1. Consider a linearized model of a physical system that you want to control given by the transfer function

\[ P(s) = \frac{s + 1}{s - \beta} \text{ where } \beta > 0. \]

Assume that a nominal value of the parameter \( \beta \) is known:

\[ \beta_{\text{nom}} = 1 \]

a) Is the nominal model stable?

b) An engineer claims that he/she has the miraculous controller scheme that can make the system behave exactly as she/he wishes. In particular, she/he proposes to use the following open loop scheme where \( K(s) = \frac{s + 1}{s + 1} \):

(i) Check that \( P_{\text{nom}} K = 1 \), hence \( y(t) = r(t) \) which means that, nominally, the output \( y \) follows the command input \( r \) for all time \( t \).

(ii) Is this a sensible solution? Explain

c) Suppose that we knew exactly the parameter \( \beta \) (say, \( \beta = 1 \)). Would you then accept the engineer's proposal? Explain. (disturbances at the input of the plant are possible).

2. Consider the unity feedback loop

where

- \( d_i \) is an input disturbance (to the plant)
- \( d_0 \) is an output disturbance (to the plant)
- \( n \) is a measurement disturbance (ex. sensor noise)
- \( r \) is the command input (to the system)
- \( y \) is the actual output of the system

a. Assuming that the rest of the inputs are zero find the transfer functions (4)

\[ H_{yx}(s) = \frac{Y(s)}{X(s)} \]

for \( x = d_i, d_0, r, n \).
b. Use the fact that the feedback system is linear to check that if all the inputs \( z \) act together then

\[
Y(s) = H_{yd_1} D_1(s) + H_{yd_0} D_0(s) + H_{yr} R(s) + H_{yn} N(s)
\]

c. Form the true error \( e = r - y \) and derive a similar expression as in part b, i.e.,

\[
E(s) = H_{ed_1} D_1(s) + H_{ed_0} D_0(s) + H_{er} R(s) + H_{en} N(s).
\]

Check that \( H_{ed_0} = -H_{er} \). Note that \( e \) is not the input to the controller \( K \).

d. Assume \( d_1, d_0 = 0 \).

Typically, we desire to have both \( H_{er}(s), H_{en}(s) \) "small" so that the system produces "small" tracking error \( E(s) \) in the presence of both input command \( r \) and sensor noise \( n \).

i) Check that

\[
H_{er}(s) + H_{en}(s) = 1.
\]

ii) What does this imply about the realization of our desires?