Lecture 13
\[ b.6 \ \Sigma F = ma \]

\[ F_r = \rho g e \text{V}_{	ext{DNA}}, \ \alpha \text{gel} \]

\[ \alpha = \text{linear + angular} \]

\[ \alpha_{\text{DNA}} = \frac{d^2 r}{dt^2} + \omega^2 r \]

at equil = 0
Warm up: We can use fluid statics in the same way that we would use potential energy – difference in pressure in these systems is only due to height change.

What is hydrostatic pressure difference between shoulders and ankle for fluid in the body?

Shoulder-ankle length = 5 ft
Mass of person = 72 kg
Density of blood = 1.056 g/cm³

Justify your answer
\[
\frac{dP(z)}{dz} + P(z) \frac{dP}{dz} - \rho g = 0 \quad \frac{dP}{dz} = -\rho g \\
\Delta P = \left(1.05 \text{e} \frac{g}{\text{cm}^3}\right) \left(3.81 \text{ m} / \text{s} \right) \left(5 \text{ ft} - 0 \text{ ft} \right) \left(\frac{305 \text{ m}}{\text{ft}}\right) \left(\frac{100 \text{ cm}}{\text{m}}\right)^3 \\
\cdot \left(\frac{1 \text{ kg}}{1000 \text{ g}}\right) \left(760 \text{ mm Hg} \right) = 119 \text{ mm Hg}
\]
We can also model collisions using conservation of momentum

\[
\sum_i \vec{p}_i - \sum_j \vec{p}_j + \sum \vec{F} = \frac{d\vec{p}_{\text{sys}}}{dt} = \vec{p}_{f} - \vec{p}_{i}
\]

\[
\vec{p}_{f} - \vec{p}_{i} = m_f \vec{v}_f - m_i \vec{v}_i
\]

For an isolated system, we can assume no external forces outside the two objects in collision act upon them, no effect of gravity in the direction of collision, and no momentum crossing the boundary.

\[
\vec{p}_{f} - \vec{p}_{i} = m_f \vec{v}_f - m_i \vec{v}_i = 0
\]
Platelets love to party and collide with each other but they also get stuck together and cause big problems downstream.

Using conservation of momentum, let’s consider them an isolated system and model the behavior.

Platelet mass = 22 pg

1. Find the velocities
2. Balance momentum in the x and y directions
3. Solve for the velocity of the adhered platelets
\[ V_A = 10 \text{ mm/s} \]
\[ V_B = 8.5 \text{ mm/s} \]

\[ V_f = 1.6 \text{ mm/s} \hat{i} - 2.5 \text{ mm/s} \hat{j} \]

\[ V_A = 10 \text{ mm/s} \hat{i} \]
\[ V_B = -8.5 \text{ mm/s}, \cos \theta \]

\[ x : 0 = m_{tot} V_{fAx} - (m_A V_{Ax} + m_B V_{Bx}) - 8.5 \text{ mm/s} \sin 30^\circ \hat{j} \]
\[ y : 0 = m_{tot} V_{fAy} - (m_A V_{Ay} + m_B V_{By}) \]

\[ x : 0 = 44 \text{ pg} V_{fAx} - (22 \times 10^-3) - (22 \times 8.5 \cos 30^\circ) \]
\[ y : 0 = 44 \text{ pg} V_{fAy} - 22 \times 8.5 \sin 30^\circ \]

\[ V_{fA} = 1.6 \text{ mm/s} \hat{i} - 2.5 \text{ mm/s} \hat{j} \]

\[ V_{f} = 1.6 \text{ mm/s} \hat{i} - 2.5 \text{ mm/s} \hat{j} \]
In a perfect elastic collision, kinetic energy is also conserved

\[ E_{f}^{\text{sys}} - E_{i}^{\text{sys}} = 0 \]

We also can define a coefficient of restitution, \( e \), for a collision between two particles A and B

\[ e = \frac{v_{\text{separation}}}{v_{\text{approach}}} = \frac{v_{B,f} - v_{A,f}}{v_{A,0} - v_{B,0}} \]

This coefficient is a way of describing how elastic a collision is

1 = perfectly elastic

0 = perfectly plastic
Your friend Kenny is super clumsy and fell off his bike while waiting for the light to change. He hits the ground at 6.3 m/s.

Luckily, he was wearing a helmet but it was loose fitting such that when he fell, the helmet and head hit then the helmet rebounds and hits his head again.

Estimate the mass of your head at 5 kg and the helmet at 330 g

The coefficient of restitution for the first collision is 0.82 and the second is 0.17. Does this make sense?
Your friend Kenny is super clumsy and fell off his bike while waiting for the light to change. He hits the ground at 6.3 m/s.

Separate the collisions out into system 1: helmet to ground and system 2: helmet to head.

System 1: What is the final velocity of the helmet in the first collision?

System 2: What are the final velocities of the helmet and head?
**Part 1**

\[ e = 0.82 = \frac{U_{Hf} - U_{Df}}{U_{G_{0}} - U_{H_{0}}} = \frac{U_{Hf}}{0 - U_{H_{0}}} \]

\[ U_{Hf} = 0.82(0 - (-6.3)) = 5.2 \text{ m/s} \]

**Part 2**

\[ e = 0.17 = \frac{U_{Hf} - U_{Df}}{U_{D_{0}} - U_{H_{0}}} = \frac{U_{Hf} - U_{Df}}{-6.3 - 5.2} \]

1. \( -2.0 \text{ m/s} = U_{Hf} - U_{Df} \)

\[ 0 = \vec{p}_{f} - \vec{p}_{0} \]

\[ 0 = m_{h} \vec{v}_{Hf} + m_{D} \vec{v}_{Df} - (m_{H} \vec{v}_{H_{0}} + m_{D} \vec{v}_{D_{0}}) \]

\[ = (0.33)(\vec{v}_{Hf}) + 5 \text{ kg} \left( \vec{v}_{Df} \right) - (0.33(5.2) \text{ j} + 5(-6.3) \text{ j}) \]

2. \( -29.8 \text{ kg m/s} = 0.33 \vec{v}_{Hf} + 5 \vec{v}_{Df} \) \text{ plug in and solve} \]

\[ \vec{v}_{Df} = -5.5 \text{ m/s \ j} \]

\[ \vec{v}_{Hf} = -7.5 \text{ m/s \ j} \]