1. **Set Operations**

Suppose you were given the following sets:

\[
\begin{align*}
A &= \{\text{Piano, (Violin, Viola, Cello), Guitar}\} \\
B &= \{(\text{Flute, Piccolo}), \text{Cymbals}\} \\
C &= \{\text{Piano, Flute}\} \\
D &= \{(\text{Violin, Viola, Cello}), (\text{Flute, Piccolo})\}
\end{align*}
\]

List the elements of the set or find the values for the following expressions:

(a) \(|A|\)

(b) \(A \cup D\)

(c) \(A \cap C\)

(d) \(B \cap C\)

(e) \(A - (B - C)\)

(f) \((B \cap D) \times C\)

(g) \(A \times \emptyset\)

(h) \(C \times \{\emptyset\}\)
2. **Counting with sets**
In our role-playing game, an evil character may be an elf or a troll, it may be red, green, brown, or black, and it may have scales or hair. A good character may be an elf or a human or a lion, it may be green, brown, or blue, and it has hair or fur. How many different character types do we have?
Suppose we have a 26 character alphabet. How many 6-letter strings start with PRE or end with TH?

3. **Euclidean algorithm**
Trace the execution of the Euclidean algorithm for computing GCD on the inputs $a = 837$ and $b = 2015$. That is, give a table showing the values of the main variables $(x, y, r)$ for each pass through the loop. Explicitly indicate what the output value is.

4. **Direct Proof Using Congruence mod k**
In the book, you will find several equivalent ways to define congruence mod $k$. For this problem, use the following definition: for any integers $x$ and $y$ and any positive integer $m$, $x \equiv y \pmod{m}$ if there is an integer $k$ such that $x = y + km$.
Using this definition prove that, for all integers $a, b, c, p, q$ where $p$ and $q$ are positive, if $a \equiv b \pmod{p}$ and $c \equiv b \pmod{q}$ and $q|p$, then $a - 2c \equiv (-b) \pmod{q}$.

5. **Equivalence classes**
Let $A = \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} - \{(0, 0)\}$, i.e. pairs of non-negative reals in which no more than one of the two numbers is zero.
Consider the equivalence relation $\sim$ on $A$ defined by

$$(x, y) \sim (p, q) \iff (xy)(p + q) = (pq)(x + y)$$

(a) List four elements of $[(3, 1)]$. Hint: what equation do you get if you set $(x, y)$ to $(3, 1)$ and $q = 2p$?

(b) Give two other distinct equivalence classes that are not equal to $[(3, 1)]$.

(c) Describe the members of $[(0, 4)]$. 

6. Relation properties

\[ \sim \text{ is the relation on } \mathbb{R} \text{ such that } x \sim y \text{ if and only if } xy = 1 \]

7. Proofs on Relations

(a) Define a relation \( \sim \) on the set of all functions from \( \mathbb{R} \) to \( \mathbb{R} \) by the rule \( f \sim g \) if and only if \( \exists k \in \mathbb{R} \) such that \( f(x) = g(x) \) for every \( x \geq k \). Prove that \( \sim \) is an equivalence relation.