Exam Policy:

- open book
- open notes
- calculators allowed; no cell phones

Some Remarks:

- There are 2 problems. Read all questions and point values.
- Write your name on every sheet of paper you hand in.
- Best of luck
Consider the unity feedback system drawn where

\[ P(s) = \frac{1}{(s^2 + 4)} \]

Suppose we use a controller \( K(s) = \frac{k(s+1)}{(s-p)}, \quad k > 0 \).

1. With \( z = -1 \) determine for what \( k \) and \( p \) the closed loop is stable.

2. For large \( k \) (i.e., \( k \to \infty \)) and with \( z = -1 \) and \( p = -3 \) determine what the closed loop poles are; what is the (approximate) settling time \( t_s \) and overshoot \( M_p \) of the step response?

3. With \( z = -1 \) and \( p = -3 \), find the range of \( k \) to lead to a steady state error of the step response less than 10%.

4. Sketch roughly the locus of the closed loop poles as \( k \) varies. Be as specific as you can if (and only if) time allows!!

5. Let \( z = -2\sqrt{3} \) and suppose we want a stable closed loop with two of the closed loop poles at \(-4\sqrt{3} \pm j2\). Is it possible to find \( p \) and \( k > 0 \) to do it? Explain why yes or no.

1. \( G(s) = (s-p)(s^2+4) + k(s+1) \)
   \[ = s^3 + 4s - ps^2 - 4p + ks + k \]
   \[ = s^3 + (-p)s^2 + (k+4)s + k - 4p \]
   Need \(-p > 0\), \( k+4 > 0\), \( k-4p > 0\)

2. \((-p)(k+4) > 1(k-4p)\) or
   \[-pk - 4p > k - 4p\] or
   \[-pk - k > 0\] or \( k(-p-1) > 0\)

\(1, 2 \Rightarrow \left[ p < -1 \right] \quad (k > 0)\)

2. 2 asymptotes centered at
   \[ \alpha = \frac{\Sigma p_i - \Sigma a_i}{n-m} = \frac{-3 - (-1)}{2} = -1 \]
Hence as $k \to \infty$ CL poles (b) along asymptotes f on top of zero at -1 i.e.,

$-1 \pm j \infty$, $-1 \implies$

$t_s = \frac{4.6}{1} = 4.6$ secs,

$\eta_0 = 100\%$ ($\eta = \sin^{2}\theta \mid \theta \to 0$)

3. $H(\omega) = \frac{1}{1+pk}$, $e(\omega) = \frac{1}{1+\rho(\omega)k(\omega)} = \frac{1}{1+\frac{1}{4}k^{1/3}} = \frac{12}{12+k}$

$|e(\omega)| \leq \frac{1}{10} \iff 12 \cdot 10 \leq 12 + k \iff k \geq 410$

$k \geq 108$

4. Can check no break points

5. Angle condition:

$\phi_2 = 90 + 60 = 150$

$\phi_1 = 180$

$\psi - \phi = (\phi_1 + \phi_2) = 180^\circ + 1360^\circ$

$\sigma = 150^\circ$

Note that as $\psi = 60 + 90 = 150^\circ$

$\phi$ should be $0^\circ$ hence there is no such $a_p$. 
PROBLEM 2 (20pts)

Consider the unity feedback system drawn and assume that $K$ is a stabilizing controller. True or False?

1. If the system is type-3, it produces zero steady state errors to ramp commands $r$ ($d_i = d_o = 0$).
2. If the system is type-2, it produces zero steady state errors to ramp disturbances $d_i$ at the input of the plant ($r = d_o = 0$).
3. If the system is type-1, it produces zero steady state errors to step disturbances $d_o$ at the output of the plant ($r = d_i = 0$).
4. Let the closed loop transfer function form $r$ to $y$ be $H_{yr}$ and suppose $z$ is a zero of $P(s)$ in the left half plane, i.e., Re($z$) < 0. Then, $H_{yr}(s)$ has always a zero at the same location $z$.

1. T
2. F
3. T
4. F