BIOE 298, SECTIONS MFI & B

PRACTICE EXAM 1

You have 80 minutes to complete this exam.
You may use notes or printouts from the course website,
but no electronic resources.
PRACTICE EXAM 1

PART I (40 points; 4 points each)

(1) True or False. The matrix \(\begin{pmatrix} 3 & 2 & 1 \\ 1 & 0 & -1 \end{pmatrix}\) has an inverse.

(2) True or False. There exists a real number \(\theta\) such that \(\begin{pmatrix} 1 \\ \theta \\ 1/2 \end{pmatrix}\) is a unit vector.

(3) We said (many times) that the integers are not a field since they have additive inverses \((-a)\) for every element but not multiplicative inverses \((a^{-1})\). We can construct a set that contains both additive and multiplicative inverses using the integers by collecting \(2^i\) and \(-2^i\) for every integer \(i\):

\[
\{\ldots, \pm 2^{-2}, \pm 2^{-1}, \pm 2^0, \pm 2^1, \pm 2^2, \ldots\}
\]

Is this set a field?

(4) \(\|x\| = 8\). What is \(\|-3x\|\)?

(5) Let \(\begin{pmatrix} 0 & 1 & -2 \\ 0 & -1 & 0 \\ 3 & 2 & 1 \end{pmatrix}\) \(\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\) = \(\begin{pmatrix} 2 \\ -8 \\ 12 \end{pmatrix}\). What is \(x_2\)?
(6) True or False. If the angle between \( x = \begin{pmatrix} 2a \\ 1 \\ 0 \end{pmatrix} \) and \( y = \begin{pmatrix} 4 \\ a \\ 2 \end{pmatrix} \) is 135°, then \( x \cdot y = 7 \).

(7) Which vectors are orthogonal to \( \begin{pmatrix} 4 \\ 0 \\ 2 \\ 0 \end{pmatrix} \)?

(a) \( \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \)  
(b) \( \begin{pmatrix} 1/4 \\ 0 \\ 1/2 \\ 0 \end{pmatrix} \)  
(c) \( \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \)  
(d) \( \begin{pmatrix} 0 \\ -12 \\ 0 \\ 8 \end{pmatrix} \)

(8) True or False. \( AB \neq BA \) for all matrices \( A \) and \( B \), even if \( A \) and \( B \) are conformable.

(9) Which of the following differential equations are linear
(a) \( \frac{\partial^2 u}{\partial x \partial y} + \sin(xy)u = 4 \)
(b) \( \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial u}{\partial r} \right) = 0 \)
(c) \( \frac{d^2 u}{dx^2} + 3e^u x \frac{du}{dx} + u = 1 \)
(d) \( \frac{1}{u} \frac{du}{dt} = t \)

(10) What is the rank of the matrix \( \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \)?
Part II (30 points)

Find the inverse of the matrix $A = \begin{pmatrix} 3 & -1 \\ 2 & 0 \end{pmatrix}$

Use the inverse to solve $Ax = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ and $Ax = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$
Part III (30 points)

Write equations for the finite difference approximation for the following ODE at four nodes spanning $[0, 3]$.

$$\frac{d^2u}{dx^2} - 4u = x^2, \quad u(0) = 1, \ u(3) = 4$$

Rewrite the equations as a matrix equation of the form $Ax = y$. 