SOLN

AE 353
EXAM # 1
Wednesday, March 8, 2006

DO NOT START UNTIL INSTRUCTED BY EXAMINER

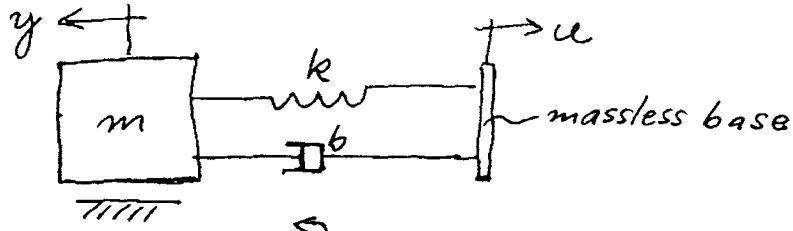
Exam Policy:

- open book and class-notes
- calculators allowed
- no use of cell-phones

Some Remarks:

- There are 3 problems. Read all questions and point values.
- Write your name on every sheet of paper you hand in.
- Max score is 100.
- Best of luck

Problem 1 (40pts)



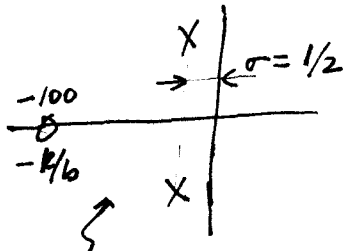
Consider the mass/spring/damper system shown below. At the positions $y = 0$, $u = 0$, the spring is relaxed.

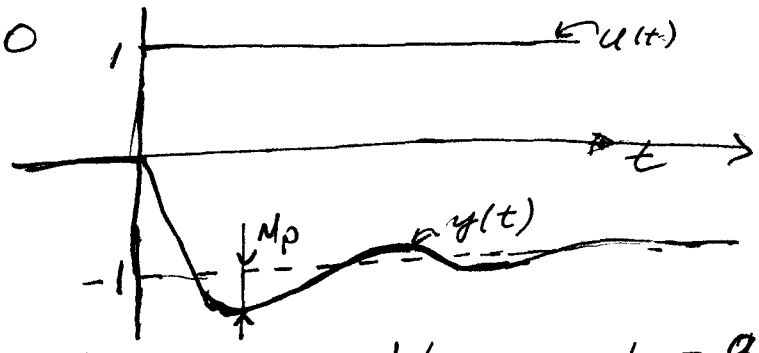
1. (10pts) Write the equations of motion.
2. (10pts) Find the transfer function, $H(s)$, from the input position u to the mass position y.
3. (10pts) Is the system stable? Explain why yes or no.
4. (10pts) Assuming that $b/m = 1$ and $k/m = 100$, roughly sketch the response of y to a unit step u and estimate the settling time of the response (using 1% criterion)

1. $$m\ddot{y} = -k(y+u) - b(\dot{y}+\dot{u})$$
$$m\ddot{y} + b\dot{y} + ky = -b\dot{u} - ku$$

2. $$H(s) = \frac{-bs - k}{ms^2+bs+k} = \frac{-b/m\, s - k/m}{s^2 + b/m\, s + k/m}$$

3. yes ; $m, b, k > 0$

4. 



lightly damped poles + zero at far left

$t_s \approx \frac{4.8}{\sigma}$, $2\sigma = b/m$ $\rightarrow t_s \approx 9.6$ secs

Note : $$H(s) = \frac{-k/m}{s^2+\frac{b}{m}s+\frac{k}{m}} - \frac{b}{k}\, s\, \frac{k/m}{s^2+\frac{b}{m}s+k/m}$$

$\omega_n^2 = k/m \rightarrow \omega_n = 10$

$2\zeta\omega_n = b/m \rightarrow$

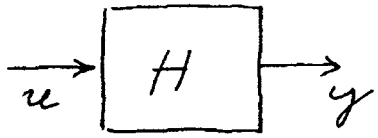
$\rightarrow \zeta = .05$

$\downarrow$

$M_p \approx 90\%$

as $b/k \ll 1$ then response of $H(s) \approx$ response of $-\dfrac{k/m}{s^2+\frac{b}{m}s+k/m}$

Problem 2 (30pts)



Consider a first order system $H(s)$. When the input is a unit step, i.e., $u(t) = 1(t)$, the steady state value of the output $y(t)$ equals 2, while the slope of the response of $y(t)$ at $t = 0^+$ equals 2. Find $H(s)$.

$$H(s) = \frac{b}{s+a}$$

$$H(0) = 2 = \frac{b}{a}$$

$$2 = \dot{y}(0^+) = s(s\hat{y}(s))\Big|_{s\to\infty} = s^2 H(s)\frac{1}{s}\Big|_{s\to\infty} = \frac{sb}{s+a}\Big|_{s\to\infty} =$$

$$= b$$

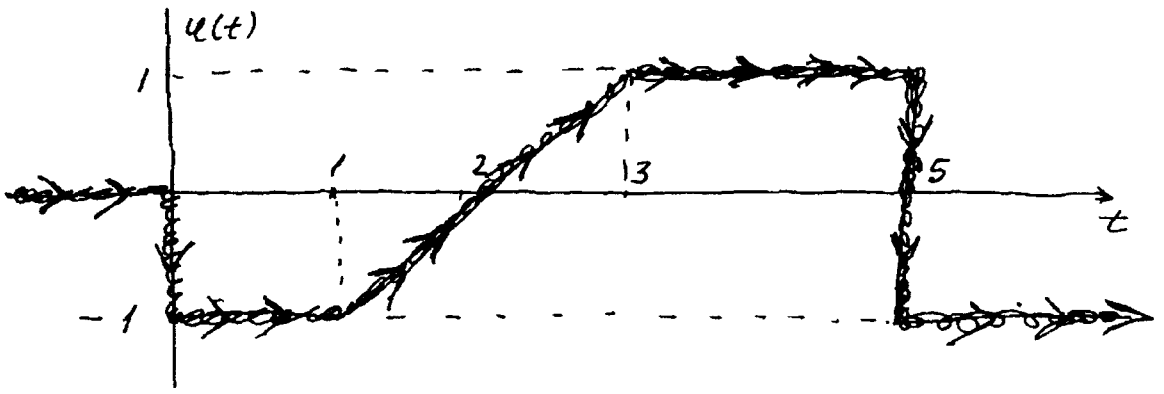
$$\therefore\ b = 2,\ a = 1$$

Problem 3 (30pts)

For the signal $u(t)$ sketched below, find its Laplace transform. If this u is an input to a stable system H with

$$H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + 1}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + 1}$$

with $m \leq n$, what will the value of the output $y = Hu$ be at steady-state?

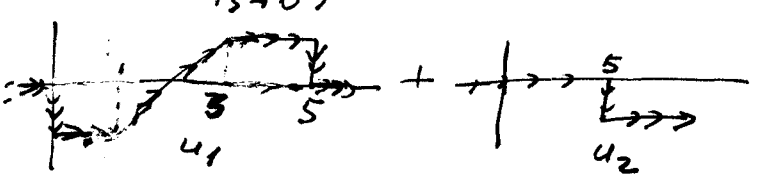


$$u(t) = -1(t) + 1(t-1) - r(t-3) - 2 \times 1(t-5)$$

$$\hat{u}(s) = -\frac{1}{s} + e^{-s}\frac{1}{s^2} - e^{-3s}\frac{1}{s^2} - 2e^{-5s}\frac{1}{s}$$

$$y(\infty) = -H(0) = -1 \quad \left(= s\,H(s)\,\hat{u}(s)\big|_{s\to 0}\right)$$

to see this split u:



$u = u_1 + u_2$

u_1 does not contribute to steady state value

u_2 is a negative delayed step