# CS125 : Introduction to Computer Science 

Lecture Notes \#11<br>Procedural Composition and Abstraction

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## Lecture 11 : Procedural Composition and Abstraction

Solving a problem...with help

Imagine I have a math problem I would like to solve. We'll use an arithmetic arithmetic problem, just to keep things simple, even if it will seem a little silly.

So, let's imagine that I am faced with calculating the following value, but don't know how, i.e. perhaps I have no idea how to do certain arithmetic operations. Specifically, we will assume that I know how to do addition, but that I do not know how to do subtraction, multiplication, or division.

```
(((2 + 5)*((3 + 10) - 1)) / 6) + 7
```

Since I can perform addition, I can evaluate the values of the first two additions in the problem, thus giving me:

```
I have: (((2 + 5) * ((3 + 10) - 1)) / 6) + 7
```

[I calculate]

I have: ((7 * (13-1)) / 6) + 7
but I cannot perform the third addition, since I first need to know the value of the expression

$$
((7 *(13-1)) / 6)
$$

since that value is one of the operands of the addition. Since I do not know how to perform subtraction, multiplication, or division, I cannot evaluate that expression, and thus cannot obtain the first operand of the remaining addition.

So, now it's time to call on friends for help. Let's imagine I have a friend, named FriendA, who knows how to do subtraction. Let's send FriendA the subtraction part of our problem. The chain of events so far is as follows:

I have: $(((2+5) *((3+10)-1)) / 6)+7$
[I calculate]
I have: ((7 * (13-1)) / 6) + 7

```
    --------> I send FriendA: (13 - 1)
```

    FriendA has: (13-1)
    FriendA isn't seeing the entire problem, but FriendA doesn't need to! All FriendA cares about, is that I have sent a subtraction problem. FriendA doesn't need to know why I need the subtraction done, in order to do the subtraction. FriendA can simply calculate the result, and send it back to me. Likewise, I don't need to know how FriendA does the work - I just need to send the problem to FriendA, and receive the solution in return.

```
I have: (((2 + 5) * ((3 + 10) - 1)) / 6) + 7
[I calculate]
I have: ((7 * (13 - 1)) / 6) + 7
    -------> I send FriendA: (13 - 1)
    FriendA has: (13 - 1)
    [FriendA calculates]
    FriendA has: 12
    <------- FriendA sends back 12
```

and now, thanks to FriendA, I can replace the expression (13 - 1) with the expression 12:
I have: $(((2+5) *((3+10)-1)) / 6)+7$
[I calculate]
I have: $((7 *(13-1)) / 6)+7$
-------> I send FriendA: (13 - 1)
FriendA has: (13-1)
[FriendA calculates]
FriendA has: 12
<------- FriendA sends back 12
I have: $((7 \times 12) / 6)+7$

Now, we've still got a multiplication and a division to deal with. I have no idea how to do those (or so we are pretending), but I have a friend, FriendB, who claims to know how to do multiplication and division. So, I send FriendB that part of the expression.

```
I have: (((2 + 5) * ((3 + 10) - 1)) / 6) + 7
[I calculate]
I have: ((7 * (13 - 1)) / 6) + 7
    -------> I send FriendA: (13 - 1)
    FriendA has: (13 - 1)
    [FriendA calculates]
    FriendA has: 12
    <------- FriendA sends back 12
I have: ((7 * 12) / 6) + 7
    -------> I send FriendB: ((7 * 12 / 6)
    FriendB has: ((7 * 12) / 6)
```

Now, perhaps FriendB knows how to do multiplication, but not division. FriendB lied to us!! But fortunately for FriendB, we don't need to find that out. This is because we've asked FriendB to perform a task for us, but we don't care how that task gets done. All we want is the correct answer returned to us. So, FriendB could likewise ask for help, completely unbeknownst to us.

First of all, FriendB performs the multiplication, since FriendB knows how to do multiplication:

I have: $(((2+5) *((3+10)-1)) / 6)+7$
[I calculate]

I have: $((7 *(13-1)) / 6)+7$
-------> I send FriendA: (13 - 1)

FriendA has: (13 - 1)
[FriendA calculates]

FriendA has: 12
<------- FriendA sends back 12

I have: $((7 * 12) / 6)+7$
-------> I send FriendB: ( $(7 \times 12 / 6)$

FriendB has: ((7*12) / 6)
[FriendB calculates]

FriendB has: (84 / 6)

Next, since FriendB doesn't know how to do division, FriendB asks another friend, FriendC, for help:

```
I have: (((2 + 5) * ((3 + 10) - 1)) / 6) + 7
[I calculate]
I have: ((7 * (13 - 1)) / 6) + 7
    -------> I send FriendA: (13 - 1)
    FriendA has: (13 - 1)
    [FriendA calculates]
    FriendA has: 12
    <------- FriendA sends back 12
I have: ((7 * 12) / 6) + 7
    -------> I send FriendB: ((7 * 12 / 6)
    FriendB has: ((7 * 12) / 6)
    [FriendB calculates]
    FriendB has: (84 / 6)
    ------> FriendB sends FriendC: (84 / 6)
    FriendC has: (84 / 6)
```

Note a few things here. Firstly, just as FriendA never saw the entire problem, likewise, FriendB does not get to see the entire problem, and FriendC sees even less. That does not matter! All I need from FriendB is to solve the part I sent to FriendB, just like all I needed from FriendA was to solve the part I sent to FriendA. If FriendA is asked to solve (13-1), FriendA does not need to know why I want that answer - FriendA just needs to give me the answer I am asking for. And likewise, FriendB does not need to know why I need the answer to ( ( $7 * 12$ ) / 6) - FriendB just needs to give me the answer I am asking for. And likewise, FriendC doesn't need to know why FriendB wants the answer to (84/6) - FriendC just needs to give FriendB the answer that FriendB is looking for.

But likewise, this "secrecy" works in reverse as well! When I sent FriendA part of the original problem, I didn't care how FriendA got the answer. I just wanted the answer. So maybe FriendA did all the work, or maybe FriendA asked someone else for help. Eitehr way, I don't need to care. I just need to ask FriendA for an answer to a problem, and then wait to receive that answer. And similarly, when I ask FriendB for help, I don't care how FriendB got the answer, as long as FriendB sends the answer back to me. Whether FriendB can do all the work, or has to ask someone (such as FriendC) for help, does not concern me. All I care about is that I sent a problem to FriendB and that I get the answer back.

So, finishing our example, FriendC is able to return the answer 14 to FriendB:

I have: $(((2+5) *((3+10)-1)) / 6)+7$
[I calculate]

I have: $((7 *(13-1)) / 6)+7$
-------> I send FriendA: (13 - 1)

FriendA has: (13 - 1)
[FriendA calculates]

FriendA has: 12
<------- FriendA sends back 12

I have: $((7 * 12) / 6)+7$
-------> I send FriendB: ( $(7 \times 12 / 6)$

FriendB has: ((7*12) / 6)
[FriendB calculates]

FriendB has: (84 / 6)
------> FriendB sends FriendC: (84 / 6)

FriendC has: (84 / 6)
[FriendC calculates]

FriendC has: 14
<------ FriendC sends back 14

FriendB has: 14
and now FriendB can return that result back to me:

I have: $(((2+5) *((3+10)-1)) / 6)+7$
[I calculate]

I have: $((7 *(13-1)) / 6)+7$
-------> I send FriendA: (13 - 1)

FriendA has: (13 - 1)
[FriendA calculates]

FriendA has: 12
<------- FriendA sends back 12

I have: $((7 * 12) / 6)+7$
-------> I send FriendB: ( $(7 * 12 / 6)$

FriendB has: $((7 * 12) / 6)$
[FriendB calculates]

FriendB has: (84 / 6)
------> FriendB sends FriendC: (84 / 6)

FriendC has: (84 / 6)
[FriendC calculates]

FriendC has: 14
<------ FriendC sends back 14

FriendB has: 14
<------ FriendB sends back 14

I have: $14+7$
at which point I can finally complete the last addition and finish the evaluation of the original expression:

I have: $(((2+5) *((3+10)-1)) / 6)+7$
[I calculate]

I have: $((7 *(13-1)) / 6)+7$
-------> I send FriendA: (13 - 1)

FriendA has: (13 - 1)
[FriendA calculates]

FriendA has: 12
<------- FriendA sends back 12

I have: $((7 * 12) / 6)+7$
-------> I send FriendB: $((7 * 12 / 6)$

FriendB has: ((7*12) / 6)
[FriendB calculates]

FriendB has: (84 / 6)
------> FriendB sends FriendC: (84 / 6)

FriendC has: (84 / 6)
[FriendC calculates]

FriendC has: 14
<------ FriendC sends back 14

FriendB has: 14
<------ FriendB sends back 14

I have: $14+7$
[I calculate]

I have: 21

This concept we are illustrating, is known as procedural abstraction. It is a kind of abstraction in which you hide away the details of how a particular task is done. Something/someone else worries about how the details of the work are done, and all you need to do is ask that something or someone, "please do this work for me" and then wait for the work to get done (and if you are waiting for an answer of some kind, wait for that something or someone to return your answer).

Because of the idea of procedural abstraction, it is not necessary for you to worry about every single detail of a computation. In the example above, I didn't worry about any of the details of the subtraction, multiplication, or division - for each of those tasks, I sent those problems to someone else and awaited my answer. And even though I did a little bit of the work in our example above (the addition work), there were details that I didn't worry about - I sent sections of the problem to other friends and waited for their answers, without worrying about how those friends were doing the work. In this way, I was able to get the value of a complex arithmetic expression, without knowing any arithmetic other than addition.

You have already used this same idea in your own MPs. When you have printed something to the screen, you sent the value you wanted to print, to a System.out.println or System.out.print statement. You didn't worry about the specifics of how that System.out.println or System.out.print statement did its work - you just typed the statement and trusted that the statement would do what it was supposed to do. Similarly, you didn't need to type a lot of code in order to input a value. You just typed Keyboard.readInt() or Keyboard.readDouble() or Keyboard.readChar(), and trusted that that expression would work as we said it would, and would do the work of obtaining your input value and sending it back to you, even if you have no idea how that task is actually working behind the scenes.

Again, this is how abstraction helps us. Since you are freed from having to worry about every little detail, you have more time to focus on the "big picture" and develop that further.

We can also relate this example to how the machine implements procedural abstraction. To do this, let's go through the example again, but let's pretend that there is a common sheet of scratch paper that FriendA, FriendB, FriendC, and I are using:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

I receive the initial problem, and start working in column 1 of the scratch paper:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :---: | :---: | :---: | :---: |
| $\left(\left((2+5)^{*}((3+10)-1)\right) / 6\right)+7$ |  |  |  |
|  |  |  |  |
|  |  |  |  |

After I perform the two additions I can do, I have reduced the problem further:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $(((2+5) *((3+10)-1)) / 6)+7$ |  |  |  |
| $\ldots$ some work done... |  |  |  |
| $((7 *(13-1)) / 6)+7$ |  |  |  |
|  |  |  |  |
|  |  |  |  |

At this point, I needed to ask FriendA for help. So I will write the problem I need FriendA to solve, at the top of column 2 - thus showing FriendA where that scratch work should be done and then I will pass the scratch paper to FriendA.

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $(((2+5) *((3+10)-1)) / 6)+7$ | $(13-1)$ |  |  |
| $\ldots$ some work done... |  |  |  |
| $((7 *(13-1)) / 6)+7$ |  |  |  |
|  |  |  |  |
|  |  |  |  |

Note now that I cannot do anymore work for the time being. This is because FriendA has the scratch paper! There wasn't any other arithmetic I could do anyway, but even if there was, I am forced to sit around waiting for FriendA to finish, since FriendA has the scratch paper and I do not. This is similar to how, in your programs, when you call Keyboard.readInt() or System.out.println(),
your program sits there and waits for the Keyboard.readInt() call or the System.out.println() call to finish, before continuing. Likewise, I cannot continue until FriendA finishes and gives me back the scratch paper.

Likewise, we will assume FriendA does NOT work in column 1, and we will further assume that FriendA does not even look at the work I have written in column 1. We have told FriendA to work in column 2, and so that is where FriendA works. And what FriendA does is to perform whatever scratch work is needed to figure out that $(13-1)$ is 12 :

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $\left(\left((2+5)^{*}((3+10)-1)\right) / 6\right)+7$ | $(13-1)$ |  |  |
| $\ldots$ some work done... | $\ldots$ some work done... |  |  |
| $\left(\left(7^{*}(13-1)\right) / 6\right)+7$ | 12 |  |  |
|  |  |  |  |
|  |  |  |  |

Now that FriendA has completed the calculation, FriendA must do three things. First, FriendA must notify us of the final answer, by writing the answer in our column:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $\left(\left((2+5)^{*}((3+10)-1)\right) / 6\right)+7$ | $(13-1)$ |  |  |
| $\ldots((7 *(13-1)) / 6)+7$ | $\ldots$ some work done... |  |  |
| $12<---$ answer from FriendA | 12 |  |  |
|  |  |  |  |
|  |  |  |  |

Second, FriendA should erase column 2. After all, I might want to give this scratch paper to someone else later on. If FriendA is not using that area of the scratch paper anymore, than erasing all the remarks there, so that the area is free for someone else to use, seems to be the right thing
to do. To leave all the now-useless remarks there, taking up space, would be wasteful:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $\left(\left((2+5)^{*}((3+10)-1)\right) / 6\right)+7$ |  |  |  |
| $\ldots$ some work done... |  |  |  |
| $\left(\left(7^{*}(13-1)\right) / 6\right)+7$ |  |  |  |
| $12<---$ answer from FriendA |  |  |  |
|  |  |  |  |

Finally, FriendA should give the scratch paper back to me! Now, finally, I can do some work again, just like when Keyboard.readInt () finishes its job and reports the input integer to your program. In that situation, your program can then finally resume work where it had paused while waiting for the Keyboard.readInt () call to finish, and likewise, now that I have the scratch paper again, I can make use of the value that FriendA gave me, to reduce my expression further. This work is of course done in column 1, since that's where I do my work:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $(((2+5) *((3+10)-1)) / 6)+7$ |  |  |  |
| $\ldots$ some work done... |  |  |  |
| $\left(\left(7^{*}(13-1)\right) / 6\right)+7$ |  |  |  |
| $12<---$ answer from FriendA |  |  |  |
| $\left(\left(7^{*} 12\right) / 6\right)+7$ |  |  |  |
|  |  |  |  |

Note that not only did FriendA not mess with anything in column 1 (aside from writing in the result value in the end, and noting it was the result value), but in addition, I cannot move to column 2 and mess with any of the work FriendA did. First of all, that work has been erased, and is thus completely gone; secondly, I am supposed to stay in column 1 anyway.

Now, the next step was to pass the multiplication and division to FriendB. I will again use column 2 for this, since it is the column right next to where I am working, and it is (and should be) free and available for this purpose. So, I will write the problem I am handing to FriendB, at the top of column 2, and then will hand the scratch paper to FriendB. As with FriendA, when FriendB is working on the problem, I do not have the scratch paper, and so I cannot do anything. And similarly, as with FriendA, FriendB will know to only do work in the designated column, and
will know to NOT mess with my work in column 1.

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $\left(\left((2+5)^{*}((3+10)-1)\right) / 6\right)+7$ | $\left(\left(7^{*} 12\right) / 6\right)$ |  |  |
| $\ldots$ some work done... |  |  |  |
| $((7 *(13-1)) / 6)+7$ |  |  |  |
| $12<---$ answer from FriendA |  |  |  |
| $\left(\left(7^{*} 12\right) / 6\right)+7$ |  |  |  |
|  |  |  |  |

So what we see here is that our columns are available for general use. Just because FriendA was using column 2 for scratch earlier, doesn't mean we can't use it now for FriendB. After all, FriendA was finished with the subtraction work, and with column 2, and thus erased everything from column 2! So if column 2 is sitting there unused, why shouldn't FriendB go ahead and use it? There's no reason not to! If I am working in column 1, then when it's time to have a friend help me out, I just have that friend work in the open column to my right - column 2, in this case. Regardless of whether that scratch work is scratch work for FriendA who does subtraction, or FriendB who does multiplication and division, if column 2 is the next open column, then when I pass work off to a friend, that's the column my friend will use.

FriendB will then do some multiplication work in an attempt to reduce the expression to something simpler:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $(((2+5) *((3+10)-1)) / 6)+7$ | $\left(\left(7^{*} 12\right) / 6\right)$ |  |  |
| $\ldots$ some work done... | $\ldots$ some work done... |  |  |
| $((7 *(13-1)) / 6)+7$ | $(84 / 6)$ |  |  |
| $12<---$ answer from FriendA |  |  |  |
| $\left(\left(7^{*} 12\right) / 6\right)+7$ |  |  |  |
|  |  |  |  |

And now FriendB is stuck. This is where FriendB had to turn for help to FriendC, and that is exactly what happens here. FriendB will do the same thing I did - write the problem for which help is needed, at the top of the next open column, and then pass the scratch paper to someone
who will work on that problem - in this case, FriendC:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $\left(\left((2+5)^{*}((3+10)-1)\right) / 6\right)+7$ | $\left(\left(7^{*} 12\right) / 6\right)$ | $(84 / 6)$ |  |
| $\ldots$ some work done... | $\ldots$ some work done... |  |  |
| $\left(\left(7^{*}(13-1)\right) / 6\right)+7$ | $(84 / 6)$ |  |  |
| $12<--$ answer from FriendA |  |  |  |
| $\left(\left(7^{*} 12\right) / 6\right)+7$ |  |  |  |
|  |  |  |  |

What has happened here is, FriendC needs a scratch column, just as I did, and just as FriendA and FriendB did. But right now, not only is column 1 taken, but column 2 is taken as well. This does not matter!. FriendC can use any column for scratch work; it just so happened before, that column 2 was always the next available open column. Right now, column 3 is the next available open column, so that is the column that gets used by FriendC instead. Either way, FriendC still gets a scratch area to work in. Whether it is me doing work, FriendA doing work, FriendB doing work, or FriendC doing work, none of us care which particular column we do our work in. We only care that we get some column - whichever one it might be - to do our work in. So the easiest thing to do is to just use the leftmost column that is not in use; I used column 1, people who I asked for help used column 2, and anyone they ask for help, uses column 3. If FriendC were to ask for help, then that person would use column 4 , since that is the next available column.

This is an important concept! Someone can use any column that is convenient as their scratch column. FriendA, for example, is not bound to always use column 2; FriendA only used column 2 since the person using column 1 (me) was the one who asked for help. If FriendC had subtraction to do, and asked FriendA for help, FriendA would proceed to work in column 4 - and yet doing subtraction, the same kind of work FriendA had done for me in column 2. Similarly, when you actually write Java code, any collection of Java instructions such as the instructions to make Keyboard.readInt () work or the instructions to make System.out.println() work, will use whatever area of memory is to the right of where the program had just been working. In the case of your programs so far, you called Keyboard.readInt () from main(), and so whatever area of memory main() had used for its variables, if you then call Keyboard.readInt (), Keyboard.readInt () will use the area of memory next to the area being used by main(), to do any scratch work that Keyboard.readInt() needs to do. It is the same when you call System. out.println(). Every time you ask another piece of code to do some work for you, that piece of code just uses the next available memory to the right of where you were working, to do its job - and then releases that memory once the code is done. Whatever area of memory Keyboard.readInt () is using, Keyboard.readInt () erases that memory and gives it back to the system when it is done and returning back to main(), just like FriendA erased column 2 just before handing the scratch paper back to me.

That is, your Java code will not be bound to run in a particular area of memory, but rather, will run in whatever area is most convenient. (You don't need to worry about that level of detail, but it will help you if you realize that concept.)

Continuing on with our example, FriendC will perform the work needed to figure out the result
of the division:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $(((2+5) *((3+10)-1)) / 6)+7$ | $\left(\left(7^{*} 12\right) / 6\right)$ | $(84 / 6)$ |  |
| $\ldots$ some work done... | $\ldots$ some work done... | $\ldots$ some work done... |  |
| $((7 *(13-1)) / 6)+7$ | $(84 / 6)$ | 14 |  |
| $12<---$ answer from FriendA |  |  |  |
| $\left(\left(7^{*} 12\right) / 6\right)+7$ |  |  |  |
|  |  |  |  |
|  |  |  |  |

And now FriendC will do the same thing that FriendA did earlier. First, FriendC will return the result, to the person who had asked for help - namely, FriendB in column 2:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $(((2+5) *((3+10)-1)) / 6)+7$ | $\left(\left(7^{*} 12\right) / 6\right)$ | $(84 / 6)$ |  |
| $\ldots$ some work done... | $\ldots$ some work done... | $\ldots$ some work done... |  |
| $((7 *(13-1)) / 6)+7$ | $(84 / 6)$ | 14 |  |
| $12<---$ answer from FriendA | 14 <--- answer from FriendC |  |  |
| $((7 * 12) / 6)+7$ |  |  |  |
|  |  |  |  |
|  |  |  |  |

Next, FriendC will erase all the scratch work in column 3, since now FriendC is done, and there is no sense cluttering up a column someone else could use, with scratch work that no longer serves a purpose:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $\left(\left((2+5)^{*}((3+10)-1)\right) / 6\right)+7$ | $\left(\left(7^{*} 12\right) / 6\right)$ |  |  |
| $\ldots$ some work done... | $\ldots$ some work done... |  |  |
| $((7 *(13-1)) / 6)+7$ | $(84 / 6)$ |  |  |
| $12<---$ answer from FriendA | 14 <--- answer from FriendC |  |  |
| $((7 * 12) / 6)+7$ |  |  |  |
|  |  |  |  |
|  |  |  |  |

And finally, FriendC will return the scratch paper back to FriendB, and FriendB can resume work now that both the scratch paper, and the needed result value, have been returned.

Now, all FriendB has left to do, is to recognize that the result given by FriendC, is the final
answer that FriendB needed:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $\left(\left((2+5)^{*}((3+10)-1)\right) / 6\right)+7$ | $\left(\left(7^{*} 12\right) / 6\right)$ |  |  |
| $\ldots$ some work done... | $\ldots$ some work done... |  |  |
| $\left(\left(7^{*}(13-1)\right) / 6\right)+7$ | $(84 / 6)$ |  |  |
| $12<--$ answer from FriendA | $14<---$ answer from FriendC |  |  |
| $\left(\left(7^{*} 12\right) / 6\right)+7$ | 14 |  |  |
|  |  |  |  |

And now FriendB can prepare to return back to me. First, FriendB tells me what the result is:

| Column 1 | Column 2 | Column 3 |
| :--- | :--- | :--- |
| $\left(\left((2+5)^{*}((3+10)-1)\right) / 6\right)+7$ | $\left(\left(7^{*} 12\right) / 6\right)$ | Column 4 |
| $\ldots$ some work done... | $\ldots$ some work done... |  |
| $\left(\left(7^{*}(13-1)\right) / 6\right)+7$ | $(84 / 6)$ |  |
| $12<---$ answer from FriendA | $14<---$ answer from FriendC |  |
| $\left(\left(7^{*} 12\right) / 6\right)+7$ | 14 |  |
| $14<---$ answer from FriendB |  |  |
|  |  |  |

Note that I got my result from FriendB, not FriendC. In fact, since I have no idea what FriendB did to get this result, I don't even have any idea FriendC was ever involved.

Second, FriendB erases column 2, since now FriendB is done and so there's no point in keeping column 2 cluttered up with completed work when someone else could use that column in the future:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $\left(\left((2+5)^{*}((3+10)-1)\right) / 6\right)+7$ |  |  |  |
| $\ldots$..some work done... |  |  |  |
| $((7 *(13-1)) / 6)+7$ |  |  |  |
| $12<---$ answer from FriendA |  |  |  |
| $\left(\left(7^{*} 12\right) / 6\right)+7$ |  |  |  |
| $14<---$ answer from FriendB |  |  |  |
|  |  |  |  |

And finally, FriendB hands the scratch paper back to me, and I can resume my own work where I
left off. Since I now know the result of the first addition operand, I can make a substitution:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $(((2+5) *((3+10)-1)) / 6)+7$ |  |  |  |
| $\ldots$.some work done... |  |  |  |
| $((7 *(13-1)) / 6)+7$ |  |  |  |
| 12 <--- answer from FriendA |  |  |  |
| $\left(\left(7^{*} 12\right) / 6\right)+7$ |  |  |  |
| $14<--$ answer from FriendB |  |  |  |
| $14+7$ |  |  |  |
|  |  |  |  |

And after some calculation work, I can complete the final addition - and with it, finish the evaluation of the expression:

| Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- |
| $(((2+5) *((3+10)-1)) / 6)+7$ |  |  |  |
| $\ldots$.some work done... |  |  |  |
| $((7 *(13-1)) / 6)+7$ |  |  |  |
| 12 <--- answer from FriendA |  |  |  |
| $\left(\left(7^{*} 12\right) / 6\right)+7$ |  |  |  |
| $14<--$ answer from FriendB |  |  |  |
| $14+7$ |  |  |  |
| $\ldots$.some work done... |  |  |  |
| 21 |  |  |  |

Now that I know the result of the expression, I can note it somewhere, and erase column 1 so that column 1 can be used for a different bunch of scratch work the next time I need to do some calculation work in the future:

|  | Column 1 | Column 2 | Column 3 | Column 4 |
| :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |
| <-- result :21 |  |  |  |  |

So that is an example of how different areas of memory (different columns, in our example) can get used by various sets of instructions. Any particular set of instructions (such as main() or Keyboard.readInt()) can make use of any empty area of memory. And as the instructions run, when a new area of memory is needed in order to run a new set of instructions, we just move over
to the right, to the closest-available empty area and use that. And when any set of instructions has finished, the memory that was being used by those instructions is erased and can be re-used by a different set of instructions in the future.

