ECE 329 Review for Exam 2

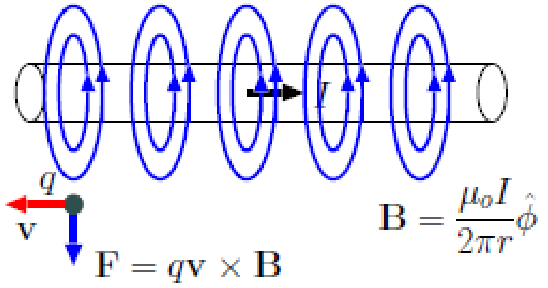
Exam 2 info

- ECEB 1002
- Thursday (10/18) 7- 8: 15 PM
- Cover Lecture 1-19
- HW 1-7

Concept list

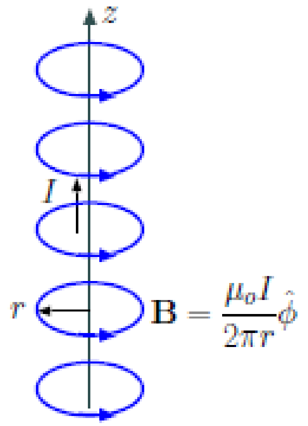
- Ampere’s law (lecture 12 & 13)
- Faraday’s law (lecture 14)
- Inductance (lecture 15)
- Charge conservation (lecture 16 & 17)
- TEM wave solutions (lecture 18 & 19)

Ampere's law



- Ic stands for the net sum of all filament currents In crossing any surface S bounded by path C

- The direction of flow is given by the "right-hand-rule"



$$\mathbf{H} \equiv \mu_o^{-1}\mathbf{B}$$

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_o I_C$$

$$I_C = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$
$$\nabla \cdot \mathbf{B} = 0$$

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S}$$

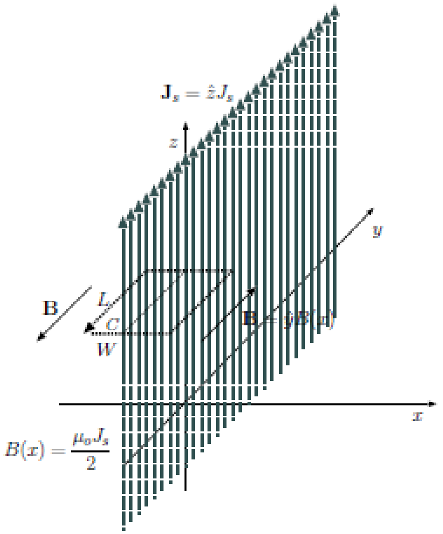
$$\mathbf{J}(x, y, z) = \mathbf{J}_s(y, z)\delta(x - x_o)$$

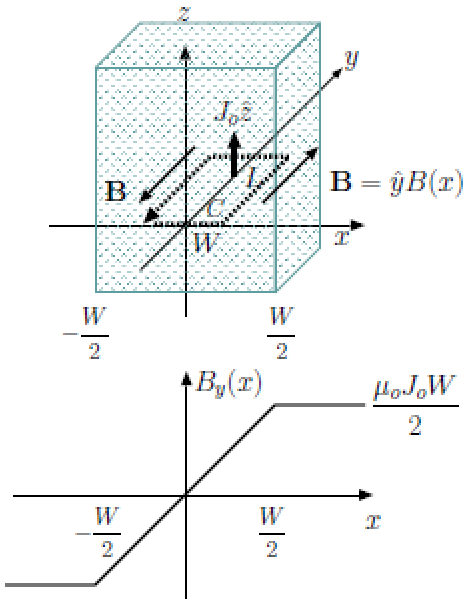
- This is the volumetric current density representation of a surface current density $J_s(x, y)$ measured in A/m units flowing on $x = x_o$ surface.

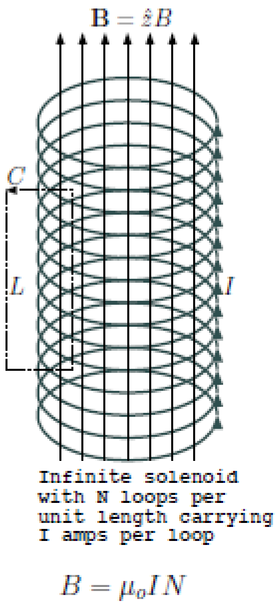
$$\mathbf{J}(x, y, z) = \hat{z}I(z)\delta(x - x_o)\delta(y - y_o)$$

- This is the line current I(z) measured in A units flowing in z-direction along a filament defined by the intersections of $x = x_o$ and $y = y_o$ surfaces.

Draw the loop







$$B(x)L + 0 - B(-x)L + 0 = \mu_o J_s L$$

$$B(x)L + 0 - B(-x)L + 0 = \mu_o J_o 2xL \;\Rightarrow\; B(x) = \mu_o J_o x$$

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_o I_C \;\Rightarrow\; LB = \mu_o I N L$$

$$\mathbf{B} = \hat{y} \frac{\mu_o J_s}{2} \mathrm{sgn}(x) \text{ and } \mathbf{H} = \hat{y} \frac{J_s}{2} \mathrm{sgn}(x)$$

$$B(x)L + 0 - B(-x)L + 0 = \mu_o J_o W L \;\Rightarrow\; B(x) = \mu_o J_o \frac{W}{2}$$

$$B = \mu_o I N \text{ and } \mathbf{H} = \hat{z} I N$$

$$\mathbf{H} = \begin{cases} \hat{y} J_o x, & |x| < \frac{W}{2} \\ \hat{y} J_o \frac{W}{2} \,\mathrm{sgn}(x), & \text{otherwise} \end{cases}$$

$$\mathbf{B} = 0 \text{ for the exterior region}$$

Faraday’s law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\Psi \equiv \int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\mathcal{E} = -\frac{d\Psi}{dt}, \qquad \text{Faraday's law}$$

$$\mathcal{E} = \oint_C \mathbf{E} \cdot d\mathbf{l} \qquad I = \frac{\mathcal{E}}{R}$$

$$\mathcal{E} = \oint_C (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l}$$

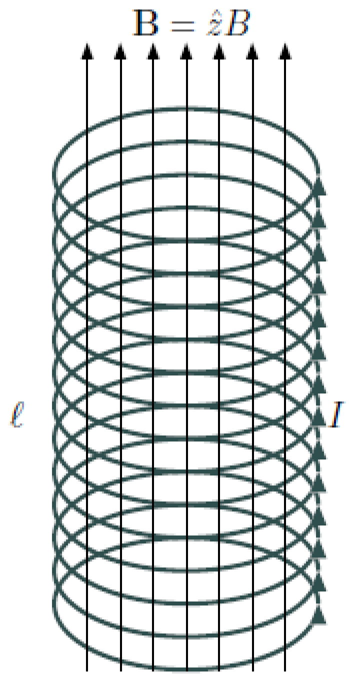
- The right hand side of the integral form equation above includes the flux of rate of change of magnetic field B over surface S.
- Ψ is the rate of change of magnetic flux.
- Induced emf (short for electro-motive force) represents the work done per unit charge moved once around path C.
- According to Faraday’s law it appears that a non-zero emf can always be caused by magnetic flux variations dΨ/dt independent of how the variations are produced — the possibilities are:

1. Fixed C, but time-varying B,
2. B =const. (in space and time), but time-varying C (rotating or changing size),
3. An inhomogeneous static B = B(r) in the measurement frame and C in motion.

Inductance

$$L \equiv \frac{n\Psi}{I} \qquad \mathcal{E} = -L\frac{dI}{dt}$$

- This is the self-inductance and emf for an inductor consisting of n-loops.

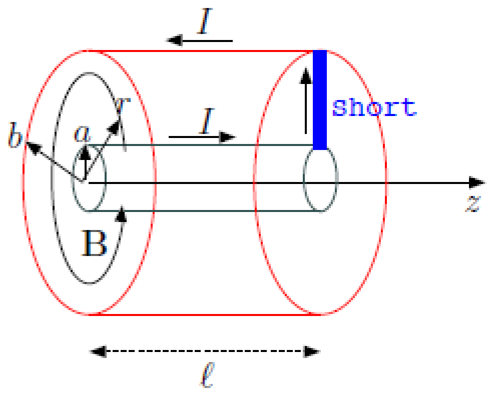


$$B = \mu_o I N$$

- **Inductance of long solenoid:**
- Consider a long solenoid with length ℓ, cross-sectional area A, density of N loops per unit length and n = Nℓ is the number of turns of the solenoid.

$$\mathbf{B} = \mu_o I N \hat{z}$$

$$L = \frac{n\Psi}{I} = \frac{N\ell(\mu_o I N)A}{I} = N^2\mu_o A\ell$$



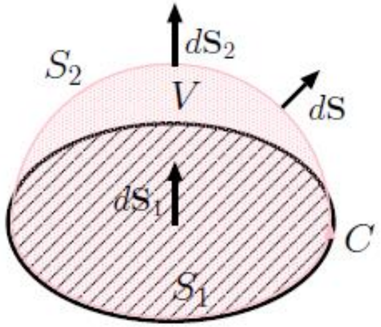
- **Inductance of shorted coax:**
- Consider a coaxial cable of some length ℓ which is “shorted” at one end (with a wire connecting the inner and outer conductors)

$$\oint_C \mathbf{B}\cdot d\mathbf{l} = \mu_o I_C$$

$$B_\phi 2\pi r = \mu_o I \qquad B_\phi = \frac{\mu_o I}{2\pi r}$$

$$\Psi = \int_S \mathbf{B}\cdot d\mathbf{S} = \ell\frac{\mu_o}{2\pi}I\int_a^b \frac{dr}{r} = \ell\frac{\mu_o}{2\pi}\ln\frac{b}{a}I \qquad L \equiv \frac{\ln\frac{b}{a}}{2\pi}\ell\mu_o$$

Charge conservation



$$\int_V \frac{\partial \rho}{\partial t} dV = -\oint_S \mathbf{J} \cdot d\mathbf{S}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{J} \qquad \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0$$

- Continuity equation is a mathematical re-statement of the principle of conservation of charge.

$$\begin{aligned} \nabla \cdot \mathbf{D} &= \rho && \text{Gauss's law} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} && \text{Faraday's law} \\ \nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, && \text{Ampere's law} \end{aligned}$$

- The modified Ampere's law postulated by Maxwell under the assumption that Gauss's law is also valid under time-varying conditions, leads to some specific predictions about how time-varying fields should behave.

- M is referred to as magnetization field; χm is a dimensionless parameter called magnetic susceptibility.

$$\begin{aligned} \hat{n} \cdot (\mathbf{D}^+ - \mathbf{D}^-) &= \rho_s \\ \hat{n} \cdot (\mathbf{B}^+ - \mathbf{B}^-) &= 0 \\ \hat{n} \times (\mathbf{E}^+ - \mathbf{E}^-) &= 0 \\ \hat{n} \times (\mathbf{H}^+ - \mathbf{H}^-) &= \mathbf{J_s} \end{aligned} \qquad \begin{aligned} \mathbf{D} &= \epsilon \mathbf{E} \\ \mathbf{B} &= \mu \mathbf{H} \end{aligned}$$

$$\mathbf{H} = \mu_o^{-1} \mathbf{B} - \mathbf{M} \qquad \mathbf{B} = \mu_o (1 + \chi_m) \mathbf{H} = \mu \mathbf{H}$$

TEM wave solutions

$$-\nabla^2 \mathbf{E} = -\mu \frac{\partial}{\partial t} \nabla \times \mathbf{H}$$

$$\nabla^2 \mathbf{E} = \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\frac{\partial^2 \mathbf{E}}{\partial x^2} + \frac{\partial^2 \mathbf{E}}{\partial y^2} + \frac{\partial^2 \mathbf{E}}{\partial z^2} = \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

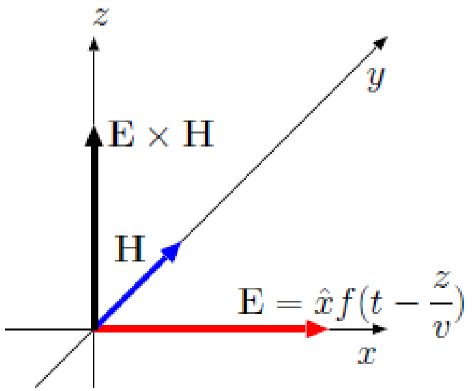
$$\mathbf{E}(\mathbf{r}, t) = \hat{x} E_x(z, t)$$

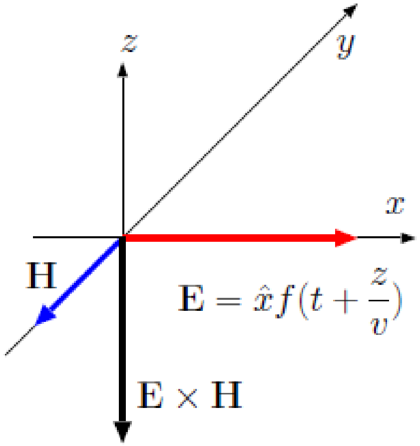
$$\frac{\partial^2 E_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2}$$

$$E_x = \cos(\omega(t \mp \frac{z}{v})) \qquad v \equiv \frac{1}{\sqrt{\mu\epsilon}}$$

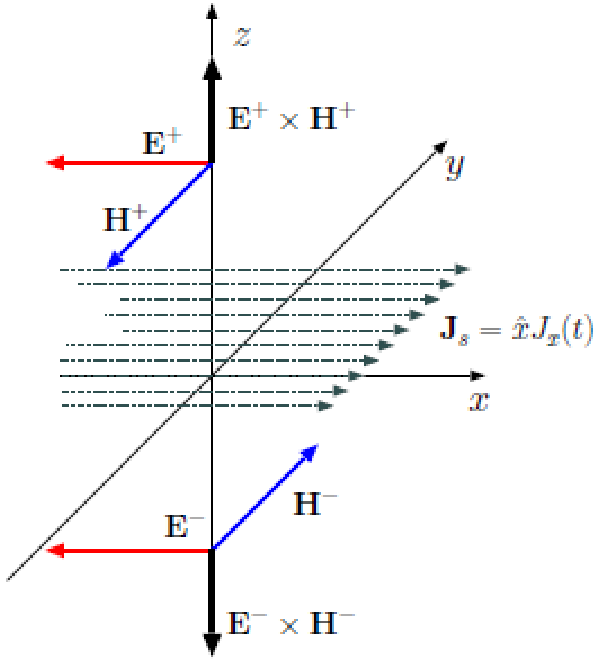
$$\mathbf{H} = \pm\hat{y} \sqrt{\frac{\epsilon}{\mu}} \cos(\omega(t \mp \frac{z}{v})) \qquad \eta \equiv \sqrt{\frac{\mu}{\epsilon}}$$

- The curl of Faraday's law combines with the Ampere's law to produce the 3D vector wave equation.
- 1D scalar wave equation is a field solution that only depends on z and t and "polarized" in x-direction.





TEM wave solutions



$$\mathbf{J}_s = \hat{x} f(t) \text{ on } z = 0 \text{ plane}$$

1. Ex and Hy waveforms are proportional to delayed versions of surface current Jx(t) at each location z above and below the current sheet, with the reference directions of E and Js opposing one another.

2. Fields E± are continuous on z = 0 surface in compliance with tangential boundary condition equations.

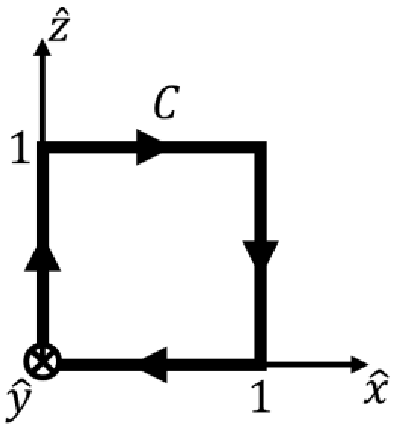
3. Fields H± exhibit a discontinuity on z = 0 surface that matches the current density of the same surface, once again in compliance with tangential boundary condition equations.

4. Opposing E and Js vectors on z = 0 plane indicate that the surface is acting as a source of radiated energy

$$\mathbf{E}^\pm = -\hat{x}\frac{\eta f(t \mp \frac{z}{v})}{2} \text{ and } \mathbf{H}^\pm = \mp\hat{y}\frac{f(t \mp \frac{z}{v})}{2} \text{ in regions } z \gtrless 0$$

Practice Problems

3. (18 points) A conducting wire of resistance $R = 5\ \Omega$ forms a 1 meter side-length square loop in the xz plane within a magnetic flux density $\mathbf{B} = 5y\cos(\omega t)\hat{x} + 5x\sin(\omega t)\hat{y} + 10e^{-t}\hat{z}$ Wb/m^2.



a) (12 pts) Find the emf $\mathcal{E}$ around the closed path C in the direction shown above.

Practice Problems

b) (4 pts) What is the magnitude of the loop current?

Your Answer (include appropriate units): $I =$

c) (2 pts) (CIRCLE ONE): At $t = 0$, the current flows $\begin{array}{c} ALONG \\ OPPOSITE \end{array}$ the path C.

Practice Problems

a) Three infinitely long conducting wires, all oriented parallel to the $\hat{z}$ axis but not centered on the $\hat{z}$ axis, carry currents $I_0\hat{z}$, $I_0\hat{z}$, and $-2I_0\hat{z}$, respectively. C is a circular contour of arbitrary radius r that encloses all three wires.

   i. TRUE or FALSE: $\oint_C \mathbf{H} \cdot \mathbf{dl} = 0$

   ii. TRUE or FALSE: $|\mathbf{H}| = 0$ everywhere along the contour C.

   iii. TRUE or FALSE: $H_z = 0$ everywhere along the contour C.

Practice Problems

(v) In free space, the magnetic field for a uniform plane wave is given by $\mathbf{H} = -2sin\,(\omega t - \beta x)\,\hat{y}\;[\frac{A}{m}]$. Noting that $\eta_0 \approx 120\pi\;[\Omega]$, the corresponding electric field is given by:

(a) $\mathbf{E} = -240\pi sin\,(\omega t - \beta x)\,\hat{z}\;[\frac{V}{m}]$.

(b) $\mathbf{E} = -240\pi cos\,(\omega t - \beta x)\,\hat{z}\;[\frac{V}{m}]$.

(c) $\mathbf{E} = 240\pi sin\,(\omega t - \beta x)\,\hat{z}\;[\frac{V}{m}]$.

(d) $\mathbf{E} = 240\pi cos\,(\omega t - \beta x)\,\hat{z}\;[\frac{V}{m}]$.

(e) None of these.