I. Hajj

ECE 552
HW # 4
Solution

Fall 2017

Prob. 1

(a) LU factorization requires $\frac{n^3-n}{3}$ operations (see notes)

Solving $LUB = I \Rightarrow B = A^{-1}$

- Forward Substitution taking advantage of leading zeroes in right-hand side vectors. For the j-th vector:

  # of operations $= \frac{(n-j+1)^2 + (n-j+1)}{2}$

  Total $= \sum_{j=1}^{n} \frac{(n-j+1)^2 + (n-j+1)}{2} = \sum_{k=1}^{n} \frac{k^2+k}{2}$, $k = n-j+1$

  $= \frac{1}{6}(n^3+3n^2+2n)$

- Backward Substitution for one vector $= \frac{n^2-n}{2}$

  Total $= \frac{n(n^2-n)}{2}$

Total of Forward & Backward Substitutions =

$$\frac{1}{6}(n^3+3n^2+2n) + \frac{n^3-n^2}{2} = \frac{2}{3}n^3 + \frac{n}{3} \qquad (*)$$

Total of LU factorization + Forward & Backward Substitution.

$$\frac{n^3-n}{3} + \frac{2}{3}n^3 + \frac{n}{3} = n^3$$

(b) From (*) above $\frac{2}{3}n^3 + \frac{n}{3}$

HW #4 (cont.)

(2)

(a) $pdq^T = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} [1] [1 \; 1 \; \cdots \; 1 \; 1]$

(b) $pdq^T = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} [1] [0 \; \cdots \; 0 \; 1]$

(c) $pdq^T = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} [1] [1 \; 1 \; \cdots \; 1 \; 1]$

(3) Change (from the stamp):

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & +\Delta a_{22} & 0 & +\Delta a_{21} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\Delta a_{12} & 0 & -\Delta a_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ i_1 \\ i_V \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta a_{22} & \Delta a_{21} \\ -\Delta a_{12} & -\Delta a_{11} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$