AE 353
EXAM # 1
Wednesday, March 8, 2006

DO NOT START UNTIL INSTRUCTED BY EXAMINER

Exam Policy:

- open book and class-notes
- calculators allowed
- no use of cell-phones

Some Remarks:

- There are 3 problems. Read all questions and point values.
- Write your name on every sheet of paper you hand in.
- Max score is 100.
- Best of luck
Problem 1 (40pts)

Consider the mass/spring/damper system shown below. At the positions $y = 0$, $u = 0$, the spring is relaxed.

1. (10pts) Write the equations of motion.

2. (10pts) Find the transfer function, $H(s)$, from the input position $u$ to the mass position $y$.

3. (10pts) Is the system stable? Explain why yes or no.

4. (10pts) Assuming that $b/m = 1$ and $k/m = 100$, roughly sketch the response of $y$ to a unit step $u$ and estimate the settling time of the response (using 1% criterion)

\[ m \dddot{y} = -k(y + u) - b \dot{y} + u \]
\[ m \dddot{y} + b \dot{y} + ky = -bu - ku \]

\[ H(s) = \frac{-bs + k}{ms^2 + bs + k} = \frac{-b/m s - k/m}{s^2 + b/m s + k/m} \]

3. yes; $m$, $b$, $k > 0$

4. \[ X(t) \quad \text{lightly damped poles + zero at far left} \]

\[ t_s \approx \frac{4.8}{\sigma} \]
\[ 9 \sigma = b/m \quad \rightarrow t_s \approx 9.6 \text{secs} \]

\[ \text{Note:} \quad H(s) = \frac{-b/m}{s^2 + b/m s + k/m} - \frac{b}{k} \frac{s}{s^2 + b/m s + k/m} \]

As $b/k \ll 1$ then $H(s) \approx \text{response of} \quad \frac{k/m}{s^2 + b/m s + k/m}$
Problem 2 (30pts)

Consider a first order system $H(s)$. When the input is a unit step, i.e., $u(t) = 1(t)$, the steady state value of the output $y(t)$ equals 2, while the slope of the response of $y(t)$ at $t = 0^+$ equals 2. Find $H(s)$.

\[ H(s) = \frac{b}{s + a} \]

\[ H(0) = 2 = \frac{b}{a} \]

\[ 2 = y(0^+) = \left. s(s \hat{y}(s)) \right|_{s \to \infty} = s^2 \left. \frac{H(s)}{s} \right|_{s \to \infty} = \left. \frac{5b}{s} \right|_{s \to \infty} = b \]

\[ \therefore b = 2, \quad a = 1 \]
Problem 3 (30pts)

For the signal \( u(t) \) sketched below, find its Laplace transform. If this \( u \) is an input to a stable system \( H \) with

\[
H(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + 1}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + 1}
\]

with \( m \leq n \), what will the value of the output \( y = Hu \) be at steady-state?

\[
\begin{align*}
  u(t) &= -1(t) + 1(t-1) - r(t-2) - 2s\delta(t-5) \\
  &\quad + e^{-s} \frac{1}{s^2} - e^{-3s} \frac{1}{s^2} - 2 e^{-6s} \frac{1}{s}
  \\
  \hat{u}(s) &= -\frac{1}{s} + e^{-s} \frac{1}{s^2} - e^{-3s} \frac{1}{s^2} - 2 e^{-6s} \frac{1}{s}
\end{align*}
\]

\[
y(\infty) = -\lim_{t \to \infty} u(t) = -1 (= \frac{sH(s) \hat{u}(s)}{s+0})
\]

to see this split \( u \):

\[
u = u_1 + u_2
\]

\( u_1 \) does not contribute to steady state value

\( u_2 \) is a negative delayed step