I. Hajj

ECE 552, **Fall** 2017

HW #9

(*Due* 18 *December* 2017)

Problem 9.1: Consider the following linear multistep formula:

$$x_n = \frac{4}{3}x_{n-1} - \frac{1}{3}x_{n-2} + \frac{2}{3}h\dot{x}_n$$

(a) Write the difference equation that is obtained by applying the formula to a test equation $\dot{x} = \lambda x$, and write the characteristic polynomial of the difference equation.

(b) Sketch the boundary of the region of stability in the complex $h\lambda - plane$. Calculate the coordinates of at least four points on the boundary, and indicate which region is stable and which region is unstable. Explain all your answers.

(c) Would you use this formula for the transient analysis of stiff systems? Why?

Problem 9.2: Consider the following linear multistep formula:

$$x_n = -4x_{n-1} + 5x_{n-2} + h(-2\dot{x}_n + 8\dot{x}_{n-1})$$

(a) Determine the number of steps k of the formula. Explain your answer.

(b) What is the *order* of the formula? Why?

(c) Is the formula explicit or implicit? Why?

(d) Is the formula consistent? Why?

(e) Is the formula zero-stable? Why?

(f) Is the formula convergent? Why?

(g) Write the difference equation that is obtained by applying the formula to a test equation $\dot{x} = \lambda x$, and write the characteristic polynomial of the difference equation.

(h) Sketch the boundary of the region of stability in the complex $h\lambda - plane$. Calculate the coordinates of at least four points on the boundary, and indicate which region is stable and which region is unstable. Explain all your answers.

(i) Would you use this formula for the transient analysis of stiff systems? Why?

Problem 9.3: Given the state equations of a linear system:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{u}(t)$$

where $\mathbf{A}$ is a 4×4 real matrix. The eigenvalues of $\mathbf{A}$ are found in four different cases to be:

(a) $\lambda_1 = -2$, $\lambda_2 = -4$, $\lambda_3 = -4 + j4$, $\lambda_4 = -4 - j4$
(b) $\lambda_1 = +2$, $\lambda_2 = +4$, $\lambda_3 = +j4$, $\lambda_4 = -j4$
(c) $\lambda_1 = -2$, $\lambda_2 = -4$, $\lambda_3 = +j4$, $\lambda_4 = -j4$
(d) $\lambda_1 = -2j$, $\lambda_2 = -2j$, $\lambda_3 = +j2$, $\lambda_4 = +j2$

Suppose (1) the F. E. Formula, (2) the B. E. Formula, and (3) the T. R. are applied to compute numerically the time response of the system.

For each case and for each integration formula, determine the range of the time step $h > 0$ so that the numerical solution behaves in the same manner as the analytical solution (from the stability point of view). Note that there are twelve cases to consider. Explain your answers.

Problem 9.4: Consider the three-terminal nonlinear capacitor, with the terminal nodes labeled i, j, and k. Its characteristics are specified as:

$$q_1 = v_1^3 + v_2$$

$$q_2 = v_1 + v_2^3$$

(a) Use the B. E. formula to derive the companion model equations of the capacitor at time t_n, given initial conditions $v_{1,n-1}$ and $v_{2,n-1}$.

(b) derive the linearized model equations obtained in (a) using Taylor Series expansion at the iteration point $v_{1,n}^{(k)} = 1V$, $v_{2,n}^{(k)} = 2V$.

(c) Construct the stamp of the linearized equations derived in (b) for use in the modified nodal equation formulation.

(d) Repeat (a), (b), and (c) by using a second order backward differentiation formula with constant timestep h. Assume initial conditions $v_{1,n-1}$, $v_{1,n-2}$, and $v_{2,n-1}$, $v_{2,n-2}$ are known.