1. Consider a system \( G \) with input \( u \) and output \( y \).

   a) Suppose that \( G \) is a time-delay, i.e., the output \( y \) is exactly the input \( u \) but delayed by \( T \) seconds, where \( T \) is constant.
   Is this a linear system?
   Is \( G \) a time invariant system?

   b) Suppose \( G \) is given by the input-output relation \( y = mu + b \) where \( m, b \) constants.
   Is this a linear system?

2. Verify the following properties of the Laplace Xform

   a. \( \mathcal{L}(f(t-T)) = e^{-sT} \mathcal{L}(f(t)) = e^{-sT}F(s) \)

   b. \( \mathcal{L}(e^{-at}f(t)) = F(s+a) \)

3. a) Given that

   \[ F(s) = \mathcal{L}(f(t)) = \frac{w}{s^2 + w^2}, \quad f(t) = \begin{cases} \sin wt, & t \geq 0 \\ 0 & t < 0 \end{cases} \]

   show that

   \[ G(s) = \mathcal{L}(g(t)) = \frac{s}{s^2 + w^2}, \quad g(t) = \begin{cases} \cos wt, & t \geq 0 \\ 0 & t < 0 \end{cases} \]

   by using a suitable property of Laplace transform.

   c) Use (if applicable) the initial value and the final value theorem to obtain

   \[ f(0^+), \quad f(\infty), \quad g(0^+), \quad g(\infty). \]

4. Find \( \mathcal{L}(f(t)) \) for the following \( f \) without direct computation. Rely strictly on the properties of the transform and assume only that \( \mathcal{L}(1(t)) = 1/s \) is known (i.e., the Laplace transform of the step function).

   ![Plot](image)

   **Hint:** Decompose appropriately \( f \).