Functions

Lecture 9
A function maps each element in the domain to an element in the co-domain.

Example: \( f : \text{A} \times \text{I} \rightarrow \{0,1,2,3,4,5\} \)
Composition

Composition of two functions $f$ and $g$: $g \circ f$

$g \circ f(x) \equiv g(f(x))$
Composition

Composition of two functions $f$ and $g$: $g \circ f$

$g \circ f(x) = g(f(x))$

$g \circ f : \text{Domain}(f) \rightarrow \text{Co-domain}(g)$
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$g \circ f(x) \triangleq g(f(x))$

$g \circ f : \text{Domain}(f) \rightarrow \text{Co-domain}(g)$
Types of Functions

- **$f(x) = x$**
- **$f(x) = |x/5|$**
- **$f(x) = 5x$**
- **$f(x) = x^2$**
Types of Functions

- Function: every column has exactly one cell “on”
- Onto Function (surjection): Every row has at least one cell “on”
- One-to-One function (injection): Every row has at most one cell “on”
- Bijection: Every row has exactly one cell “on”
One-to-One Functions
One-to-One Functions

A function $f:A \rightarrow B$ is one-to-one if $\forall x, x' \in A \ f(x) = f(x') \rightarrow x = x'$
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e.g. $f : \mathbb{Z} \rightarrow \mathbb{Z}$ defined as $f(x) = 5x$ is one-to-one

In fact, any strictly increasing function is one-to-one
One-to-One Functions

A function \( f: A \rightarrow B \) is one-to-one if \( \forall x, x' \in A \ f(x) = f(x') \rightarrow x = x' \)

e.g. \( f : \mathbb{Z} \rightarrow \mathbb{Z} \) defined as \( f(x) = x^2 \) is not one-to-one

e.g. \( f : \mathbb{Z}^+ \rightarrow \mathbb{Z}^+ \) defined as \( f(x) = x^2 \) is one-to-one

\( f(x) = 5x \) is one-to-one

In fact, any strictly increasing function is one-to-one

And, any strictly decreasing function too is one-to-one
Question

Below $f: \mathbb{Z} \to \mathbb{Z}$. Pick the right choice.

A. $f$ s.t. $f(x)=\lfloor x/5 \rfloor$ is increasing, but not one-to-one
B. $f$ s.t. $f(x) = x + |x|$ is increasing and one-to-one
C. $f$ s.t. $f(x)=1$ is neither increasing nor one-to-one
D. $f$ is one-to-one $\rightarrow$ $f$ is increasing or decreasing
E. None of the above
Question

Below f: \( \mathbb{Z} \rightarrow \mathbb{Z} \). Pick the right choice.

A. f s.t. \( f(x) = \lfloor x/5 \rfloor \) is increasing, but not one-to-one
B. f s.t. \( f(x) = x + |x| \) is increasing and one-to-one
C. f s.t. \( f(x) = 1 \) is neither increasing nor one-to-one
D. f is one-to-one \( \rightarrow \) f is increasing or decreasing
E. None of the above
One-to-One Functions
One-to-One Functions

One-to-one functions are “invertible”
One-to-One Functions

One-to-one functions are "invertible"

\[ f \text{ invertible: } \exists g \text{ s.t. } g \circ f \equiv \text{Id} \]
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Suppose $f: A \to B$ is one-to-one
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One-to-One Functions

One-to-one functions are "invertible"

Suppose $f: A \rightarrow B$ is one-to-one

Let $g: B \rightarrow A$ be defined as follows:
- for $y \in \text{Im}(f)$, $g(y) = x$ s.t. $f(x) = y$ (well-defined)
- for $y \notin \text{Im}(f)$, $g(y) = 0$ (or some arbitrary element in $A$)

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\( \forall y \in \text{Im}(f) \exists ! x \in A \) \( f(x) = y \)

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[ETPT]
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Then \( g \circ f \equiv \text{Id}_A \), where \( \text{Id}_A: A \rightarrow A \) is the identity function over \( A \)

\[ \forall y \in \text{Im}(f) \exists ! x \in A \quad f(x) = y \]

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One-to-One Functions

- One-to-one functions are "invertible"
- Suppose \( f: A \to B \) is one-to-one
- Let \( g: B \to A \) be defined as follows:
  - for \( y \in \text{Im}(f) \), \( g(y) = x \) s.t. \( f(x) = y \) (well-defined)
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- Then \( g \circ f \equiv \text{Id}_A \), where \( \text{Id}_A: A \to A \) is the identity function over \( A \)
- If \( \text{Im}(f) \subset B \), then \( f \circ g \not\equiv \text{Id}_B \)

\( f \) invertible:
\[ \exists g \text{ s.t. } g \circ f \equiv \text{Id} \]

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\( g \) not one-to-one \( \rightarrow \) \( g \) not invertible
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Then $g \circ f \equiv \text{Id}_A$, where $\text{Id}_A: A \to A$ is the identity function over $A$

If $\text{Im}(f) \subseteq B$, then $f \circ g \not\equiv \text{Id}_B$

$g$ not one-to-one $\to$ $g$ not invertible

$f$ invertible: $\exists g$ s.t. $g \circ f \equiv \text{Id}$

$\forall y \in \text{Im}(f) \exists ! x \in A \ f(x) = y$

$g$ invertible $\to$ $g$ one-to-one

Suppose $h \circ g \equiv \text{Id}$

$g(x_1) = g(x_2) \to h(g(x_1)) = h(g(x_2)) \to x_1 = x_2$ (since $h \circ g \equiv \text{Id}$)

i.e., $\forall x_1, x_2 \ g(x_1) = g(x_2) \to x_1 = x_2$
Below $f: \mathbb{Z} \rightarrow \mathbb{Z}$ and $g: \mathbb{Z} \rightarrow \mathbb{Z}$ defined as follows:

$f(x) = 5x$, and $g(x) = \lfloor x/5 \rfloor$. Then:

A. $f \circ g$ is identity over $\mathbb{Z}$
B. $g \circ f$ is identity over $\mathbb{Z}$
C. $f \circ g$ is not well-defined
D. $g \circ f$ is not well-defined
E. None of the above
Question

Below $f: \mathbb{Z} \to \mathbb{Z}$ and $g: \mathbb{Z} \to \mathbb{Z}$ defined as follows:

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E. None of the above
Question

Below f: \( \mathbb{Z} \to \mathbb{Z} \) and g: \( \mathbb{Z} \to \mathbb{Z} \) defined as follows:

\[ f(x) = 5x, \quad \text{and} \quad g(x) = \lfloor x/5 \rfloor. \]

Then:

A. \( f \circ g \) is identity over \( \mathbb{Z} \)
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Bijections
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They are totally awesome!
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**Bijection:** both onto (surjection) and one-to-one (injection)
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- Every row and every column has exactly one cell “on”
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  - Every element in the co-domain has exactly one “pre-image”
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- If $f: A \rightarrow B$, $f^{-1}: B \rightarrow A$ such that
  $f^{-1} \circ f: A \rightarrow A$ and $f \circ f^{-1}: B \rightarrow B$
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If $f:A \rightarrow B$, $f^{-1}:B \rightarrow A$ such that

$\circ f: A \rightarrow A$ and $f \circ f^{-1}: B \rightarrow B$

* Both $f$ and $f^{-1}$ are invertible, and the inverses are unique
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- Both \( f \) and \( f^{-1} \) are invertible, and the inverses are unique
- \( (f^{-1})^{-1} = f \)
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- Every row and every column has **exactly** one cell “on”
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If $f: A \rightarrow B$, $f^{-1}: B \rightarrow A$ such that $f^{-1} \circ f: A \rightarrow A$ and $f \circ f^{-1}: B \rightarrow B$

Both $f$ and $f^{-1}$ are invertible, and the inverses are unique

$(f^{-1})^{-1} = f$

If $A$, $B$ **finite** sets and there is a bijection $f: A \rightarrow B$, then $|A| = |B|$
Bijections

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**Bijection**: both onto (surjection) and one-to-one (injection)

- Every row and every column has exactly one cell “on”
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If \( f : A \to B \), \( f^{-1} : B \to A \) such that

\[ f^{-1} \circ f : A \to A \] and \( f \circ f^{-1} : B \to B \)

Both \( f \) and \( f^{-1} \) are invertible, and the inverses are unique

\( (f^{-1})^{-1} = f \)

If \( A, B \) finite sets and there is a bijection \( f : A \to B \), then \( |A| = |B| \)

If \( A, B \) finite and \( |A| = |B| \), then there is a bijection from \( A \) to \( B \)
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- Every row and every column has **exactly** one cell “on”
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If \( f: A \rightarrow B \), \( f^{-1}: B \rightarrow A \) such that
  \( f^{-1} \circ f: A \rightarrow A \) and \( f \circ f^{-1}: B \rightarrow B \)

Both \( f \) and \( f^{-1} \) are invertible, and the inverses are unique

\( (f^{-1})^{-1} = f \)

If \( A, B \) **finite** sets and there is a bijection \( f: A \rightarrow B \), then \( |A| = |B| \)

If \( A, B \) finite and \( |A| = |B| \), then there is a bijection from \( A \) to \( B \)

If \( |A| = |B| = n \), there are \( n! \) such bijections (coming up)
Bijections: Infinite sets
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- e.g., \( f: \mathbb{Z} \rightarrow \mathbb{Z} \) defined as \( f(x) = -x \)
Bijections: Infinite sets

- e.g., $f: \mathbb{Z} \rightarrow \mathbb{Z}$ defined as $f(x) = -x$

- $f$ is a bijection
Bijections: Infinite sets

- e.g., \( f: \mathbb{Z} \rightarrow \mathbb{Z} \) defined as \( f(x) = -x \)
- \( f \) is a bijection
- A set \( S \) is **countably infinite** if there is a bijection from \( S \) to \( \mathbb{Z} \)
Bijections: Infinite sets

- e.g., $f: \mathbb{Z} \to \mathbb{Z}$ defined as $f(x) = -x$
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- A set $S$ is **countably infinite** if there is a bijection from $S$ to $\mathbb{Z}$

- e.g., set of even integers $\mathbb{E}$. $f: \mathbb{E} \to \mathbb{Z}$ where $f(x) = x/2$, is a bijection
Bijections: Infinite sets

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  "Two countably infinite sets are only as numerous as one"
Bijections: Infinite sets

- e.g., $f: \mathbb{Z} \to \mathbb{Z}$ defined as $f(x) = -x$
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- A set $S$ is **countably infinite** if there is a bijection from $S$ to $\mathbb{Z}$

- e.g., set of even integers $\mathbb{E}$. $f: \mathbb{E} \to \mathbb{Z}$ where $f(x) = x/2$, is a bijection
  - “Two countably infinite sets are only as numerous as one”

- e.g., there is a bijection $f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$
Bijections: Infinite sets

- e.g., $f: \mathbb{Z} \to \mathbb{Z}$ defined as $f(x) = -x$
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- A set $S$ is **countably infinite** if there is a bijection from $S$ to $\mathbb{Z}$
  - e.g., set of even integers $E$. $f: E \to \mathbb{Z}$ where $f(x) = x/2$, is a bijection

  - “Two countably infinite sets are only as numerous as one”

- e.g., there is a bijection $f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$

  - “Countably infinitely many countably infinite sets are only as numerous as one”
Composition
Composition

Composition “respects onto-ness”
Composition

Composition “respects onto-ness”

If \( f \) and \( g \) are onto, \( g \circ f \) is onto as well
Composition

Composition “respects onto-ness”

If $f$ and $g$ are onto, $g \circ f$ is onto as well

Composition “respects one-to-one-ness”
Composition

Composition “respects onto-ness”

If f and g are onto, \( g \circ f \) is onto as well

Composition “respects one-to-one-ness”

If f and g are one-to-one, \( g \circ f \) is one-to-one as well
Composition

Composition “respects onto-ness”

- If $f$ and $g$ are onto, $g \circ f$ is onto as well

Composition “respects one-to-one-ness”

- If $f$ and $g$ are one-to-one, $g \circ f$ is one-to-one as well

Hence composition respects bijections
Composition

Composition “respects onto-ness”
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Composition “respects one-to-one-ness”
- If \( f \) and \( g \) are one-to-one, \( g \circ f \) is one-to-one as well

Hence composition respects bijections
- If \( f \) and \( g \) are bijections, \( g \circ f \) is a bijection as well
Permutation of a string
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More precisely, permutation of the positions (indices) of characters in a string
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e.g., $\pi_{53214}(\text{hello}) = \text{lleoh}$
Permutation of a string

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\( \pi_{53214}(\text{hello}) = \text{lleoh} \)
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- e.g., $\pi_{35142}(\text{lleoh}) = \text{ehlol}$
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As bijections from $\{1,2,3,4,5\}$ to itself, permutations compose
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As bijections from $\{1,2,3,4,5\}$ to itself, permutations compose

- e.g., $\pi_{35142} \circ \pi_{53214} = \pi_{21534}$
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Today

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Today

- One-to-one functions
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  - One-to-one functions are invertible
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- Composing permutations