Contents

Introduction 5

0.1 Notation 5

1 Fields and Vectors 9

1.1 Algebra 9

1.2 The Field Axioms 9

1.2.1 Common Fields in Mathematics 11

1.3 Vector Addition 12

1.4 Vector Multiplication is not Elementwise 13

1.4.1 Do We Need Multiplication? 13

1.5 Linear Systems 13

1.6 Vector Norms 14

1.6.1 Normalized (Unit) Vectors 16

1.7 Scalar Vector Multiplication 16

1.8 Inner (Dot) Products 17

1.8.1 Computing the Dot Product 17

1.8.2 Dot Product Summary 19

2 Matrices 21

2.1 Matrix Multiplication 21

2.1.1 Generalized Multiplication 22
2.2 Identity Matrix  23
2.3 Matrix Transpose  23
2.4 Solving Linear Systems  24
2.5 Gaussian Elimination  27
2.6 Computational Complexity of Gaussian Elimination  28

3 The Finite Difference Method  31
3.1 Finite Differences  31
3.2 Linear Differential Equations  32
3.3 Discretizing a Linear Differential Equation  33
3.4 Boundary Conditions  34

4 Solvability and Rank  37
4.1 Matrix Inverses  37