(1) Is the vector \( \begin{pmatrix} -2 \\ 1 \\ -7 \end{pmatrix} \) a unit vector? If not, normalize it.

(2) Are the vectors \( \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \) and \( \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix} \) orthogonal? If not, what is the angle between them?

(3) Let \( A = \begin{pmatrix} 1 & 4 & 2 \\ 7 & 3 & 2 \end{pmatrix} \), \( B = \begin{pmatrix} 2 & -1 \\ -3 & 3 \\ 5 & 2 \end{pmatrix} \), and \( C = \begin{pmatrix} 5 & 2 & -1 \\ 0 & -3 & 2 \\ 1 & -1 & 1 \end{pmatrix} \).
   
   (a) Compute \( AB \).
   
   (b) For each of the following, tell if the expression is conformable; if so, indicate the dimensions of the resulting matrix.
   
   - \( ABC \)
   - \( A^T B^T \)
   - \( B^T A^T \)
   - \( ACB \)
   - \( B^T BC \)
   - \( BB^T C \)

(4) Solve the system of equations using Gaussian elimination.

\[
\begin{align*}
    x_1 - 2x_2 + x_3 &= 3 \\
    x_1 + x_3 &= 2 \\
    x_2 + 3x_3 &= 7
\end{align*}
\]

(5) Find the inverse matrix for the coefficient matrix of the above system. Use it to solve the equations below:

\[
\begin{align*}
    x_1 - 2x_2 + x_3 &= 2 \\
    x_1 + x_3 &= 2 \\
    x_2 + 3x_3 &= 3
\end{align*}
\]
(6) Write a system of equations that approximates the following ODE at five points spanning [0, 2]:

\[ 3 \cos(x) \frac{du}{dx} = 0, \quad u(2) = 4 \]

**Part II: Machine Problem (60 points)**

At steady state, the temperature \( T \) in a fluid subject to heat transfer by conduction and convection is governed by the differential equation

\[ \alpha \frac{d^2 T}{dx^2} - v_x \frac{dT}{dx} = 0 \]

where \( \alpha \) is the heat diffusivity and \( v_x \) is the \( x \) component of the velocity of the fluid. If \( v_x > 0 \), there is fluid flow in the positive \( x \) direction. If \( v_x < 0 \), there is fluid flow in the negative \( x \) direction.

We want to calculate temperatures on a 1 cm slab with boundary conditions \( T(0 \text{ cm}) = 37^\circ C \) and \( T(1 \text{ cm}) = 25^\circ C \). We know that the heat diffusivity decays exponentially, i.e. \( \alpha = 0.3e^{-x} \text{ cm/s}^2 \). Use a finite difference approximation to solve for the temperature \( T \) at 11 points spanning the domain [0, 1] cm under three conditions:

1. no convection \( (v_x = 0 \text{ cm/s}) \)
2. forward convection \( (v_x = 0.75 \text{ cm/s}) \)
3. backward convection \( (v_x = -0.75 \text{ cm/s}) \).

Plot the temperature profiles for all three conditions on the same plot and comment on the effect of convection. Include your Matlab code with your submission.