-5	-4	-3	-2	-1
0	1	2	3	4	5	6
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<u>$11 \equiv 18 \pmod{7}$</u>

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Each column is an equivalence class of the relation $\cdot \equiv \cdot \pmod{m}$

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modulus=7

$11 \equiv 18 \pmod{7}$

$11 \equiv -10 \pmod{7}$

Modular Arithmetic

- $[a]_m$: the set of all elements x, s.t. $a \equiv x \pmod m$
- Modular addition: $[a]_m +_m [b]_m \triangleq [a+b]_m$
- Modular multiplication: $[a]_m \times_m [b]_m \triangleq [a \cdot b]_m$
- Multiplicative Inverse! a has a multiplicative inverse modulo m iff a is co-prime with m.
  - $\gcd(a,m)=1 \leftrightarrow \exists u,v\ au+mv=1 \leftrightarrow \exists u\ [a]_m \times_m [u]_m = [1]_m$
  - e.g. $[2]_9 \times_9 [5]_9 = [1]_9$ so $[2]_9^{-1} = [5]_9$ and $[5]_9^{-1} = [2]_9$
  - For a prime modulus p, all except $[0]_p$ have inverses!

Question

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- Let $a = 31^3 + 374 \times 12 + 5 \times 121$. Then

  A. $a \equiv 0 \pmod 3$
  B. $a \equiv 1 \pmod 3$
  C. $a \equiv 2 \pmod 3$
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$\equiv 1^3 + 0 + 2\times1 \pmod 3$

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- Let $a = 11^7$

  A. $a \equiv 0 \pmod{12}$
  B. $a \equiv 1 \pmod{12}$
  C. $a \equiv 7 \pmod{12}$
  D. $a \equiv 11 \pmod{12}$
  E. None of the above

Question

- Let $a = 11^7$

  A. $a \equiv 0 \pmod{12}$
  B. $a \equiv 1 \pmod{12}$
  C. $a \equiv 7 \pmod{12}$
  D. $a \equiv 11 \pmod{12}$
  E. None of the above

$(-1)^7 \equiv -1 \pmod{12}$