Prof. P. Voulgaris

AE 353
Homework # 3

1. Find the inverse Laplace transform of the following functions

$$a) \quad Y(s) = \frac{s^2+9s+19}{(s+1)(s+2)(s+4)}$$

$$b) \quad Y(s) = \frac{s(s+2)}{(s+1)^2}.$$

2. Solve the following differential eqn by means of the Laplace transform.

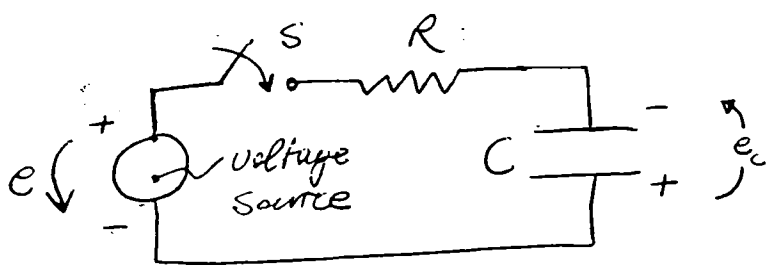
$$\ddot{y} + 5\dot{y} + 4y = u$$

with

$$a) \quad u(t) = e^{-2t}, t \geq 0, \text{ IC's } \equiv 0$$
$$b) \quad u(t) = \delta(t), \text{ IC's } \equiv 0$$
$$c) \quad u(t) = 0, t \geq 0, y(0) = 0, \quad \dot{y}(0) = 1.$$

How is your answer to b) related to your answer to c)?

3. Consider the electrical system below



a. Write the governing differential equation when the switch S is closed.

b. Suppose that the capacitor C is initially charged at a voltage of 1 volt before the switch closes the circuit at $t = 0$. What is the system's free response (i.e., $e = 0$) if $RC = 1sec^{-1}$?

c. Let now $e(t) = 1(t)$. What is the forced response? (I.e., IC's $\equiv 0$). What is the total response? What is the steady state value of e_c? (I.e., $e_c(t = \infty)$).

d. Assume IC's $\equiv 0$ and let $e(t) = \delta(t)$. Find the response $e_c(t)$. Is this different from what you found in part a)? Explain. How is the response in part c) related to your answer in part d)?

e. Assume $e(t) = 1(t)$ and that the capacitor is initially charged at 1Volt. What is the response?

4. Let

$$Y(s) = \frac{8}{(s+1)(s^2+25)}$$

represent the Laplace transform of the signal $y(t)$, $t \geq 0$ which is response of a LTI system to a unit impulse $u(t) = \delta(t)$.

a Determine $y(t)$. What is the transient response? What is the steady-sate response?

b Use the initial value theorem to obtain $y(0)$. Check with your answer in part a).
Is the final value theorem applicable for $Y(s)$? Explain.

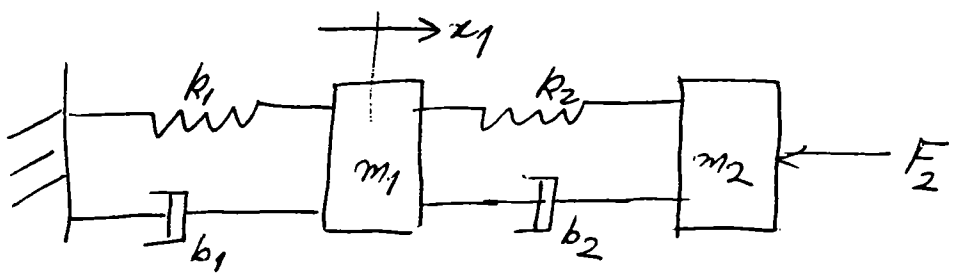
c What is the differential equation that governs the dynamics of the system?

d What is the Laplace transform of the system's output due to a unit impulse at $t = 2$? (IC's $\equiv 0$).
(I.e., $u(t) = \delta(t-2)$).

e What is the system's response to a unit step input? (I.e., $u(t) = 1(t)$, IC's $\equiv 0$).

5. Find the transfer function from F_2 to x_1 of the system in problem 1, homework 1, when $F_1 \equiv 0$, $m_1 = m_2 = 1$, $K_2 = b_2 = 1$, $K_1 = .5$, $b_1 = 1.5$.

Recall:



Reading: Appendix A, Ch 3: 3.1.1 ÷ 3.1.7, 3.9