Use COMSOL Simulations to Numerically Solve

(a) Consider flow through a vertical pipe (e.g. a blood vessel) of length $L$ and circular cross section of radius $R$. If the pipe is now coated with a thin layer of hydrophobic material such that fluid flowing through it no longer sticks to the walls of the pipe, the mass flow rate is expected to be enhanced for a given pressure. Instead of the “no-slip” condition, the flow now becomes such that the fluid slips at a velocity $v_b$ at the wall of the pipe. Plot: (i) The velocity profile of the fluid in the pipe, (ii) mass flow rate of the fluid, and (iii) assume that the same mass flows through a pipe of length $L$ and radius $R_{\text{eff}}$ but with a no-slip or stick boundary condition. Calculate $Re_{\text{eff}}$ for the fluid in terms of the sample pressure drop, length and Radius of the original pipe $R$. Assume $V_b$ to be $0.1V_{\text{max}}$.

(b) In a second case, the pipe in part (a) is partially occluded by a solid cylindrical object or radius $R_1$ such that the central axis of the pipe and occluding cylinder are coincident. Calculate (i) the velocity profile of the fluid between the concentric cylinders and (ii) mass flow rate. Note that the answer may be best expressed with the parameters of the total radius of the pipe $R$ and the fraction of the radius that is occluded, $\beta = \frac{R_1}{R}$. Hint: you will have to assume that a point in the middle of the cylinders, $\lambda R (R_1 < \lambda < R)$ is such that shear stress is zero at that point. Assume $\beta = 0.7$ for this exercise.

Please note:

A. Compare profiles to those predicted by theory in HW 1. Write a short paragraph or provide plots to show that the expected dependence (e.g. $r^2$ of velocity) is observed.

B. You will need values of diameters of vessels (e.g. blood vessels), flow rates and pressure drop. Find these from the literature or from Conservation Principles book.