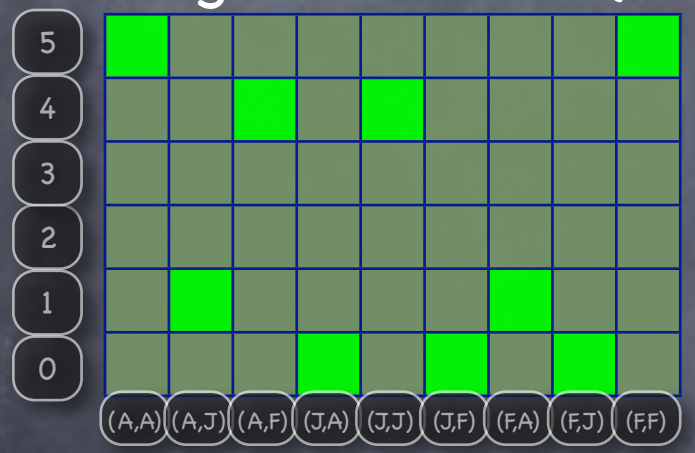
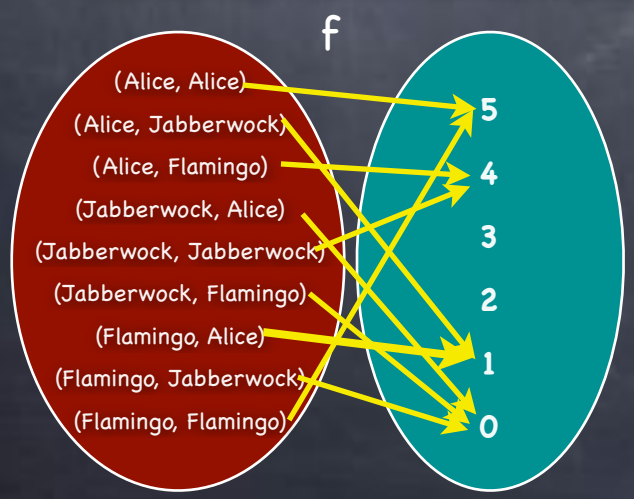
Functions

Lecture 9

Function

- A function maps each element in the domain to an element in the co-domain
  - eg: f: $AIW^2 \rightarrow \{0,1,2,3,4,5\}$

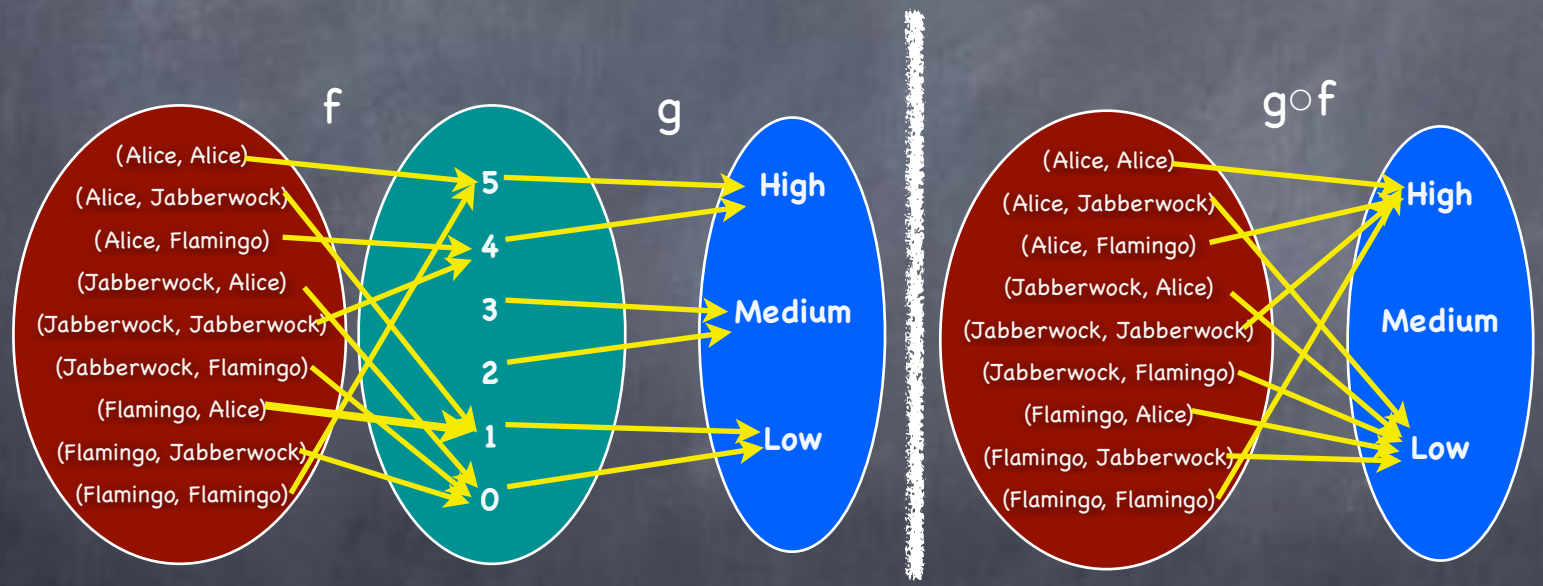




x∈Domain	f(x)∈Co-Domain
(Alice, Alice)	5
(Alice, Jabberwock)	1
(Alice, Flamingo)	4
(Jabberwock, Alice)	0
(Jabberwock, Jabberwock)	4
(Jabberwock, Flamingo)	0
(Flamingo, Alice)	1
(Flamingo, Jabberwock)	0
(Flamingo, Flamingo)	5

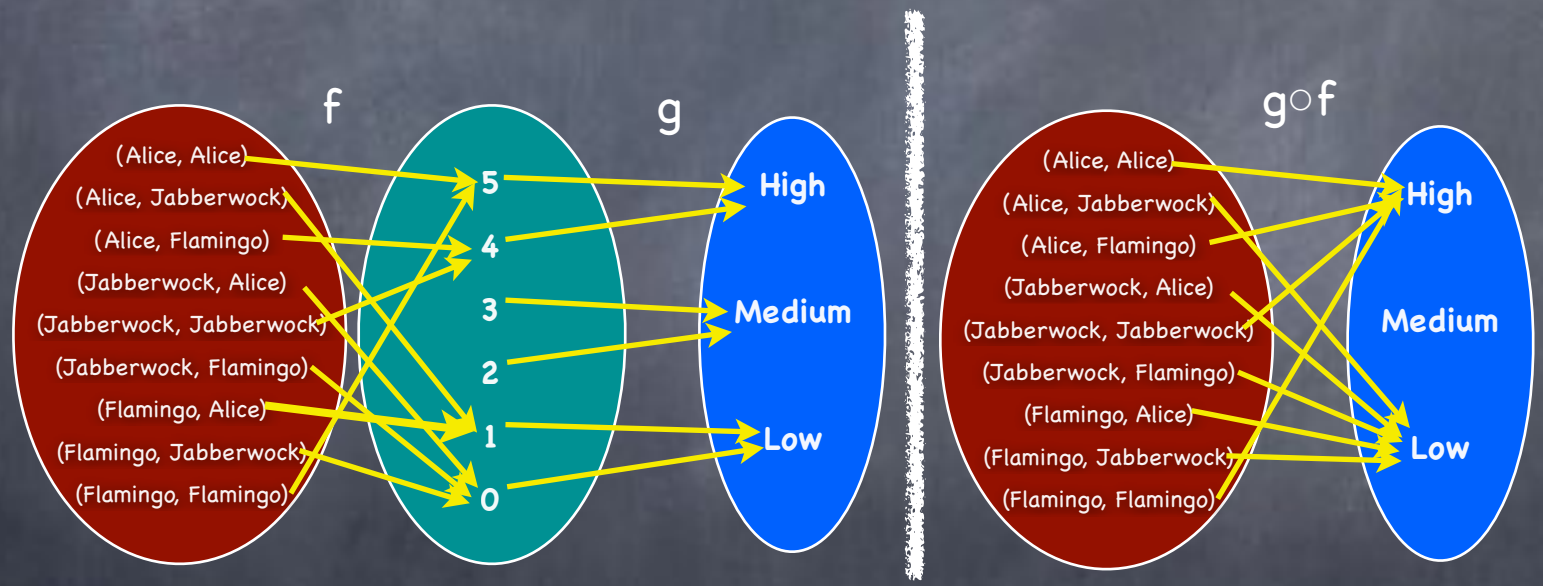
Composition

- Composition of two functions f and g: $g \circ f$
  - $g \circ f(x) \triangleq g(f(x))$



Composition

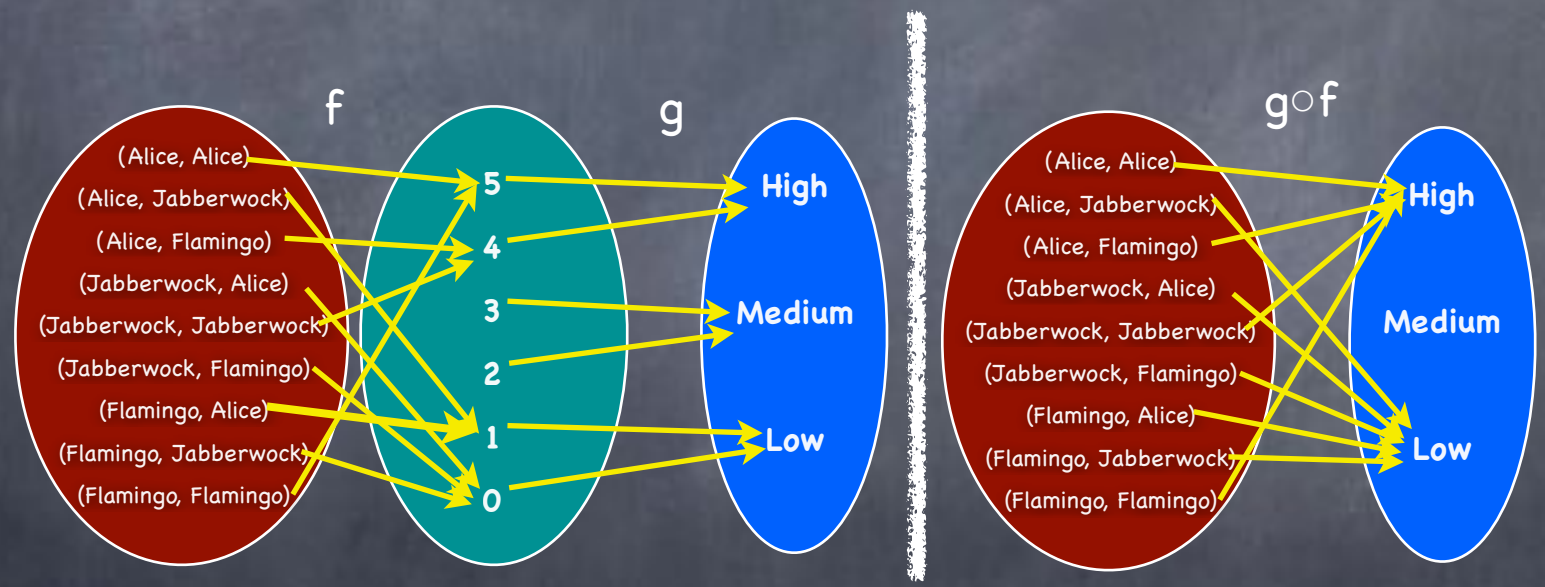
- Composition of two functions f and g: $g \circ f$
  - $g \circ f(x) \triangleq g(f(x))$



- $g \circ f$: Domain(f) → Co-domain(g)

Composition

- Composition of two functions f and g: $g \circ f$
  - $g \circ f(x) \triangleq g(f(x))$

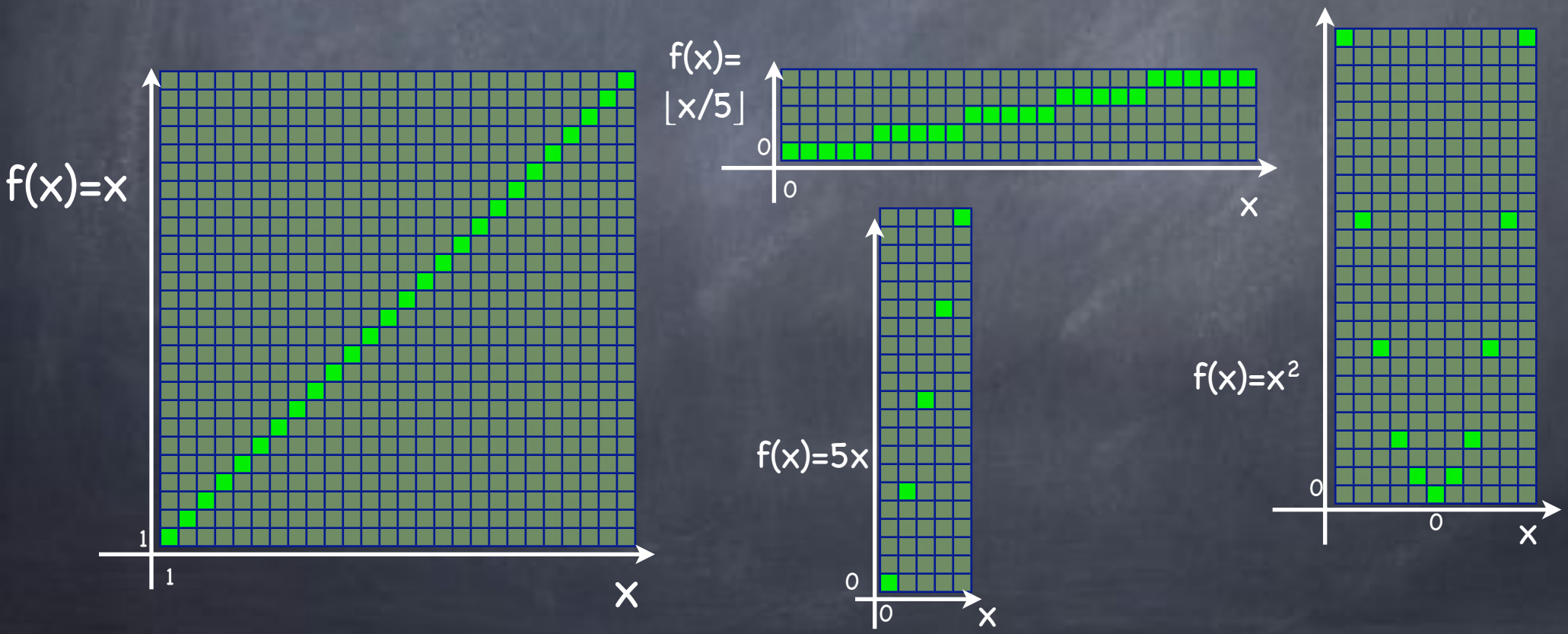


- $g \circ f$: Domain(f) → Co-domain(g)

output input

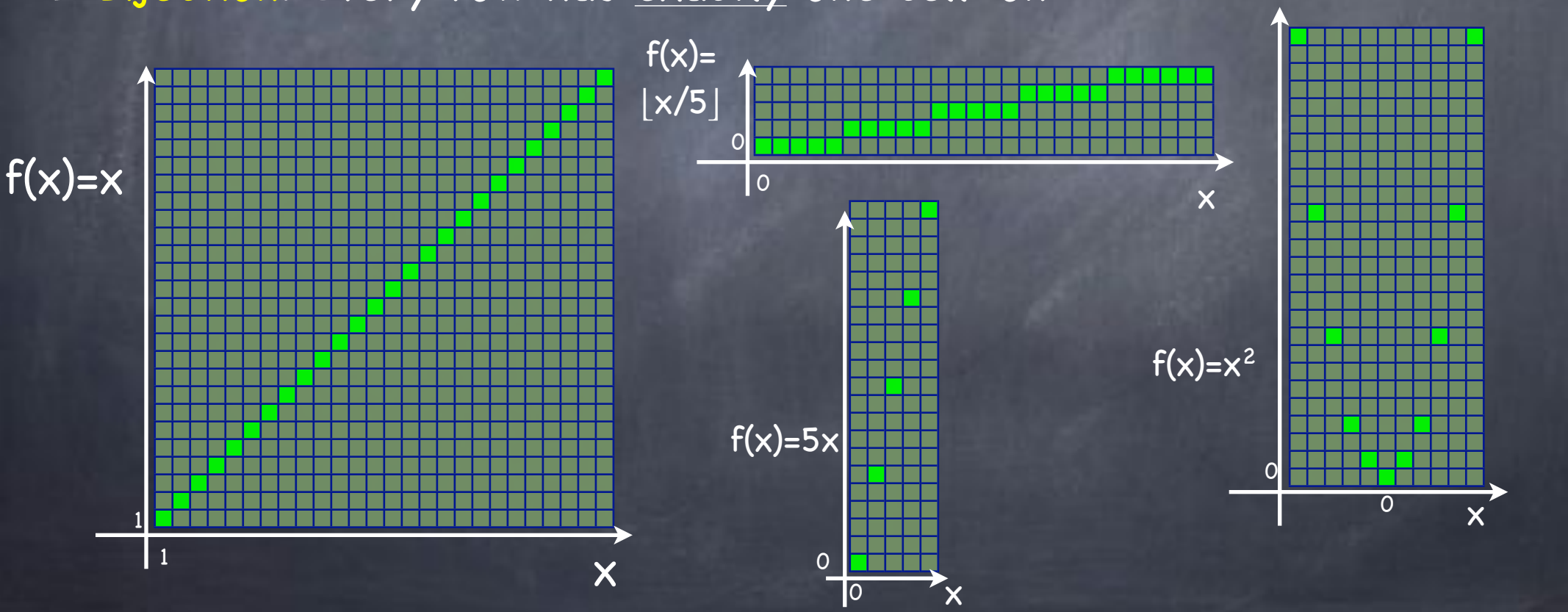
$g \circ f$

Types of Functions



Types of Functions

- Function: every column has exactly one cell "on"
- Onto Function (surjection): Every row has <u>at least</u> one cell "on"
- One-to-One function (injection): Every row has <u>at most</u> one cell "on"
- Bijection: Every row has <u>exactly</u> one cell "on"



One-to-One Functions

One-to-One Functions

- A function $f:A \to B$ is one-to-one if $\forall x,x' \in A \;\; f(x)=f(x') \to x=x'$

One-to-One Functions

- A function $f:A \to B$ is one-to-one if $\forall x,x' \in A \quad f(x)=f(x') \to x=x'$
- e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=x^2$ is <u>not</u> one-to-one

One-to-One Functions

- A function $f:A \to B$ is one-to-one if $\forall x,x' \in A \quad f(x)=f(x') \to x=x'$
- e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=x^2$ is <u>not</u> one-to-one
  - e.g. $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ defined as $f(x)=x^2$ is one-to-one

One-to-One Functions

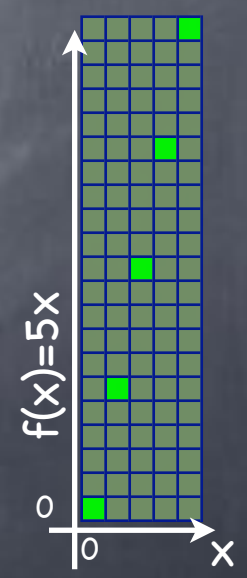
- A function $f:A \to B$ is one-to-one if $\forall x,x' \in A \quad f(x)=f(x') \to x=x'$
- e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=x^2$ is <u>not</u> one-to-one
  - e.g. $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ defined as $f(x)=x^2$ is one-to-one
- e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=\lfloor x/5 \rfloor$ is <u>not</u> one-to-one

One-to-One Functions

- A function $f:A \to B$ is one-to-one if $\forall x,x' \in A \quad f(x)=f(x') \to x=x'$
- e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=x^2$ is <u>not</u> one-to-one
  - e.g. $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ defined as $f(x)=x^2$ is one-to-one
- e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=\lfloor x/5 \rfloor$ is <u>not</u> one-to-one
  - e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=5x$ is one-to-one

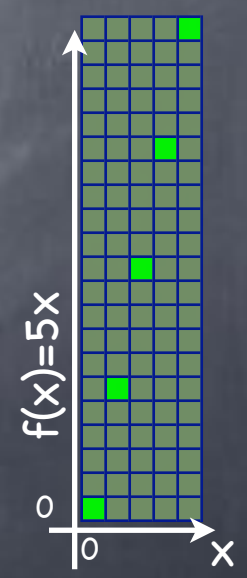
One-to-One Functions

- A function $f:A \to B$ is one-to-one if $\forall x,x' \in A \quad f(x)=f(x') \to x=x'$
- e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=x^2$ is <u>not</u> one-to-one
  - e.g. $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ defined as $f(x)=x^2$ is one-to-one
- e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=\lfloor x/5 \rfloor$ is <u>not</u> one-to-one
  - e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=5x$ is one-to-one



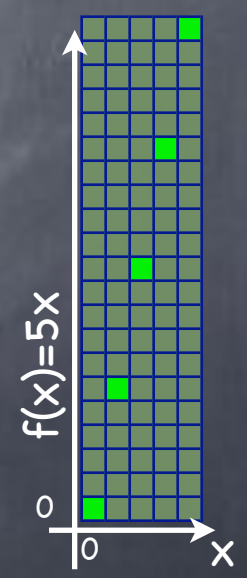
One-to-One Functions

- A function $f:A \to B$ is one-to-one if $\forall x,x' \in A \quad f(x)=f(x') \to x=x'$
- e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=x^2$ is <u>not</u> one-to-one
  - e.g. $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ defined as $f(x)=x^2$ is one-to-one
- e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=\lfloor x/5 \rfloor$ is <u>not</u> one-to-one
  - e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=5x$ is one-to-one
- In fact, any <u>strictly increasing function</u> is one-to-one



One-to-One Functions

- A function $f:A \to B$ is one-to-one if $\forall x,x' \in A \quad f(x)=f(x') \to x=x'$
- e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=x^2$ is <u>not</u> one-to-one
  - e.g. $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ defined as $f(x)=x^2$ is one-to-one
- e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=\lfloor x/5 \rfloor$ is <u>not</u> one-to-one
  - e.g. $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x)=5x$ is one-to-one
- In fact, any <u>strictly increasing function</u> is one-to-one
  - And, any <u>strictly decreasing function</u> too is one-to-one



Question

- Below f: $\mathbb{Z} \to \mathbb{Z}$. Pick the right choice.

  A. f s.t. $f(x)=\lfloor x/5 \rfloor$ is increasing, but not one-to-one
  B. f s.t. $f(x) = x + |x|$ is increasing and one-to-one
  C. f s.t. $f(x)=1$ is neither increasing nor one-to-one
  D. f is one-to-one $\rightarrow$ f is increasing or decreasing
  E. None of the above

Question

- Below f: $\mathbb{Z} \to \mathbb{Z}$. Pick the right choice.

f is increasing, but not strictly increasing

A. f s.t. $f(x)=\lfloor x/5 \rfloor$ is increasing, but not one-to-one
B. f s.t. $f(x) = x + |x|$ is increasing and one-to-one
C. f s.t. $f(x)=1$ is neither increasing nor one-to-one
D. f is one-to-one $\rightarrow$ f is increasing or decreasing
E. None of the above

One-to-One Functions

[ETPT]

One-to-One Functions

- One-to-one functions are "invertible"

[ETPT]

One-to-One Functions

- One-to-one functions are "<u>invertible</u>"

f invertible:
$\exists g$ s.t. $g \circ f \equiv Id$

[ETPT]

One-to-One Functions

- One-to-one functions are "<u>invertible</u>"
- Suppose $f:A \to B$ is one-to-one

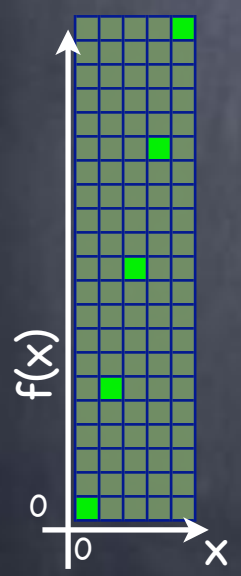
f invertible:
$\exists g$ s.t. $g \circ f \equiv Id$

[ETPT]

One-to-One Functions

- One-to-one functions are "<u>invertible</u>"
- Suppose $f:A \rightarrow B$ is one-to-one

f invertible:
$\exists g$ s.t. $g \circ f \equiv Id$

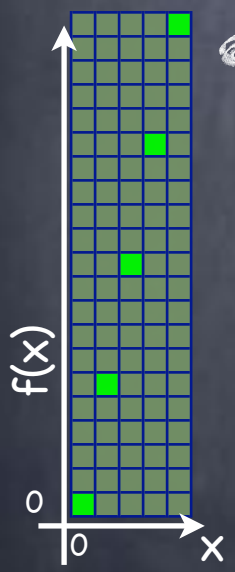


[ETPT]

One-to-One Functions

f invertible:
$\exists g$ s.t. $g \circ f \equiv Id$

- One-to-one functions are "invertible"
- Suppose $f:A \rightarrow B$ is one-to-one
  - Let $g:B \rightarrow A$ be defined as follows:
    for $y \in Im(f)$, $g(y)=x$ s.t. $f(x)=y$ (well-defined)
    for $y \notin Im(f)$, $g(y) = 0$ (or some arbitrary element in A)



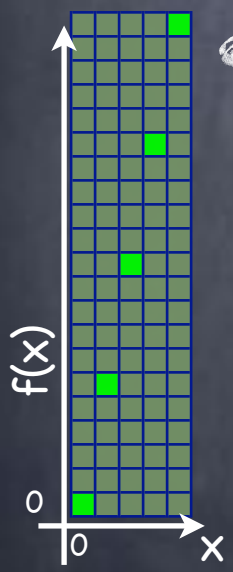
[ETPT]

One-to-One Functions

- One-to-one functions are "invertible"
- Suppose $f:A \to B$ is one-to-one
  - Let $g:B \to A$ be defined as follows:
    for $y \in Im(f)$, $g(y)=x$ s.t. $f(x)=y$ (well-defined)
    for $y \notin Im(f)$, $g(y) = 0$ (or some arbitrary element in A)

f invertible:
$\exists g$ s.t. $g \circ f \equiv Id$

$\forall y \in Im(f)\ \exists! x \in A \quad f(x) = y$



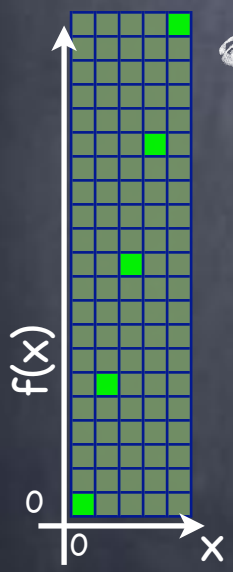
[ETPT]

One-to-One Functions

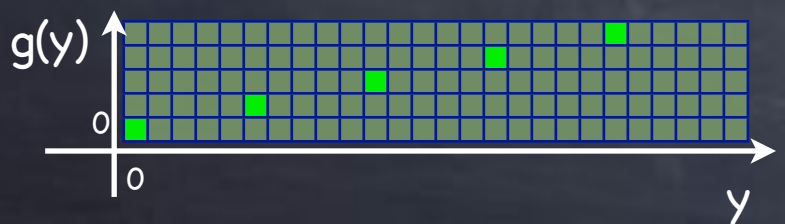
- One-to-one functions are "invertible"
- Suppose $f:A \to B$ is one-to-one
  - Let $g:B \to A$ be defined as follows:
    for $y \in Im(f)$, $g(y)=x$ s.t. $f(x)=y$ (well-defined)
    for $y \notin Im(f)$, $g(y) = 0$ (or some arbitrary element in A)

f invertible:
$\exists g$ s.t. $g \circ f \equiv Id$

$\forall y \in Im(f)\ \exists! x \in A \quad f(x) = y$



[ETPT]

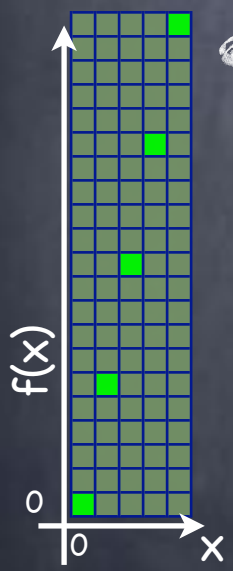


One-to-One Functions

- One-to-one functions are "invertible"
- Suppose $f:A \to B$ is one-to-one
  - Let $g:B \to A$ be defined as follows:
    for $y \in Im(f)$, $g(y)=x$ s.t. $f(x)=y$ (well-defined)
    for $y \notin Im(f)$, $g(y) = 0$ (or some arbitrary element in A)

f invertible:
$\exists g$ s.t. $g \circ f \equiv Id$

$\forall y \in Im(f)\ \exists! x \in A \quad f(x) = y$



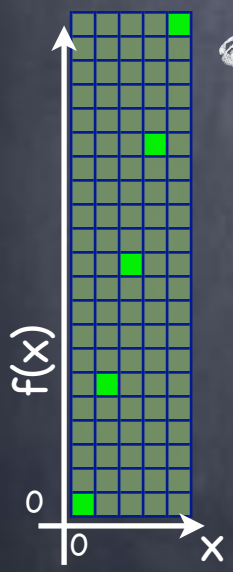
[ETPT]

One-to-One Functions

f invertible:
$\exists g$ s.t. $g \circ f \equiv Id$

- One-to-one functions are "invertible"
- Suppose $f:A \to B$ is one-to-one

$\forall y \in Im(f)\ \exists! x \in A \quad f(x) = y$



  - Let $g:B \to A$ be defined as follows:
    for $y \in Im(f)$, $g(y)=x$ s.t. $f(x)=y$ (well-defined)
    for $y \notin Im(f)$, $g(y) = 0$ (or some arbitrary element in A)
    - Then $g \circ f \equiv Id_A$, where $Id_A:A \to A$ is the identity function over A

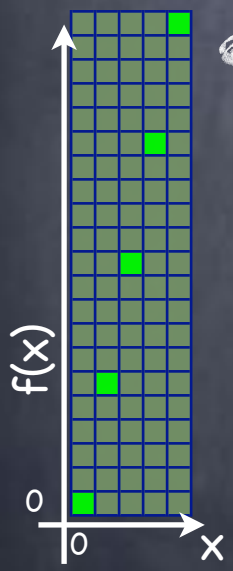
[ETPT]

One-to-One Functions

f invertible:
$\exists g$ s.t. $g \circ f \equiv Id$

- One-to-one functions are "invertible"
- Suppose $f:A \to B$ is one-to-one

$\forall y \in Im(f)\ \exists! x \in A \quad f(x) = y$



  - Let $g:B \to A$ be defined as follows:
    for $y \in Im(f)$, $g(y)=x$ s.t. $f(x)=y$ (well-defined)
    for $y \notin Im(f)$, $g(y) = 0$ (or some arbitrary element in A)
    - Then $g \circ f \equiv Id_A$, where $Id_A:A \to A$ is the identity function over A
    - If $Im(f) \subset B$, then $f \circ g \not\equiv Id_B$

[ETPT]

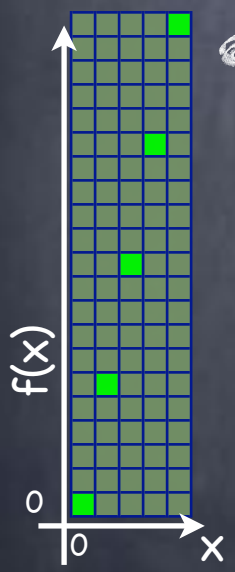
One-to-One Functions

- One-to-one functions are "<u>invertible</u>"

f invertible:
$\exists g$ s.t. $g \circ f \equiv Id$

- Suppose $f:A \to B$ is one-to-one

$\forall y \in Im(f)\ \exists! x \in A \quad f(x) = y$



  - Let $g:B \to A$ be defined as follows:
    for $y \in Im(f)$, $g(y)=x$ s.t. $f(x)=y$ (well-defined)
    for $y \notin Im(f)$, $g(y) = 0$ (or some arbitrary element in A)
    - Then $g \circ f \equiv Id_A$, where $Id_A:A \to A$ is the identity function over A
    - If $Im(f) \subset B$, then $f \circ g \not\equiv Id_B$

[ETPT]

  - g not one-to-one → g not invertible

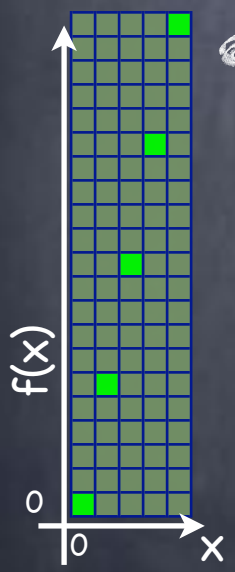
One-to-One Functions

- One-to-one functions are "invertible"

f invertible:
$\exists g$ s.t. $g \circ f \equiv Id$

- Suppose $f:A \to B$ is one-to-one

$\forall y \in Im(f)\ \exists! x \in A \quad f(x) = y$



  - Let $g:B \to A$ be defined as follows:
    for $y \in Im(f)$, $g(y)=x$ s.t. $f(x)=y$ (well-defined)
    for $y \notin Im(f)$, $g(y) = 0$ (or some arbitrary element in A)
    - Then $g \circ f \equiv Id_A$, where $Id_A:A \to A$ is the identity function over A
    - If $Im(f) \subset B$, then $f \circ g \not\equiv Id_B$
  - g not one-to-one → g not invertible

[ETPT]
g invertible → g one-to-one
Suppose $h \circ g \equiv Id$
$g(x_1)=g(x_2) \to h(g(x_1)) = h(g(x_2))$
$\to x_1=x_2$ (since $h \circ g \equiv Id$)
i.e., $\forall x_1,x_2\ g(x_1)=g(x_2) \to x_1= x_2$

Question

- Below $f:\mathbb{Z}\rightarrow\mathbb{Z}$ and $g:\mathbb{Z}\rightarrow\mathbb{Z}$ defined as follows:

  $f(x) = 5x$, and $g(x)=\lfloor x/5 \rfloor$. Then:

  A. $f\circ g$ is identity over $\mathbb{Z}$
  B. $g\circ f$ is identity over $\mathbb{Z}$
  C. $f\circ g$ is not well-defined
  D. $g\circ f$ is not well-defined
  E. None of the above

Question

- Below $f:\mathbb{Z}\to\mathbb{Z}$ and $g:\mathbb{Z}\to\mathbb{Z}$ defined as follows:

  $f(x) = 5x$, and $g(x)=\lfloor x/5 \rfloor$. Then:

  A. $f\circ g$ is identity over $\mathbb{Z}$

  $f\circ g(x) = 5 \lfloor x/5 \rfloor \in [x-4,x]$

  B. $g\circ f$ is identity over $\mathbb{Z}$

  C. $f\circ g$ is not well-defined

  D. $g\circ f$ is not well-defined

  E. None of the above

Question

- Below $f:\mathbb{Z}\to\mathbb{Z}$ and $g:\mathbb{Z}\to\mathbb{Z}$ defined as follows:

  $f(x) = 5x$, and $g(x)=\lfloor x/5 \rfloor$. Then:

  A. $f\circ g$ is identity over $\mathbb{Z}$

  $f\circ g(x) = 5 \lfloor x/5 \rfloor \in [x-4,x]$

  B. $g\circ f$ is identity over $\mathbb{Z}$

  $g\circ f(x) = \lfloor 5x/5 \rfloor = x$

  C. $f\circ g$ is not well-defined

  D. $g\circ f$ is not well-defined

  E. None of the above

Bijections

Bijections

- They are totally awesome!

Bijections

- They are totally awesome!
- Bijection: both onto (surjection) and one-to-one (injection)

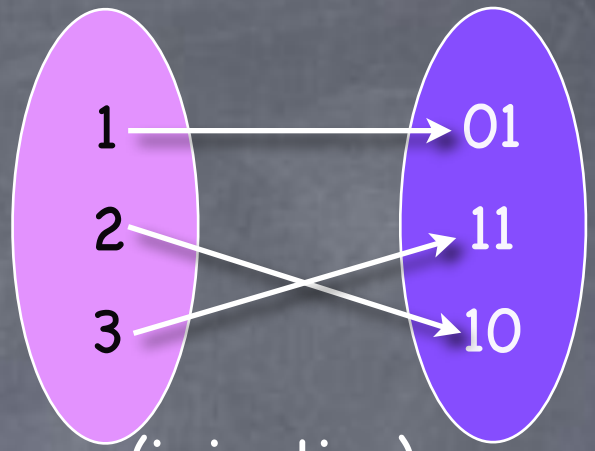
Bijections

- They are totally awesome!
- Bijection: both onto (surjection) and one-to-one (injection)
  - Every row and every column has <u>exactly</u> one cell "on"

Bijections

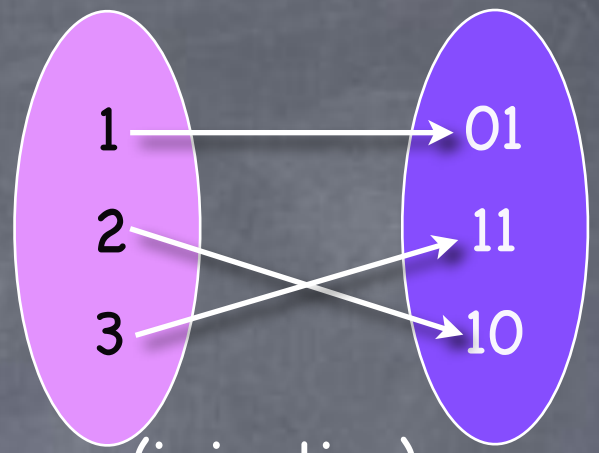
- They are totally awesome!
- Bijection: both onto (surjection) and one-to-one (injection)
  - Every row and every column has exactly one cell "on"
  - Every element in the co-domain has exactly one "pre-image"

Bijections

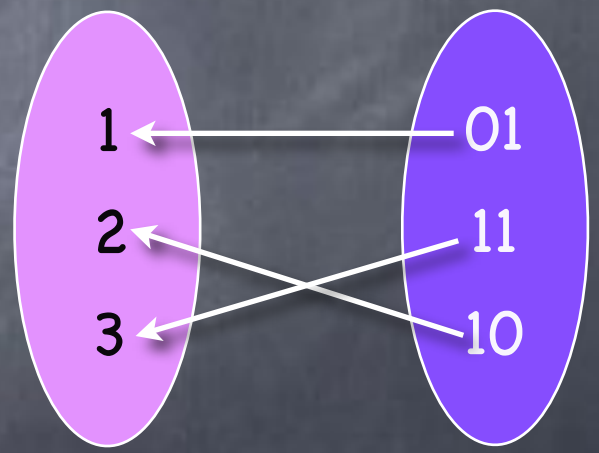


- They are totally awesome!
- Bijection: both onto (surjection) and one-to-one (injection)
  - Every row and every column has <u>exactly</u> one cell "on"
  - Every element in the co-domain has exactly one "pre-image"

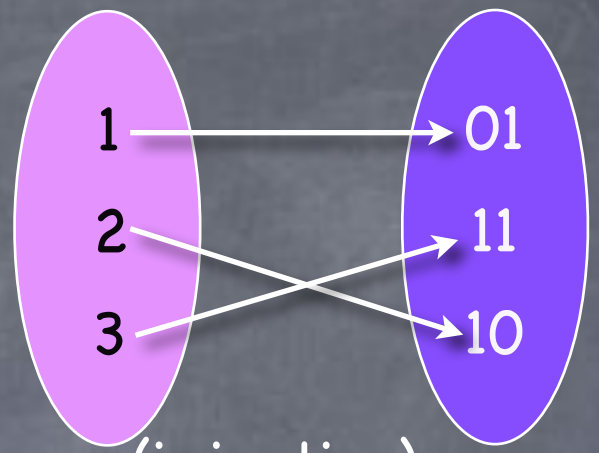
Bijections



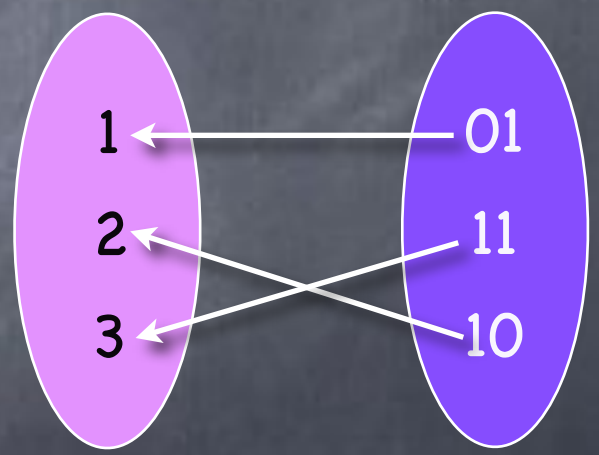
- They are totally awesome!
- Bijection: both onto (surjection) and one-to-one (injection)
  - Every row and every column has <u>exactly</u> one cell "on"
  - Every element in the co-domain has exactly one "pre-image"



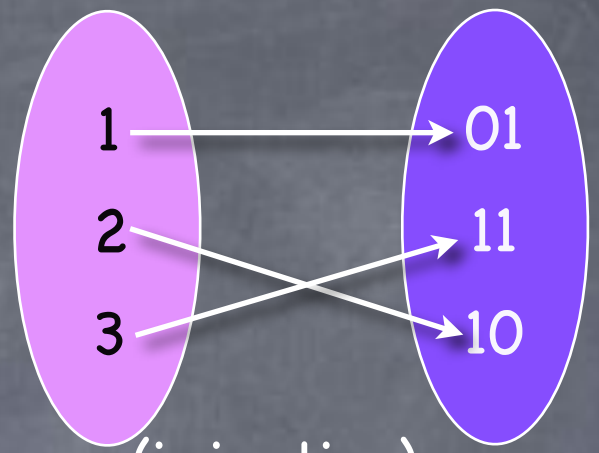
Bijections



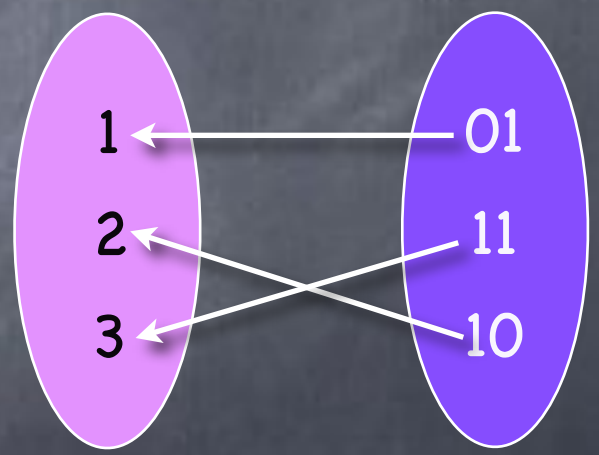
- They are totally awesome!
- Bijection: both onto (surjection) and one-to-one (injection)
  - Every row and every column has <u>exactly</u> one cell "on"
  - Every element in the co-domain has exactly one "pre-image"
    - If $f:A \rightarrow B$, $f^{-1}:B \rightarrow A$ such that $f^{-1} \circ f: A \rightarrow A$ and $f \circ f^{-1}: B \rightarrow B$



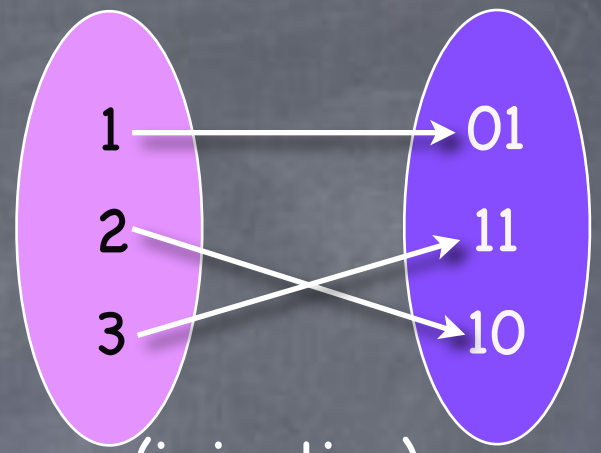
Bijections



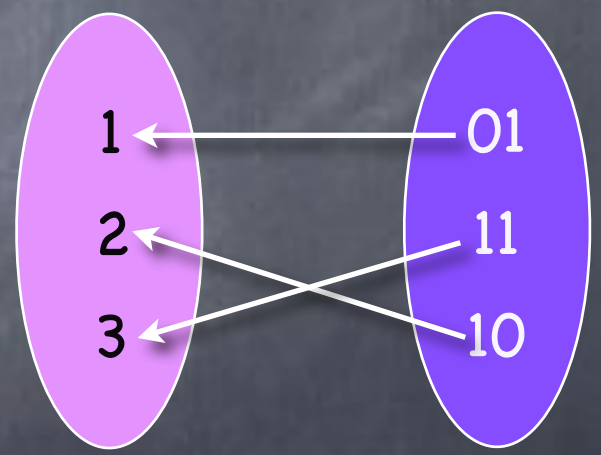
- They are totally awesome!
- Bijection: both onto (surjection) and one-to-one (injection)
  - Every row and every column has <u>exactly</u> one cell "on"
  - Every element in the co-domain has exactly one "pre-image"
    - If $f:A \rightarrow B$, $f^{-1}:B \rightarrow A$ such that $f^{-1} \circ f: A \rightarrow A$ and $f \circ f^{-1}: B \rightarrow B$
    - Both f and f^{-1} are invertible, and the inverses are unique



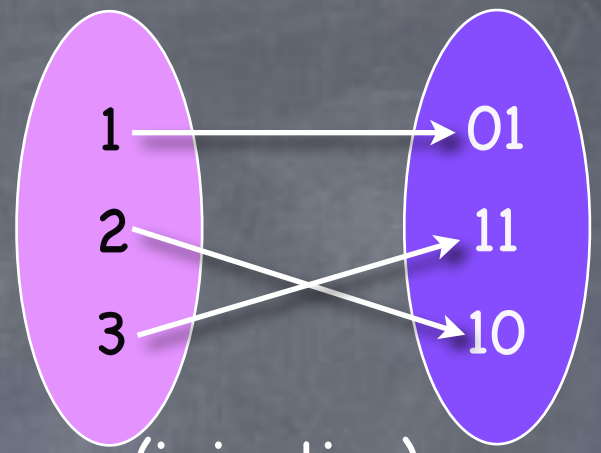
Bijections



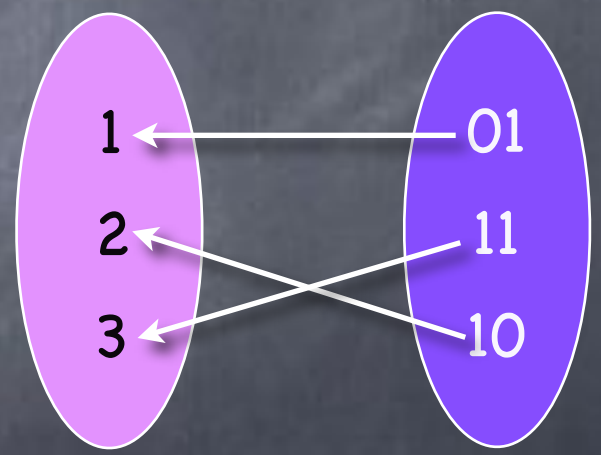
- They are totally awesome!
- Bijection: both onto (surjection) and one-to-one (injection)
  - Every row and every column has <u>exactly</u> one cell "on"
  - Every element in the co-domain has exactly one "pre-image"
    - If $f:A \rightarrow B$, $f^{-1}:B \rightarrow A$ such that $f^{-1} \circ f: A \rightarrow A$ and $f \circ f^{-1}: B \rightarrow B$
    - Both f and f^{-1} are invertible, and the inverses are unique
    - $(f^{-1})^{-1} = f$



Bijections

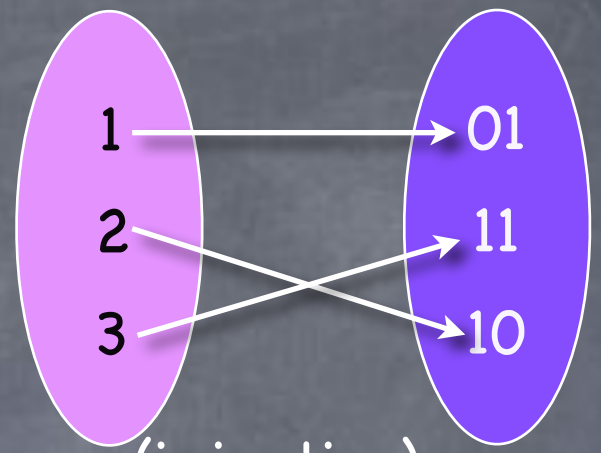


- They are totally awesome!
- Bijection: both onto (surjection) and one-to-one (injection)
  - Every row and every column has <u>exactly</u> one cell "on"
  - Every element in the co-domain has exactly one "pre-image"
    - If $f:A \to B$, $f^{-1}:B \to A$ such that $f^{-1} \circ f: A \to A$ and $f \circ f^{-1}: B \to B$
    - Both f and f^{-1} are invertible, and the inverses are unique
    - $(f^{-1})^{-1} = f$

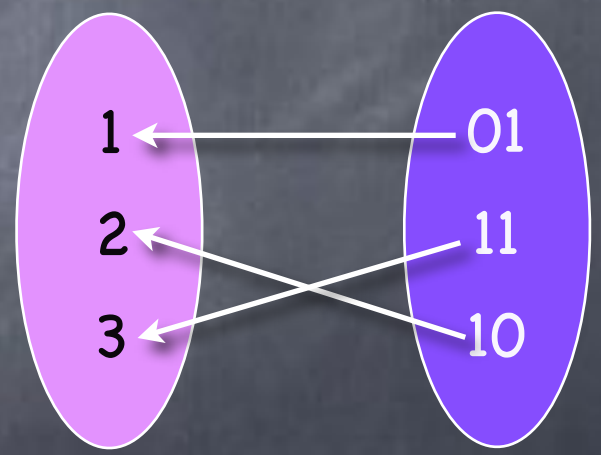


- If A, B <u>finite</u> sets and there is a bijection $f:A \to B$, then $|A|=|B|$

Bijections

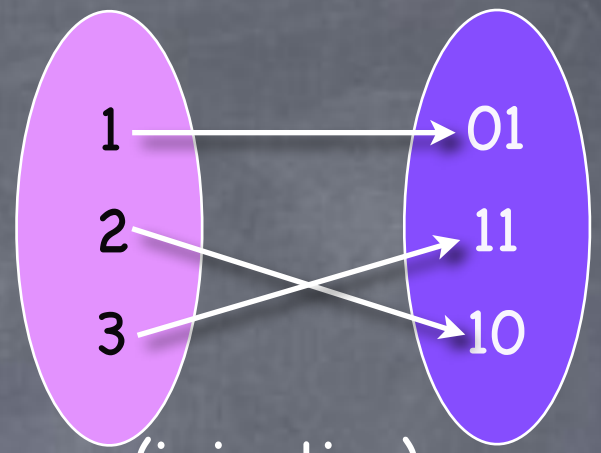


- They are totally awesome!
- Bijection: both onto (surjection) and one-to-one (injection)
  - Every row and every column has <u>exactly</u> one cell "on"
  - Every element in the co-domain has exactly one "pre-image"
    - If $f:A \to B$, $f^{-1}:B \to A$ such that $f^{-1} \circ f: A \to A$ and $f \circ f^{-1}: B \to B$
    - Both f and f^{-1} are invertible, and the inverses are unique
    - $(f^{-1})^{-1} = f$

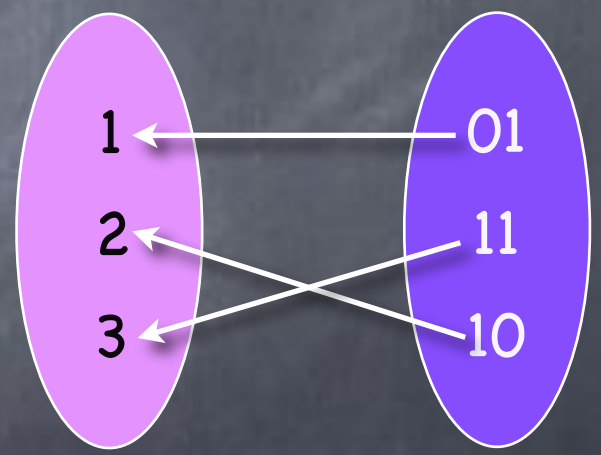


- If A, B <u>finite</u> sets and there is a bijection $f:A \to B$, then $|A|=|B|$
- If A, B finite and $|A|=|B|$, then there is a bijection from A to B

Bijections



- They are totally awesome!
- Bijection: both onto (surjection) and one-to-one (injection)
  - Every row and every column has <u>exactly</u> one cell "on"
  - Every element in the co-domain has exactly one "pre-image"
    - If $f:A \to B$, $f^{-1}:B \to A$ such that $f^{-1} \circ f: A \to A$ and $f \circ f^{-1}: B \to B$
    - Both f and f^{-1} are invertible, and the inverses are unique
    - $(f^{-1})^{-1} = f$



- If A, B <u>finite</u> sets and there is a bijection $f:A \to B$, then $|A|=|B|$
- If A, B finite and $|A|=|B|$, then there is a bijection from A to B
  - If $|A|=|B|=n$, there are $n!$ such bijections (coming up)

Bijections: Infinite sets

Bijections: Infinite sets

- e.g., f: $\mathbb{Z} \rightarrow \mathbb{Z}$ defined as f(x) = -x

Bijections: Infinite sets

- e.g., f: $\mathbb{Z} \rightarrow \mathbb{Z}$ defined as f(x) = -x
  - f is a bijection

Bijections: Infinite sets

- e.g., f: $\mathbb{Z} \rightarrow \mathbb{Z}$ defined as f(x) = -x
  - f is a bijection
- A set S is countably infinite if there is a bijection from S to $\mathbb{Z}$

Bijections: Infinite sets

- e.g., f: $\mathbb{Z} \rightarrow \mathbb{Z}$ defined as f(x) = -x
  - f is a bijection
- A set S is countably infinite if there is a bijection from S to $\mathbb{Z}$
- e.g., set of even integers $\mathbb{E}$. f: $\mathbb{E} \rightarrow \mathbb{Z}$ where f(x)=x/2, is a bijection

Bijections: Infinite sets

- e.g., f: $\mathbb{Z} \rightarrow \mathbb{Z}$ defined as f(x) = -x
  - f is a bijection
- A set S is countably infinite if there is a bijection from S to $\mathbb{Z}$
- e.g., set of even integers $\mathbb{E}$. f: $\mathbb{E} \rightarrow \mathbb{Z}$ where f(x)=x/2, is a bijection
  - "Two countably infinite sets are only as numerous as one"

Bijections: Infinite sets

- e.g., f: $\mathbb{Z} \rightarrow \mathbb{Z}$ defined as f(x) = -x
  - f is a bijection
- A set S is countably infinite if there is a bijection from S to $\mathbb{Z}$
- e.g., set of even integers $\mathbb{E}$. f: $\mathbb{E} \rightarrow \mathbb{Z}$ where f(x)=x/2, is a bijection
  - "Two countably infinite sets are only as numerous as one"
- e.g., there is a bijection f: $\mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$

Bijections: Infinite sets

- e.g., f: $\mathbb{Z} \rightarrow \mathbb{Z}$ defined as f(x) = -x
  - f is a bijection
- A set S is countably infinite if there is a bijection from S to $\mathbb{Z}$
- e.g., set of even integers $\mathbb{E}$. f: $\mathbb{E} \rightarrow \mathbb{Z}$ where f(x)=x/2, is a bijection
  - "Two countably infinite sets are only as numerous as one"
- e.g., there is a bijection f: $\mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$
  - "Countably infinitely many countably infinite sets are only as numerous as one"

Composition

Composition

- Composition "respects onto-ness"

Composition

- Composition "respects onto-ness"
  - If f and g are onto, $g \circ f$ is onto as well

Composition

- Composition "respects onto-ness"
  - If f and g are onto, $g \circ f$ is onto as well
- Composition "respects one-to-one-ness"

Composition

- Composition "respects onto-ness"
  - If f and g are onto, $g \circ f$ is onto as well
- Composition "respects one-to-one-ness"
  - If f and g are one-to-one, $g \circ f$ is one-to-one as well

Composition

- Composition "respects onto-ness"
  - If f and g are onto, $g \circ f$ is onto as well
- Composition "respects one-to-one-ness"
  - If f and g are one-to-one, $g \circ f$ is one-to-one as well
- Hence composition respects bijections

Composition

- Composition "respects onto-ness"
  - If f and g are onto, $g \circ f$ is onto as well
- Composition "respects one-to-one-ness"
  - If f and g are one-to-one, $g \circ f$ is one-to-one as well
- Hence composition respects bijections
  - If f and g are bijections, $g \circ f$ is a bijection as well

Permutation of a string

Permutation of a string

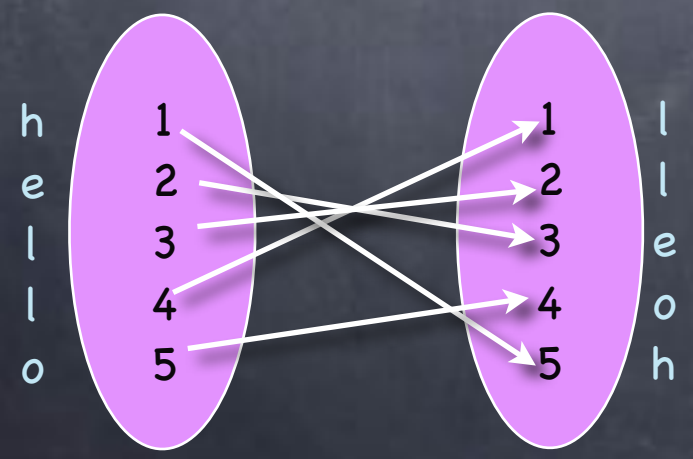
- More precisely, permutation of the positions (indices) of characters in a string

Permutation of a string

- More precisely, permutation of the positions (indices) of characters in a string
  - e.g., π_{53214}(hello) = lleoh

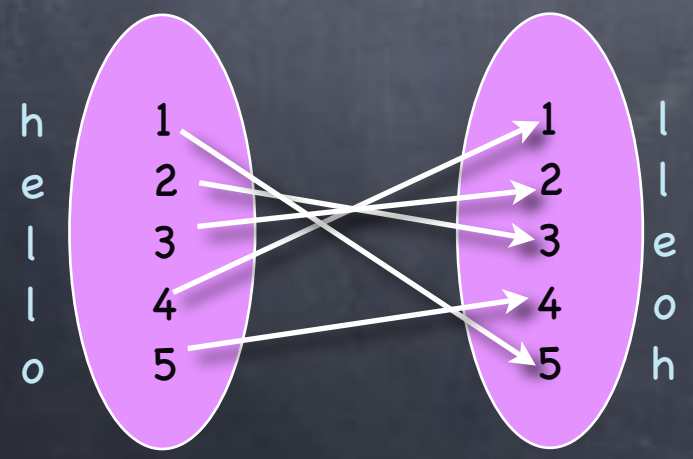
Permutation of a string

- More precisely, permutation of the positions (indices) of characters in a string
  - e.g., π_{53214}(hello) = lleoh



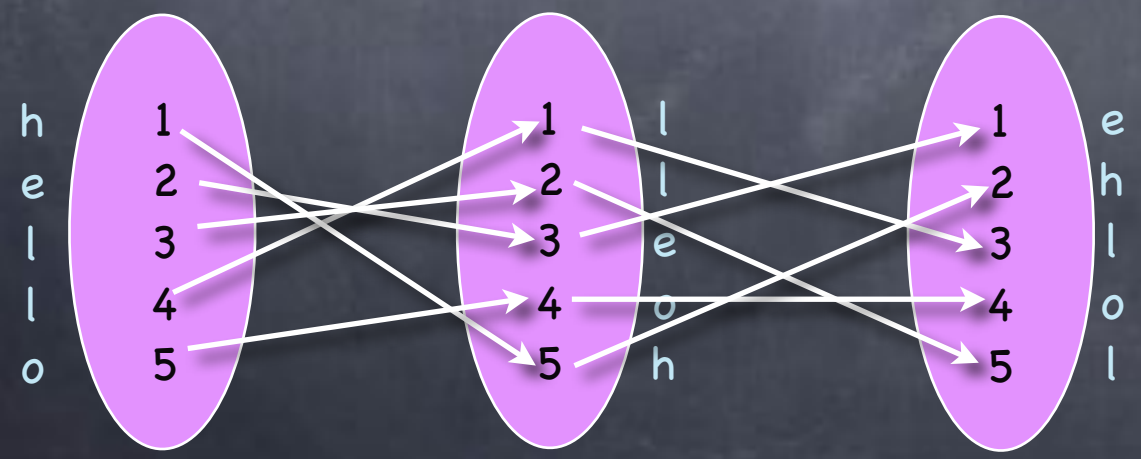
Permutation of a string

- More precisely, permutation of the positions (indices) of characters in a string
  - e.g., π_{53214}(hello) = lleoh
  - e.g., π_{35142}(lleoh) = ehlol



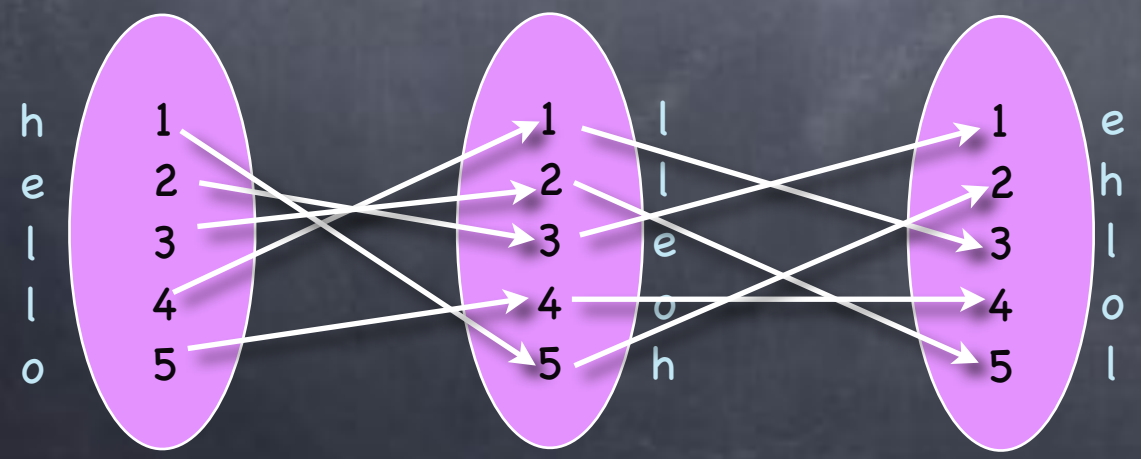
Permutation of a string

- More precisely, permutation of the positions (indices) of characters in a string
  - e.g., π_{53214}(hello) = lleoh
  - e.g., π_{35142}(lleoh) = ehlol



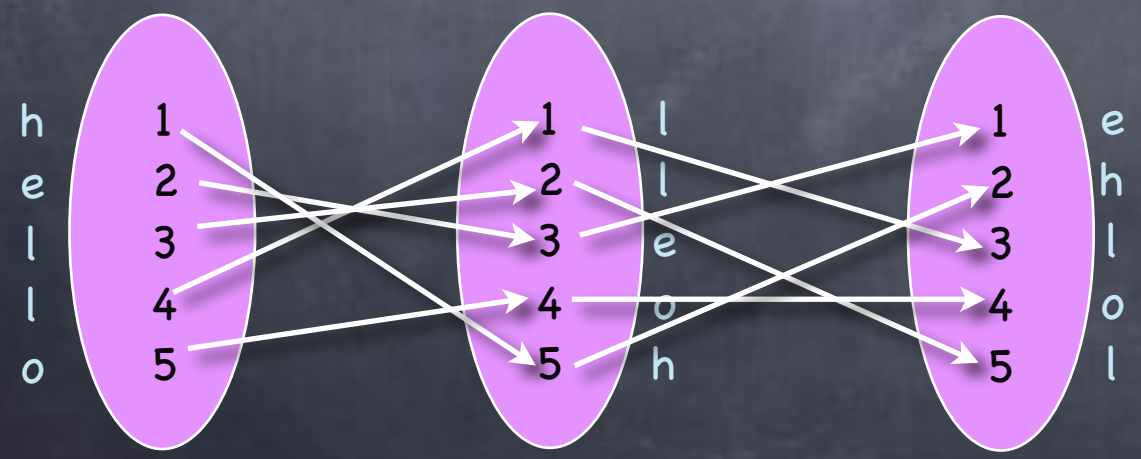
Permutation of a string

- More precisely, permutation of the positions (indices) of characters in a string
  - e.g., π_{53214}(hello) = lleoh
  - e.g., π_{35142}(lleoh) = ehlol
- As bijections from {1,2,3,4,5} to itself, permutations compose



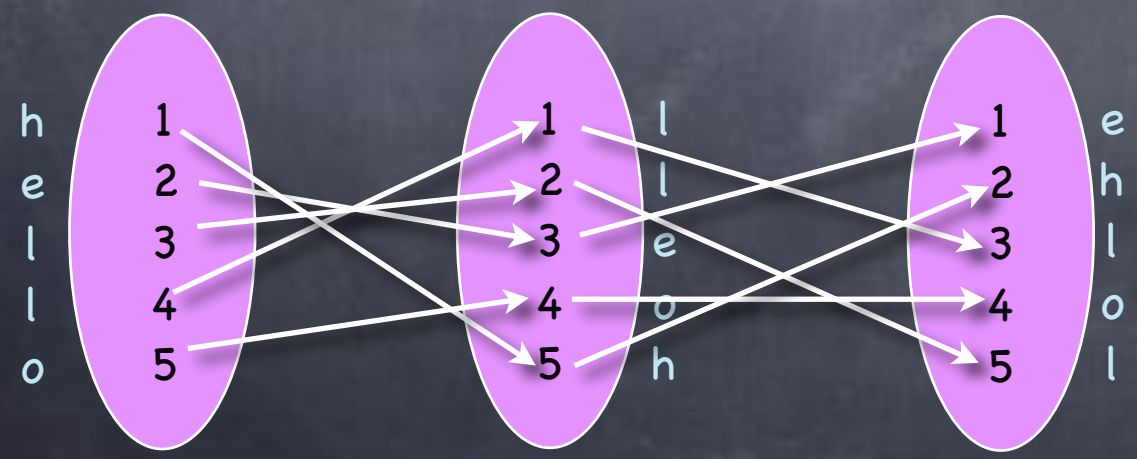
Permutation of a string

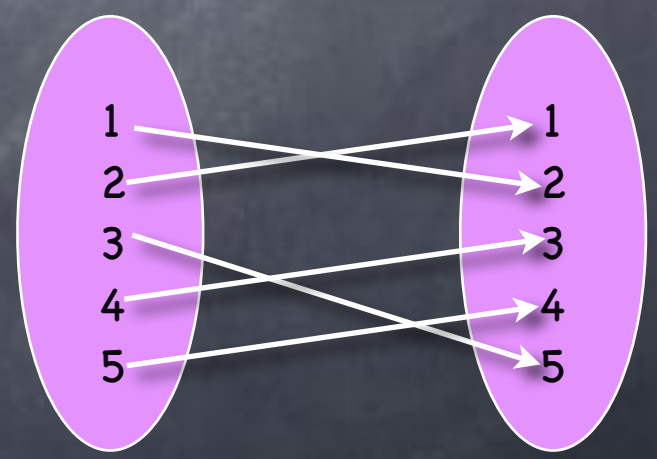
- More precisely, permutation of the positions (indices) of characters in a string
  - e.g., π_{53214}(hello) = lleoh
  - e.g., π_{35142}(lleoh) = ehlol
- As bijections from {1,2,3,4,5} to itself, permutations compose
  - e.g., $\pi_{35142} \circ \pi_{53214} = \pi_{21534}$



Permutation of a string

- More precisely, permutation of the positions (indices) of characters in a string
  - e.g., π_{53214}(hello) = lleoh
  - e.g., π_{35142}(lleoh) = ehlol
- As bijections from {1,2,3,4,5} to itself, permutations compose
  - e.g., $\pi_{35142} \circ \pi_{53214} = \pi_{21534}$

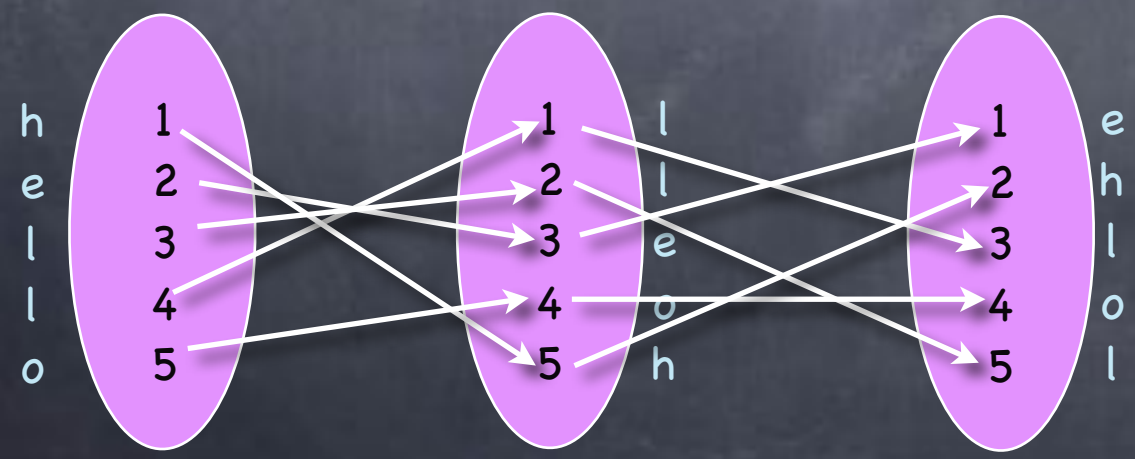


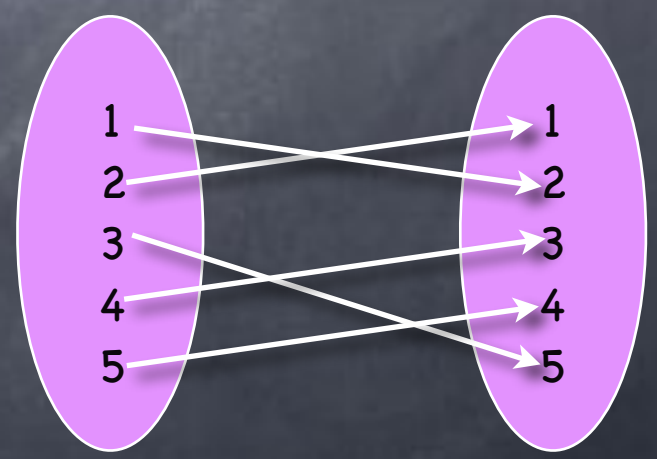


Permutation of a string

- More precisely, permutation of the positions (indices) of characters in a string
  - e.g., π_{53214}(hello) = lleoh
  - e.g., π_{35142}(lleoh) = ehlol
- As bijections from {1,2,3,4,5} to itself, permutations compose
  - e.g., $\pi_{35142} \circ \pi_{53214} = \pi_{21534}$

Could be defined as (special kinds of) bijections from alphabet^5 to itself too. Then too big to draw here.





Today

Today

- One-to-one functions

Today

- One-to-one functions
  - Strictly increasing/decreasing functions are one-to-one (but not vice versa)

Today

- One-to-one functions
  - Strictly increasing/decreasing functions are one-to-one (but not vice versa)
  - One-to-one functions are invertible

Today

- One-to-one functions
  - Strictly increasing/decreasing functions are one-to-one (but not vice versa)
  - One-to-one functions are invertible
- Bijections

Today

- One-to-one functions
  - Strictly increasing/decreasing functions are one-to-one (but not vice versa)
  - One-to-one functions are invertible
- Bijections
  - Uniquely invertible

Today

- One-to-one functions
  - Strictly increasing/decreasing functions are one-to-one (but not vice versa)
  - One-to-one functions are invertible
- Bijections
  - Uniquely invertible
  - If A, B finite and have a bijection, then $|A|=|B|$

Today

- One-to-one functions
  - Strictly increasing/decreasing functions are one-to-one (but not vice versa)
  - One-to-one functions are invertible
- Bijections
  - Uniquely invertible
  - If A, B finite and have a bijection, then $|A|=|B|$
  - Evens, rationals etc. are countably infinite (have bijections with $\mathbb{Z}$)

Today

- One-to-one functions
  - Strictly increasing/decreasing functions are one-to-one (but not vice versa)
  - One-to-one functions are invertible
- Bijections
  - Uniquely invertible
  - If A, B finite and have a bijection, then $|A|=|B|$
  - Evens, rationals etc. are countably infinite (have bijections with $\mathbb{Z}$)
- Composition: "respects" onto-ness and one-to-one-ness

Today

- One-to-one functions
  - Strictly increasing/decreasing functions are one-to-one (but not vice versa)
  - One-to-one functions are invertible
- Bijections
  - Uniquely invertible
  - If A, B finite and have a bijection, then |A|=|B|
  - Evens, rationals etc. are countably infinite (have bijections with $\mathbb{Z}$)
- Composition: "respects" onto-ness and one-to-one-ness
- Composing permutations