Question 1, 2, 3 and 4 relate to the figure below. The figure shows four processes and the point-to-point communication channels between the processes.

In Questions 1, 2 and 3, answer Yes or No, with a brief justification.

1. Assume that the system above is synchronous, and at most one process may crash. Is it possible to achieve exact consensus in this system?
   
   YES. Each pair of non-faulty processes can communicate reliable despite one crash failure. Thus, a complete network can be simulated, and we can use the f+1 round algorithm discussed in class.

2. Assume that the system above is asynchronous, and at most one process may crash. Is it possible to achieve exact consensus in this system?
   
   NO. Due to FLP impossibility result.

3. Assume that the system above is synchronous, and at most one process may be Byzantine faulty. Is it possible to achieve exact consensus in this system?
   
   NO. Need 2f+1 = 3 connectivity for consensus with f Byzantine faults.

4. Consider the Distributed Snapshot algorithm in the Chandy-Lamport paper. Does this algorithm record a consistent cut if the communication channels are not FIFO (first-in first-out)?
   
   NO. A marker message M sent after application message M1 may possibly arrive before M1.
5. In the execution below, does there exist a consistent cut that contains event \( m \), but does not contain event \( e \)? If you answer yes, show such a consistent cut in the figure.

**YES.** The **maximal** consistent cut satisfying these constraints contains events \{a,i,k,m\}.

![Diagram](image)

6. In the execution below, does there exist a consistent cut that contains event \( h \), but does not contain event \( b \)? If you answer yes, show such a consistent cut in the figure. **NO, because \( b \rightarrow h \).**

![Diagram](image)

7. Suppose that we want to implement an iterative average consensus algorithm in the system shown on the previous page. Assume that the system is **synchronous**, and **no failures** occur. Let us denote by \( X[t] \) the vector of states of the four processes, where \( X_i[t] \) is the state of process \( i \) after \( t \) iterations. Recall that the state updates can be expressed in the following matrix form: \( X[t+1] = M X[t] \) where \( M \) is a square matrix.

Present a matrix \( M \) that will result in **average consensus** as \( t \rightarrow \infty \).

Use the following matrix \( M \), where \( a < \frac{1}{2} \).

\[
\begin{array}{cccc}
1-2a & a & a & 0 \\
a & 1-2a & 0 & a \\
a & 0 & 1-2a & 0 \\
0 & a & a & 1-2a \\
\end{array}
\]

Briefly explain why the specified \( M \) will result in average consensus.

The network is strongly connected, the matrix \( M \) corresponds to this network, and \( M \) is doubly stochastic.