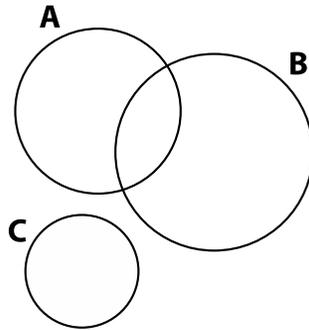


Homework #1

1. (10 points) If $P(A) = 0.2$, $P(B) = 0.2$, and A and B are mutually exclusive, are they independent?

Answer: If A and B are mutually exclusive, then $P(A \cap B) = 0$ and $P(A)P(B) = 0.04$. Therefore, A and B are not independent.

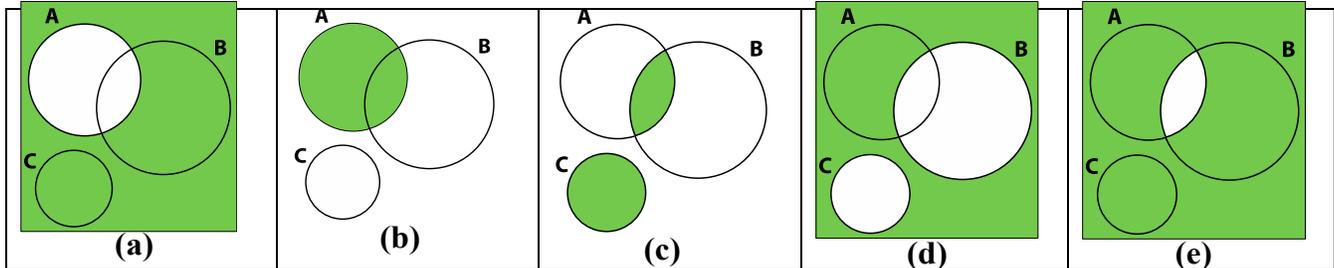
2. (10 points) Three events are shown on the Venn diagram in the following figure:



Reproduce the figure and shade the region that corresponds to each of the following events.

- (a) A' (b) $(A \cap B) \cup (A \cap B')$ (c) $(A \cap B) \cup C$ (d) $(B \cup C)'$ (e) $(A \cap B)' \cup C$

Answers are shown in green



3. (5 points) Consider the hospital emergency department data in the following table. Let A denote the event that a visit is to Hospital 1 and let B denote the event that a visit results in admittance to any hospital. Determine the number of persons in each of the following events.

Hospital	1	2	3	4	total
Total	5292	6991	5640	4329	22,252
LWBS	195	270	246	242	953
Admitted	1277	1558	666	984	4485
Not admitted	3820	5163	4728	3103	16,814

LWBS: People leave without being seen by a physician.

- (a) $A \cap B$
 (b) A'
 (c) $A \cup B$
 (d) $A \cup B'$
 (e) $A' \cap B'$

Answers:

$$a) A \cap B = 1277$$

$$b) A' = 22252 - 5292 = 16960$$

$$c) A \cup B = 195 + 1277 + 3820 + 1558 + 666 + 984 = 8500$$

$$d) A \cup B' = 195 + 270 + 246 + 242 + 3820 + 5163 + 4728 + 3103 + 1277 = 19044$$

$$e) A' \cap B' = 270 + 246 + 242 + 5163 + 4728 + 3103 = 13752$$

4. **(10 points)** There are 4 red balls and 6 white balls in a box. One draws two balls simultaneously. What is the probability that they are the same color?

Answer:

$$P(\text{they are red}) = P(\text{the first one is red}) * P(\text{the second one is red given the first one is red}) = (4/10) * (3/9) = 2/15$$

$$P(\text{they are white}) = P(\text{the first one is white}) * P(\text{the second one is white given the first one is white}) = (6/10) * (5/9) = 1/3$$

$$P(\text{they are the same color}) = P(\text{they are reds}) + P(\text{they are white}) = 2/15 + 1/3 = 7/15$$

5. **(20 points)** George has asked a professor for a recommendation for graduate school. He estimates that the probability that the letter will be strong is 0.5, the probability that the letter will be weak is 0.2, and mediocre is 0.3. he also estimates that if the letter is strong, the probability that he will get the job is 0.8; if it is weak, 0.0; and if it is mediocre, then 0.4. Given that he did get the job, what is the probability that (a) the letter was strong and (b) the letter was weak?

Answer: Let us denote S/W/M as strong/weak/mediocre recommendation letters. Also we represent the event whether or not George did get the job as Y/N.

$$\text{According to the problem, we have } P(S) = 0.5, P(W) = 0.2, P(M) = 0.3, P(Y|S) = 0.8, P(Y|W) = 0.0, P(Y|M) = 0.4$$

Applying Bayes theorem, the probability that the letter was strong given he did get the job is

$$P(S|Y) = \frac{P(Y|S)P(S)}{P(Y|S)P(S) + P(Y|M)P(M) + P(Y|W)P(W)} = \frac{0.8 * 0.5}{(0.8 * 0.5 + 0.4 * 0.3 + 0.0 * 0.2)} = 0.769$$

Obviously, the probability that the letter was weak given he did get the job is 0: $P(W|Y) = 0$

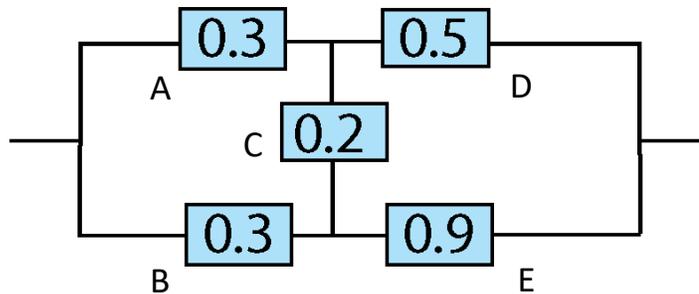
6. **(10 points)** Suppose that a bag contains ten coins, three of which are fair, the remaining seven having probability 0.6 of giving heads when flipped. A coin is taken at random from the bag and flipped five times. All five flips give heads. What's the probability that a coin is fair given the five coin flips?

Answer: Let us denote H_1 as the hypothesis that a coin is fair and H_2 that a coin is biased. The data that all five flips were heads is denoted as D. Therefore,

$$P(H_1|D) = \frac{P(D|H_1)P(H_1)}{P(D)} = \frac{0.5^5 * 0.3}{(0.5^5 * 0.3 + 0.6^5 * 0.7)} = 0.147$$

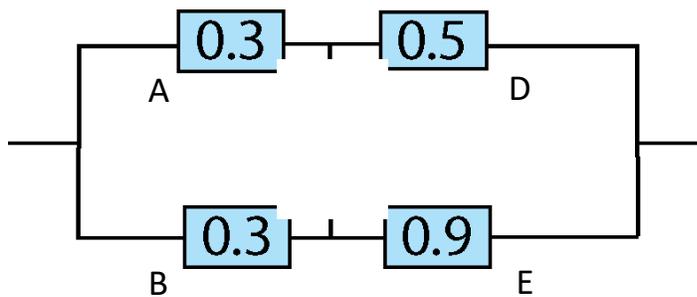
7. **(5 points)** The following circuit operates if and only if there is a path of functional devices

from left to right. The probability that each device functions is as shown. Assume that the probability that a device is functional does not depend on whether or not other devices are functional. What is the probability that the circuit operates?



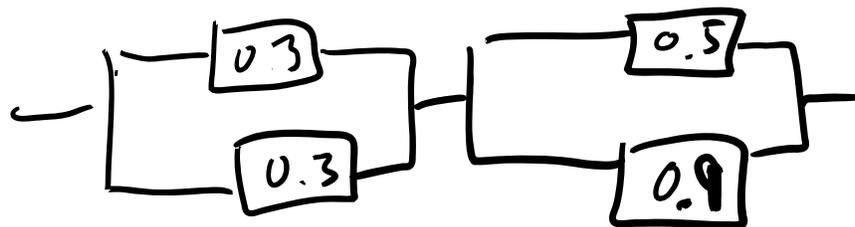
Answer:

You should treat separately two cases: when C works (which happens with probability 0.2) and when C does not (which happens with probability 0.8). In the first case the circuit can be viewed as



$$P(\text{circuit works}) = 1 - (1 - 0.3 \cdot 0.5) \cdot (1 - 0.3 \cdot 0.9) = 0.3795.$$

In the second case the circuit becomes equivalent to



$$\text{Thus } P(\text{circuit works}) = (1 - 0.7 \cdot 0.7) \cdot (1 - 0.5 \cdot 0.1) = 0.4845.$$

$$\text{The final answer is } 0.8 \cdot 0.3795 + 0.2 \cdot 0.4845 = 0.4005$$