Prof. P. Voulgaris

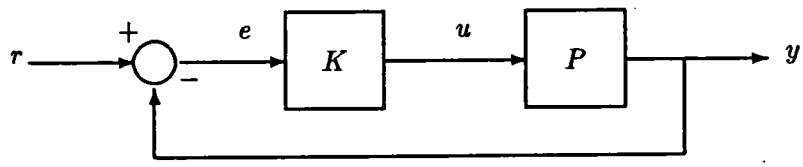
AAE 351
Homework # 7

Reading: 4.~~1, 4.2, 4.3, 4.4, 4.4.1, 4.4.3, 4.3.1, 4.3.2 in text~~ 4.1 ÷ 4.3

1. Suppose you are given the system $P(s) = \frac{1}{s-1}$.

   a. Select a proportional (P) controller $K(s) = g_P$ such that the closed loop transfer function from r to y (i.e., $H_{yr} = \frac{Y}{R}$) is a stable one and also the steady state error to a unit step command (i.e., $w(t) = 1(t)$) is less than 1%. (Assume unity feedback.)

   b. Select a PI controller (proportional + integral) such that the poles of the closed loop transfer function H_{yr} are located at $-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$



   c. What is the steady state error to a unit step command?

2. Use the Routh stability criterion to determine if the following characteristic equations represent stable systems. If the system is unstable, determine how many poles lie in the CRHP

   a. $s^3 + s^2 + 2s + 24 = 0$.

   b. $s^4 + 5s^3 + 13s^2 + 19s + 10 = 0$.

3. Consider the missile problem of homework # 4. The linearized dynamics of the system are described by the transfer function

$$G(s) = \frac{\hat{\alpha}(s)}{\hat{\delta}(s)} = \frac{k}{s^2 - a}, \quad a,\ k > 0$$

   where a, k are positive constants depending on the missile characteristics. Assume that $k = 9$, $a = 5$. To stabilize and control the angle of attack we use a controller $C(s)$ in a unity feedback configuration.

   a. Start with a P controller $C(s) = g_p$. Show that the system cannnot be stabilized for any g_p and the closest to closed loop stability that one can get is cl-poles on the imaginary axis. Select a g_p that will bring the closed loop poles on the imaginary axis and plot using MATLAB the step response to a reference command $\alpha_r(t) = 1(t)$.

   b. To stabilize the closed loop we use a PD controller $C(s) = g_p(5 + s)$, $g_p > 0$. Use the Routh criterion to find the range of g_p for which the closed loop is stable. Plot the step response to a reference command $\alpha_r(t) = 1(t)$ for the value you selected and comment on the properties of the transient and steady state characteristics.

   c. To impove on the steady-state behavior we use a PID controller

$$C(s) = g_p(5 + s + \frac{9}{s}), \quad g_p > 0.$$

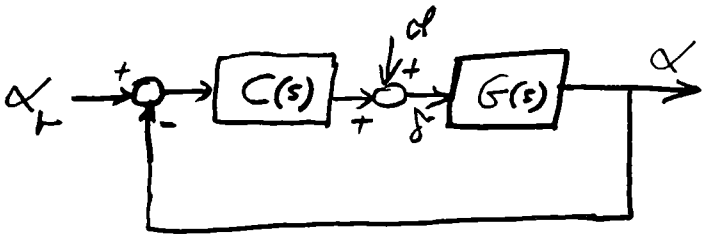
   Use the Routh criterion to find the range of g_p for which the closed loop is stable. Select g_p such that the response has the following characteristics:

   settling time $t_s \leq 1.5$

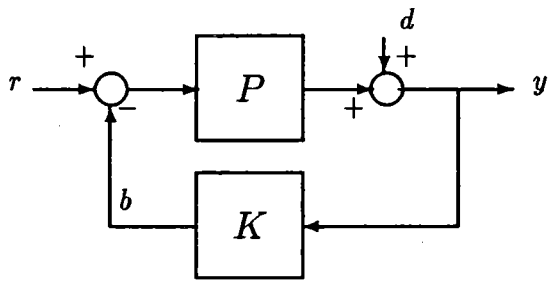
max overshoot $M_p \leq .3$

For your selection present the plot of the step response. Also, find the CL poles. (You can do this using MATLAB's "roots(.)" function to find the roots of the characteristic polynomial).

d. Assuming $\alpha_r \equiv 0$, what is the steady state error of the response to a unit step disturbance d at the input of plant G (atmospheric wind can be modelled as such) for the controllers you selected in parts b and c? Document your answers with plots of the responses.



4. (OPTIONAL) Sometimes the controller $K(s)$ in a feedback system is located in the feedback path as below



$$P = \frac{N_p}{D_p}$$
$$K = \frac{N_k}{D_k}$$

For stability of the closed loop we require that all signals in the feedback loop remain bounded if the inputs r, d are bounded.

a) Express $\hat{y}(s)$, $\hat{b}(s)$ as

$$\hat{y} = H_{yr}\hat{r} + H_{yd}\hat{d}$$
$$\hat{b} = H_{br}\hat{r} + H_{bd}\hat{d}$$

(where $\hat{}$ stands for the Laplace transform) and give the expressions for H_{yr}, H_{yd}, H_{br}, H_{bd}.

b) Check that for stability of the CL system it is necessary and sufficient that the polynomial

$$\phi(s) = D_k D_p + N_k N_p$$

has roots in the open left half plane (OLHP)