BIOE 298, SECTIONS MFI & B

HOMEWORK 2

Due 2/8/2018 by end of class. Updated 1/31/2018.

Turn in your answers to Part I on paper in class.
Upload a single PDF with your answer to Part II to Compass.

PART I (40 POINTS)

(1) Is the vector \( \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} \) a unit vector? If not, normalize it.

(2) Are the vectors \( \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \) and \( \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \) orthogonal? If not, what is the angle between them?

(3) What is the magnitude of the projection of \( \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \) onto \( \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \)?

(4) Let \( A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 0 & 1 \end{pmatrix} \), \( B = \begin{pmatrix} 0 & -1 \\ 2 & 1 \\ 4 & 0 \end{pmatrix} \), and \( C = \begin{pmatrix} 5 & 2 & -1 \\ 0 & -3 & 2 \\ 1 & -1 & 1 \end{pmatrix} \).

(a) Compute \( AB \).

(b) For each of the following, tell if the expression is conformable; if so, indicate the dimensions of the resulting matrix.

- \( ABC \)
- \( A^TB^T \)
- \( B^TA^T \)
- \( ACB \)
- \( B^TBC \)
- \( BB^TC \)

(5) Express the quadratic \( x_1^2 + 3x_1x_3 - 2x_2x_3 + 4x_3^2 \) as the product of a vector \( x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \) and an upper-triangular matrix \( Q \). (An upper-triangular matrix has no nonzero entries below the diagonal.)
(6) Solve the system of equations
\[ \begin{align*}
  x_1 - x_2 + x_3 &= 4 \\
  x_1 + 2x_3 &= 8 \\
  x_2 + 2x_3 &= 3
\end{align*} \]

(7) Write a system of equations that approximates the following ODE at five points spanning \([0,1]:\)
\[ \sin(x) \frac{du}{dx} = 0, \quad u(1) = 3 \]

**Part II: Machine Problem (60 points)**

At steady state, the temperature \(T\) in a fluid subject to heat transfer by conduction and convection is governed by the differential equation
\[ \alpha \frac{d^2T}{dx^2} - v_x \frac{dT}{dx} = 0 \]
where \(\alpha\) is the heat diffusivity and \(v_x\) is the \(x\) component of the velocity of the fluid. If \(v_x > 0\), there is fluid flow in the positive \(x\) direction. If \(v_x < 0\), there is fluid flow in the negative \(x\) direction.

We want to calculate temperatures on a 1 cm slab with boundary conditions \(T(0\text{ cm}) = 37^\circ\text{C}\) and \(T(1\text{ cm}) = 25^\circ\text{C}\). We know that the heat diffusivity decays exponentially, i.e. \(\alpha = 0.3e^{-x} \text{ cm/s}^2\). Use a finite difference approximation to solve for the temperature \(T\) at 11 points spanning the domain \([0,1]\) cm under three conditions:

1. no convection \((v_x = 0 \text{ cm/s})\)
2. forward convection \((v_x = 0.75 \text{ cm/s})\)
3. backward convection \((v_x = -0.75 \text{ cm/s}).\)

Plot the temperature profiles for all three conditions on the same plot and comment on the effect of convection. Include your Matlab code with your submission.