Soln

AE 353
EXAM # 2
Wednesday, April 26, 2006

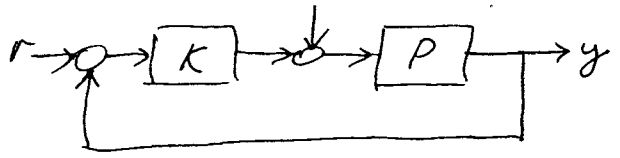
DO NOT START UNTIL INSTRUCTED BY EXAMINER

Exam Policy:

- open book
- open notes
- calculators allowed; no cell phones

Some Remarks:

- There are 2 problems. Read all questions and point values.
- Write your name on every sheet of paper you hand in.
- Best of luck



PROBLEM 1 (80pts)

Consider the unity feedback system drawn where

$$P(s) = \frac{1}{(s^2+4)}.$$

Suppose we use a controller $K(s) = \frac{k(s-z)}{(s-p)}$, $k > 0$.

16 pts each

1. With $z = -1$ determine for what k and p the closed loop is stable.
2. For large k (i.e., $k \to \infty$) and with $z = -1$ and $p = -3$ determine what the closed loop poles are; what is the (approximate) settling time t_s and overshoot M_p of the step response?
3. With $z = -1$ and $p = -3$, find the range of k to lead to a steady state error of the step response less than 10%.
4. Sketch *roughly* the locus of the closed loop poles as k varies. Be as specific as you can *if (and only if) time allows!!*
5. Let $z = -2\sqrt{3}$ and suppose we want a stable closed loop with two of the closed loop poles at $-4\sqrt{3} \pm j2$. Is it possible to find p and $k > 0$ to do it? Explain why yes or no.

1. $\varphi(s) = (s-p)(s^2+4) + k(s+1)$

$= s^3 + 4s - ps^2 - 4p + ks + k$

$= s^3 + (-p)s^2 + (k+4)s + k - 4p$

need $-p > 0$, $k+4 > 0$, $k - 4p > 0$ ①

and $(-p)(k+4) > 1 \cdot (k-4p)$ or

$-pk - 4p > k - 4p$ or

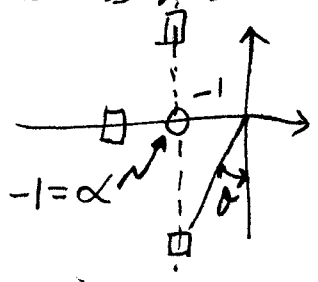
$-pk - k > 0$ or $k(-p-1) > 0$ ②

①, ② $\Rightarrow$ $\boxed{p < -1}$ ($k > 0$)

2. 2 Asymptotes centered at

$$\alpha = \frac{\sum p_i - \sum z_i}{n-m} = \frac{-3-(-1)}{2} = -1$$

Hence as $k \to \infty$ CL poles (□) along asymptotes & on top of zero at -1 i.e.,



$$-1 \pm j\infty \quad , \quad -1 \quad \Rightarrow$$

$$t_s = \frac{4.6}{1} = 4.6 \text{ secs},$$

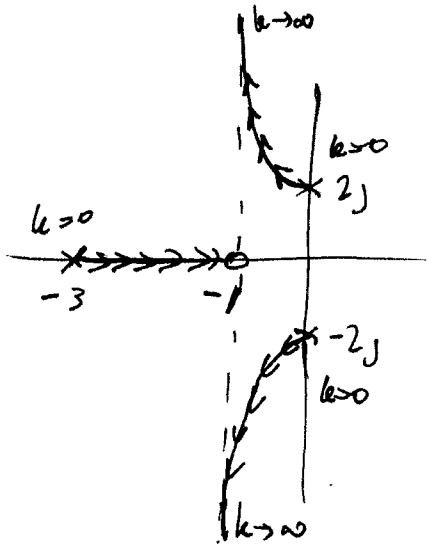
$$M_p = 100\% \quad \left(\zeta = \sin\theta \Big|_{\theta \to 0} = 0 \right)$$

3. $H_{er} = \dfrac{1}{1+PK}$, $e(\infty) = \dfrac{1}{1+P(0)\,K(0)} = \dfrac{1}{1+\frac{1}{4}k\frac{1}{3}} = \dfrac{12}{12+k}$

$$|e(\infty)| \le \frac{1}{10} \Leftrightarrow 12 \cdot 10 \le 12 + k \Leftrightarrow \cancel{k \ge 110}$$

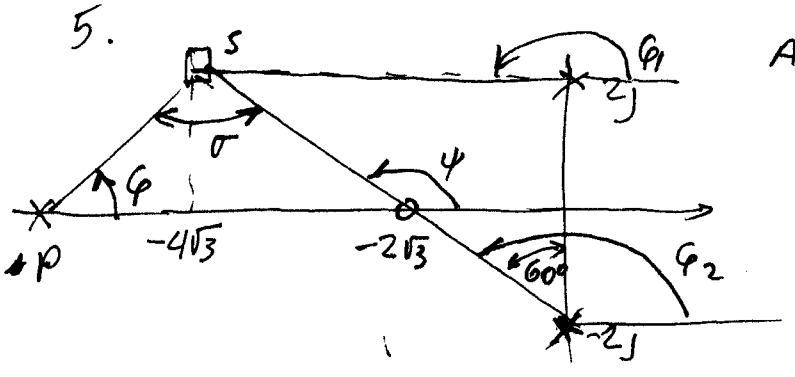
$$k > 108$$

4.



can check no break points

5.



Angle condition:

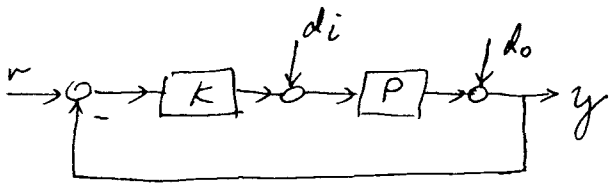
$\varphi_2 = 90 + 60 = 150$

$\varphi_1 = 180$

$$\underbrace{\psi - \varphi}_{\sigma} - (\varphi_1 + \varphi_2) = 180^\circ + l360^\circ$$

$$\sigma = 150^\circ$$

Note that as $\psi = 60 + 90 = 150^\circ$ φ should be 0° $\Rightarrow$ there is no such a p.



PROBLEM 2 (20pts)

Consider the unity feedback system drawn and assume that K is a *stabilizing* controller. True or False?

1. If the system is type-3, it produces zero steady state errors to ramp commands r ($d_i = d_o = 0$).
2. If the system is type-2, it produces zero steady state errors to ramp disturbances d_i at the input of the plant ($r = d_o = 0$).
3. If the system is type-1, it produces zero steady state errors to step disturbances d_o at the output of the plant ($r = d_i = 0$).
4. Let the closed loop transfer function form r to y be H_{yr} and suppose z is a zero of $P(s)$ in the left half plane, i.e., $\mathrm{Re}(z) < 0$. Then, $H_{yr}(s)$ has *always* a zero at the same location z.

1. T
2. F
3. T
4. F