Homework 1

Discrete Structures
CS 173 [B] : Fall 2012

Released: Tue Sep 4
Due: Wed Sep 12, 3:00 PM

0. **Take our survey!** [1 point]

   Please take the survey at `https://illinois.edu/fb/sec/1257507`.

1. **Logic in plain English.** [25 points]

   Consider the following propositions p and q.

   p: There will be a snow storm today.

   q: Today's classes will be canceled.

   (a) Express the following statements symbolically, in terms of the propositions p, q, $\neg p$, and $\neg q$, and operators $\wedge$, $\vee$ and $\rightarrow$.

      i. Neither will there be a snow storm today, nor will today's classes be canceled.
      ii. If there will be no snow storm today, then today's classes won't be canceled.
      iii. If today's classes will be canceled, then there will be a snow storm today.
      iv. Though there will be a snow storm today, today's classes won't be canceled.

      [*In this exercise, you would be ignoring any suggestion of order of events or "causality," while interpreting the plain English statements. Only the "correlation" between the two events is captured by the propositions you construct.*]

   (b) Find the largest set of statements above that can all be simultaneously true.

2. **Simplifying formulas.** [20 points]

   Every formula in two variables corresponds to a binary operator. Identify the operator in the following cases, and write down an equivalent expression.

   (Thus your answer should be one of the 16 possibilities: T, F, p, q, $\neg p$, $\neg q$, $p \oplus q$, $p \leftrightarrow q$, $p \wedge q$, $p \vee q$, $p \uparrow q$, $p \downarrow q$, $p \rightarrow q$, $q \rightarrow p$, $p \not\rightarrow q$ and $q \not\rightarrow p$.)

   You may prepare a truth table for each formula to help with the task. You could also employ the distributive property, De Morgan's law and other equivalences from the lecture.

   (a) $(p \rightarrow q) \wedge \neg q$
   (b) $p \vee \neg(q \rightarrow p)$
   (c) $(p \wedge q) \rightarrow q$
   (d) $(p \wedge q) \leftrightarrow q$
   (e) $(p \leftrightarrow q) \leftrightarrow ((p \wedge q) \vee (\neg p \wedge \neg q) \vee (p \wedge \neg p))$

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3. Is the following argument valid? Explain. [9 points]

   - If my house is less than a mile away from my office, I walk to work.
   - I walk to work.
   - Therefore, my house is less than a mile away from my office.

   [*Hint: Denote the proposition "my house is less than a mile away from my office" by p, and the proposition "I walk to work" by q. Then write down the proposition that corresponds to the AND of first two items above. Does it "imply" the last one?*]

4. **Functional Completeness.** [25 points]

   A set of operators is *functionally complete* if all logical operations can be expressed as formulas that use only these operators. In other words, all possible truth tables (over any number of propositions) can be produced by formulas that use only these operators. In the following, you may use the fact that the set $\{\vee, \wedge, \neg\}$ is functionally complete.

   (a) Is the set $\{\neg, \vee\}$ functionally complete? Explain why or why not.
   [*Hint: Can you express $p \wedge q$ using only $\neg$ and $\vee$?*]

   (b) Recall the operator NAND, or $\uparrow$, which has the following truth table:

p	q	$p \uparrow q$
F	F	T
F	T	T
T	F	T
T	T	F

   i. Express $\neg p$ as a formula that involves only the operator $\uparrow$.
   ii. Express $p \vee q$ using only the operator $\uparrow$.

   (c) Argue that $\{\uparrow\}$ is functionally complete. You may use answers to previous problems in your explanation.

5. **Predicate logic. In plain English.** [20 points]

   Suppose we define the following predicates:

   $H(x)$ tells if x is a human (i.e., $H(x) = T$ if and only if x is human).

   $R(x)$ tells if x is a "reptilian."

   $S(x)$ tells if x is from outer space.

   $C(x)$ tells if x is a cow.

   $A(x, y)$ tells if x abducts y.

   $I(x, y)$ tells if x is more intelligent than y.

   Translate each of the following into English.

   (a) $\exists x\ H(x) \wedge R(x)$
   (b) $\forall y\ R(y) \rightarrow S(y)$
   (c) $\exists x\ S(x) \wedge (\exists y\ C(y) \wedge A(x, y))$
   (d) $\forall x \exists y\ C(x) \rightarrow (H(y) \wedge I(x, y))$