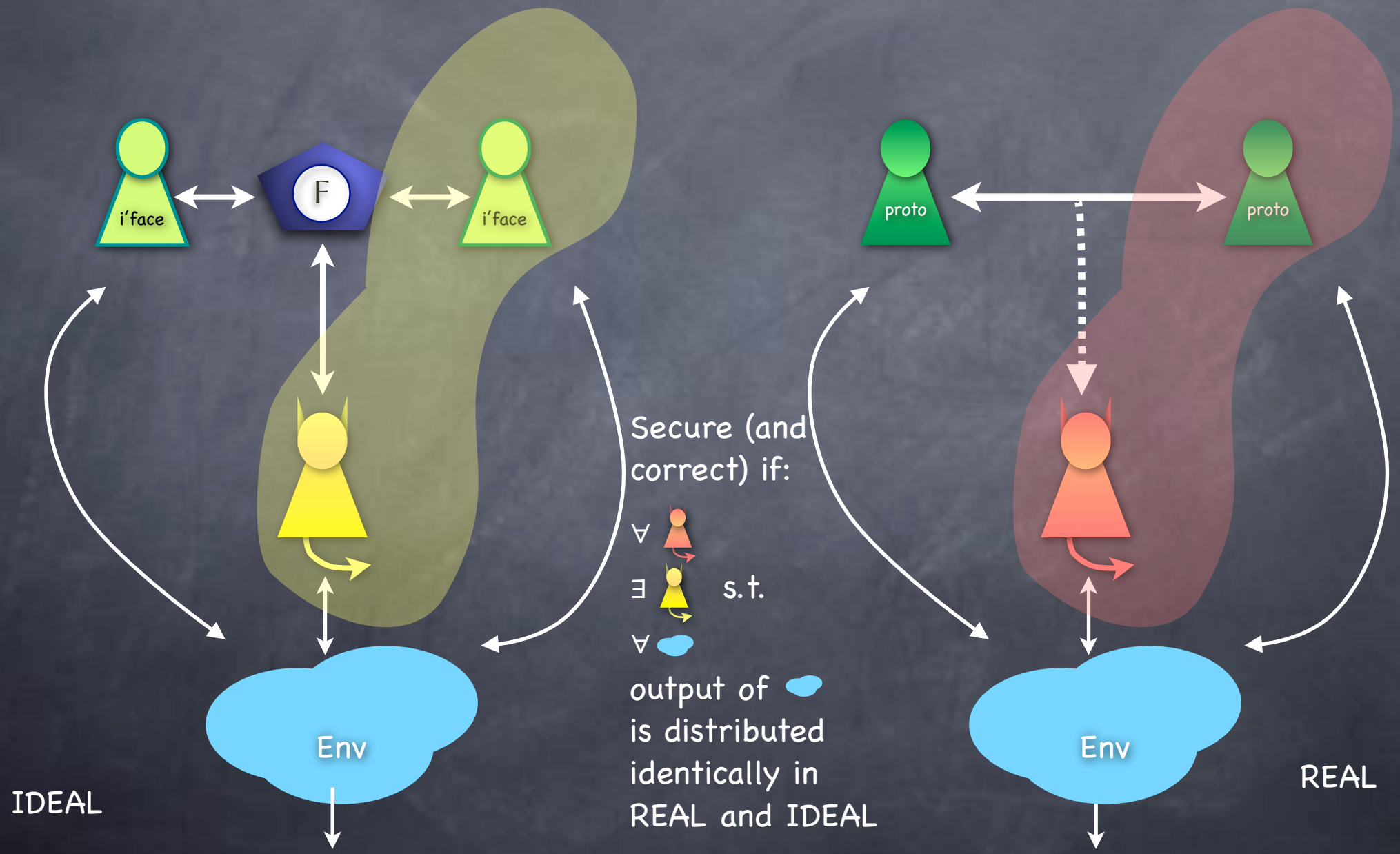
Secure 2-Party Computation

Lecture 14

Yao's Garbled Circuit

RECALL

SIM-Secure MPC



RECALL

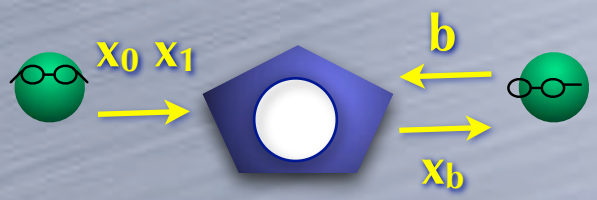
Passive Adversary

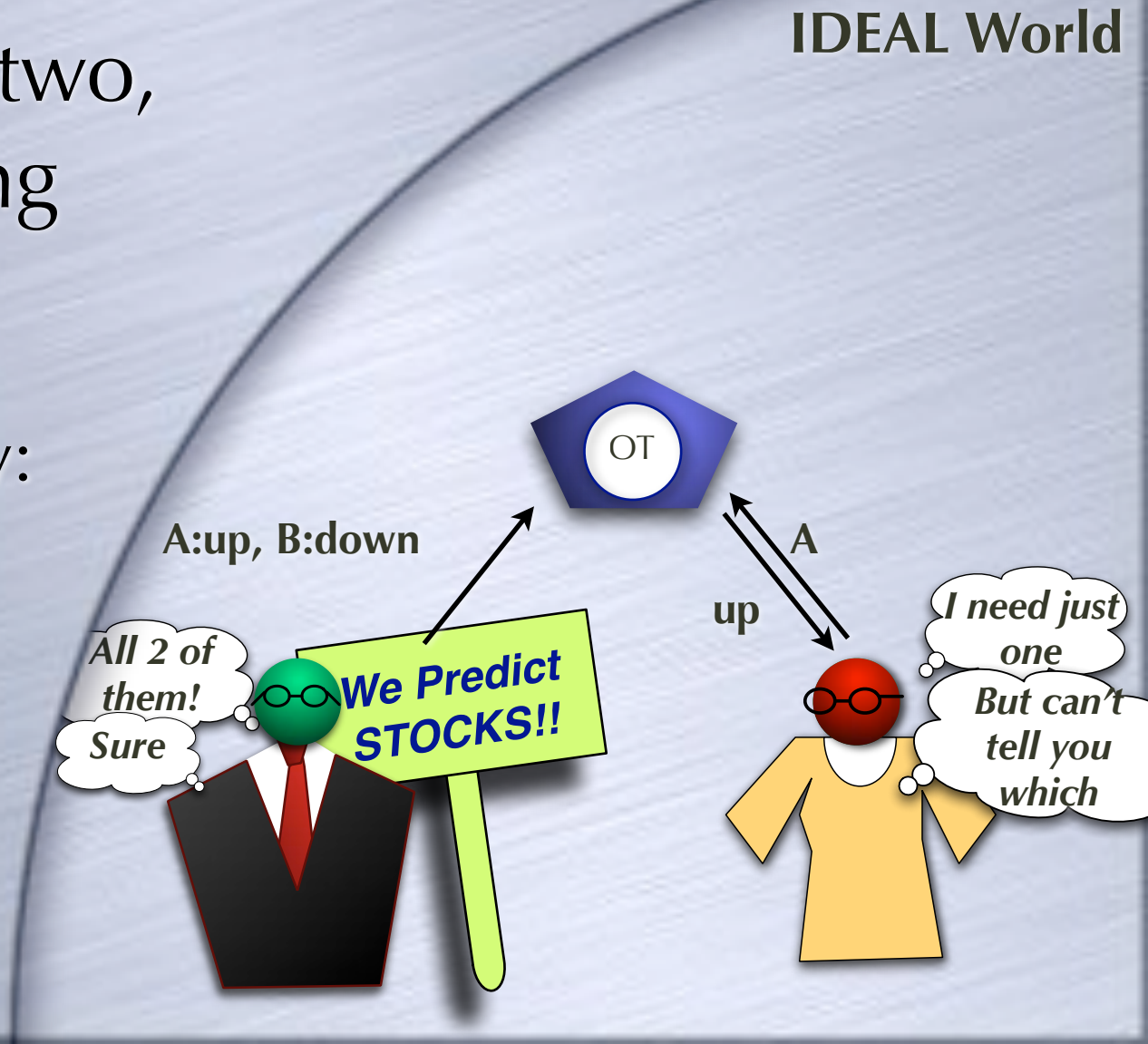
- Gets only read access to the internal state of the corrupted players (and can use that information in talking to environment)
  - Also called "Honest-But-Curious" adversary
  - Will require that simulator also corrupts passively
- Simplifies several cases
  - e.g. coin-tossing [why?], commitment [coming up]
- Oddly, sometimes security against a passive adversary is more demanding than against an active adversary
  - Active adversary: too pessimistic about what guarantee is available even in the IDEAL world
  - e.g. 2-party SFE for OR, with output going to only one party (trivial against active adversary; impossible without computational assumptions against passive adversary)

RECALL

Oblivious Transfer

- Pick one out of two, without revealing which
  - Intuitive property: transfer partial information "obliviously"





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- So, for passive security, enough to consider one-sided SFE

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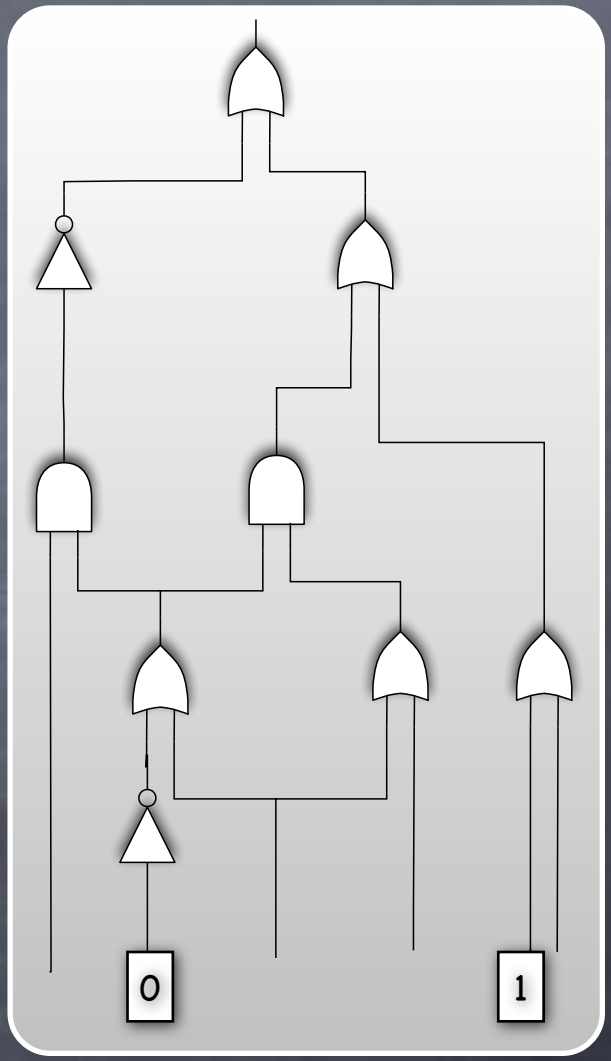
- Can we do "general" deterministic, one-sided SFE (i.e., for all functions)?

Boolean Circuits



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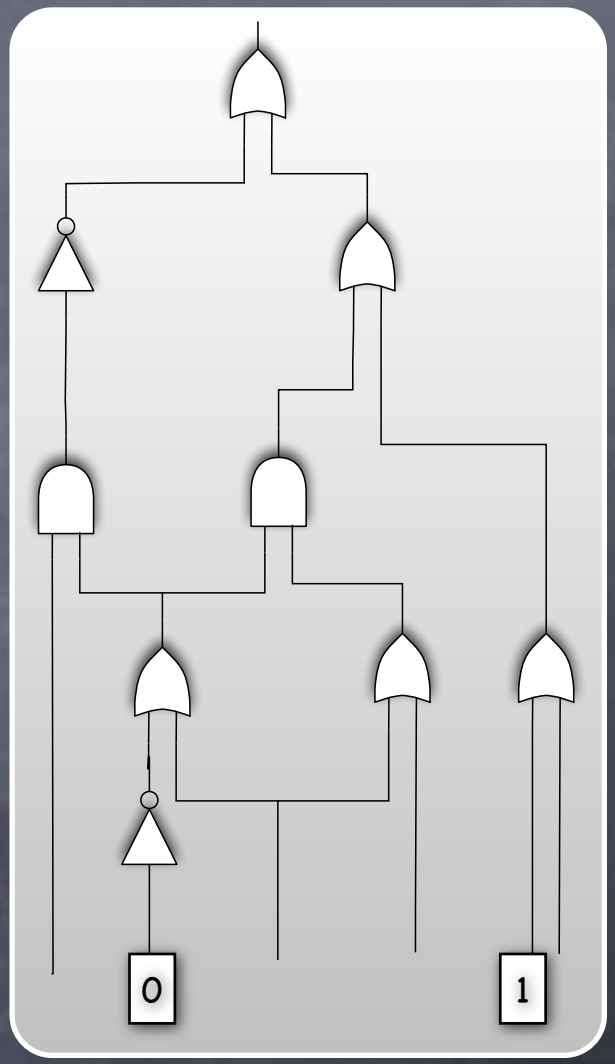

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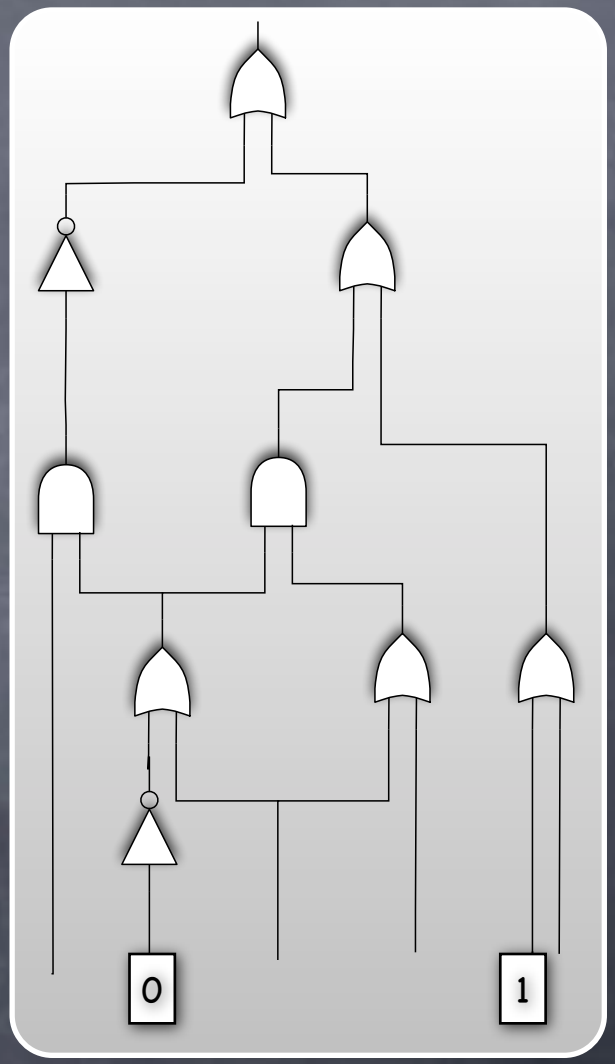

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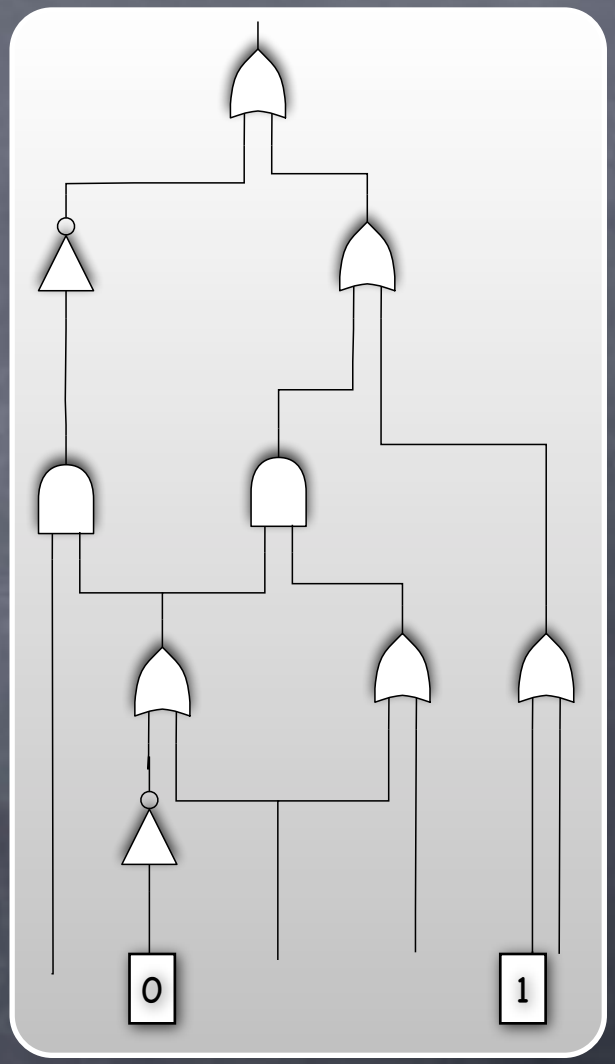

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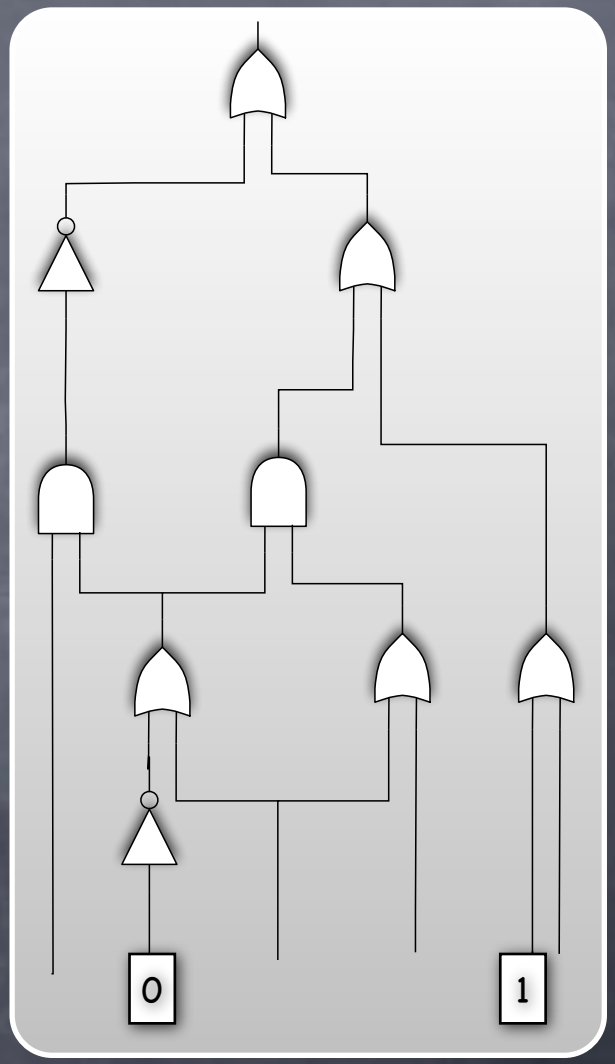

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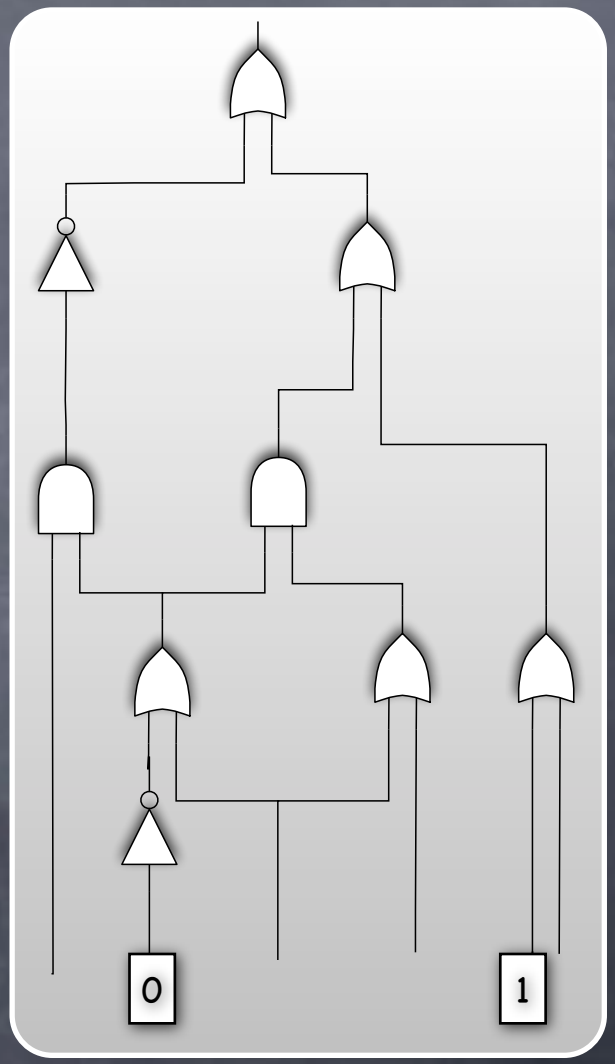

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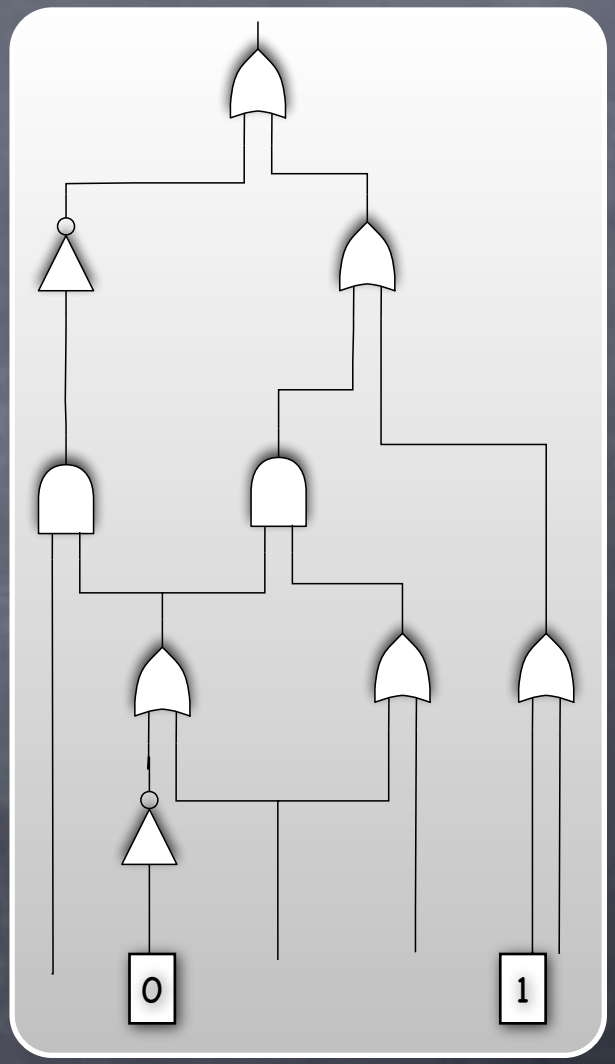

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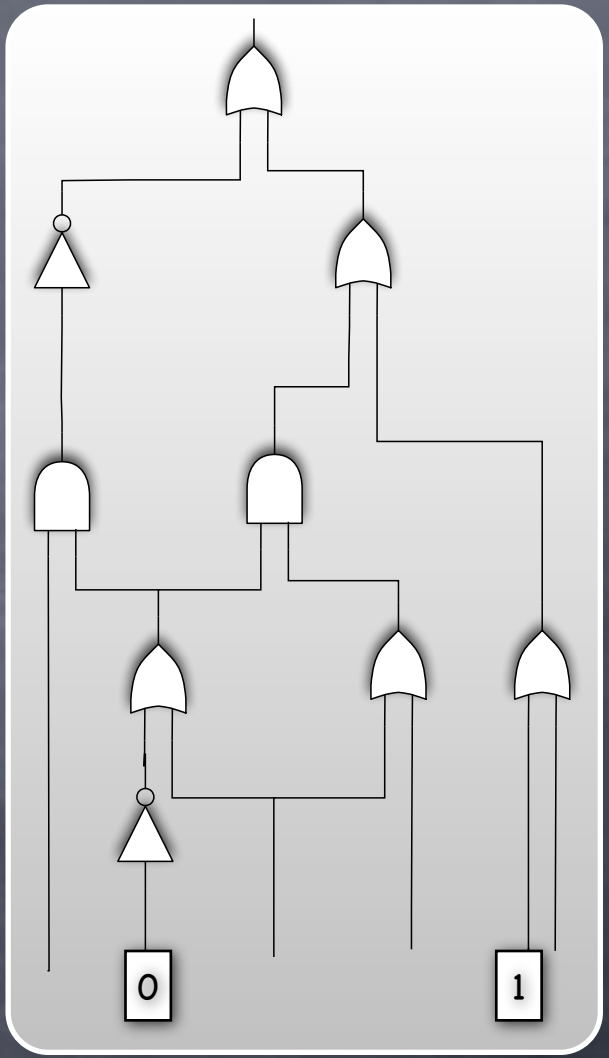
    - Note: no memory gates



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  - Often problems already described as succinct programs/circuits

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  - Any ideas?

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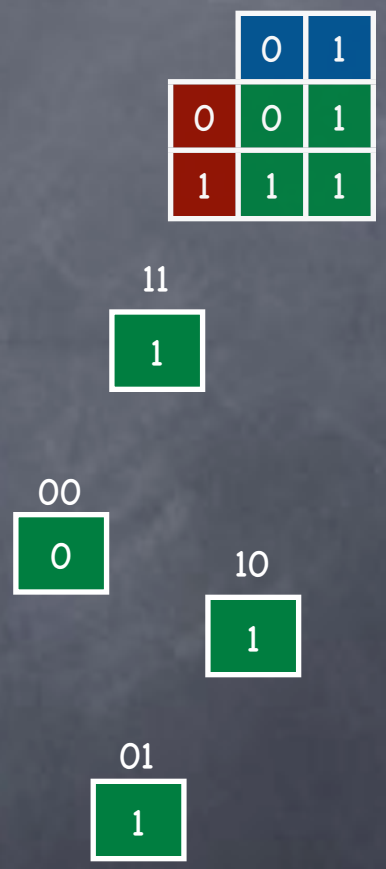

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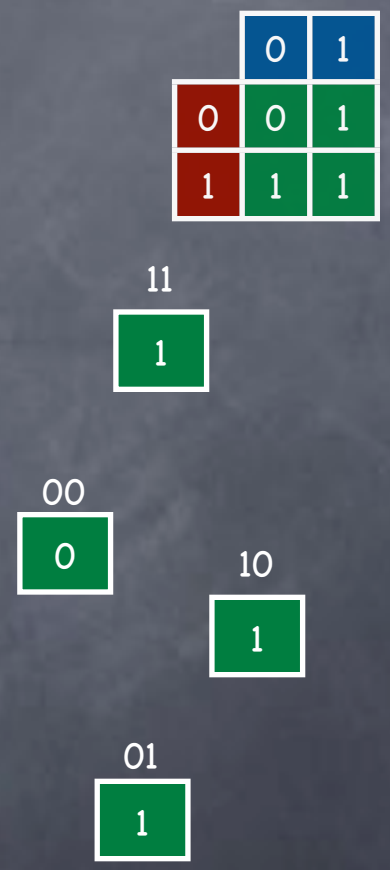

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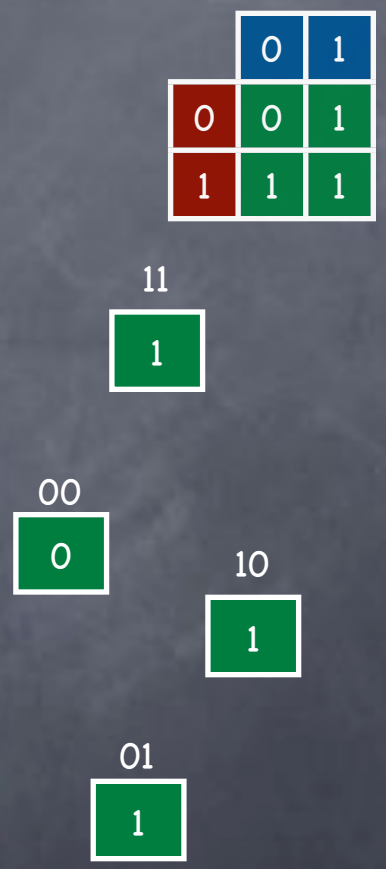

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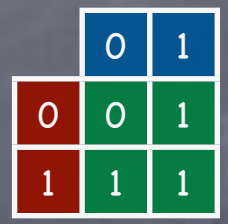

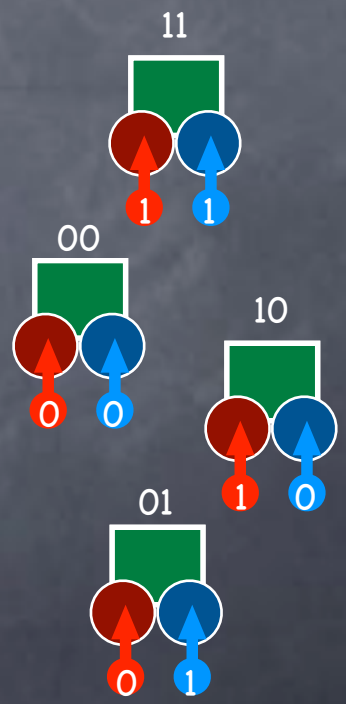

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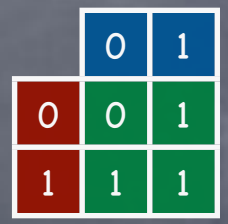

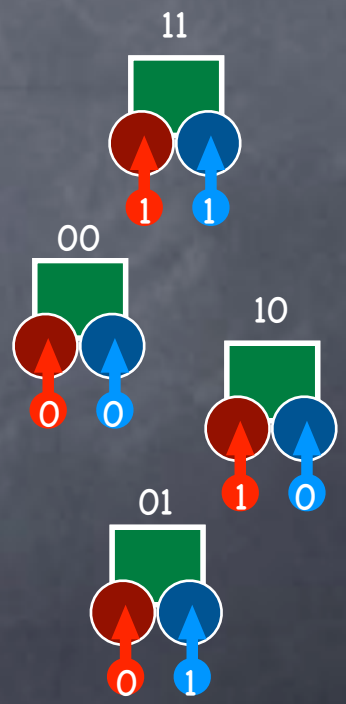

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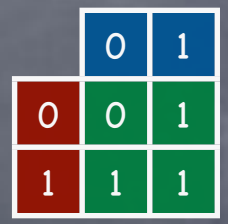

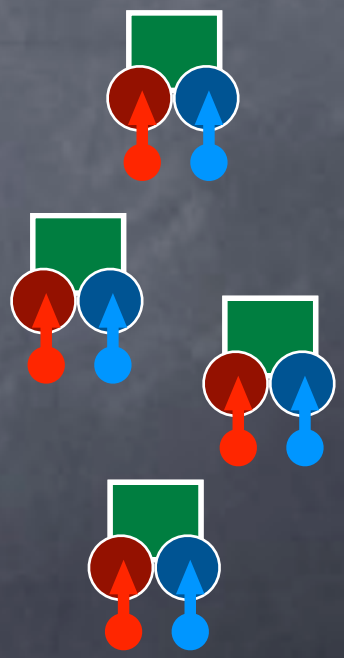

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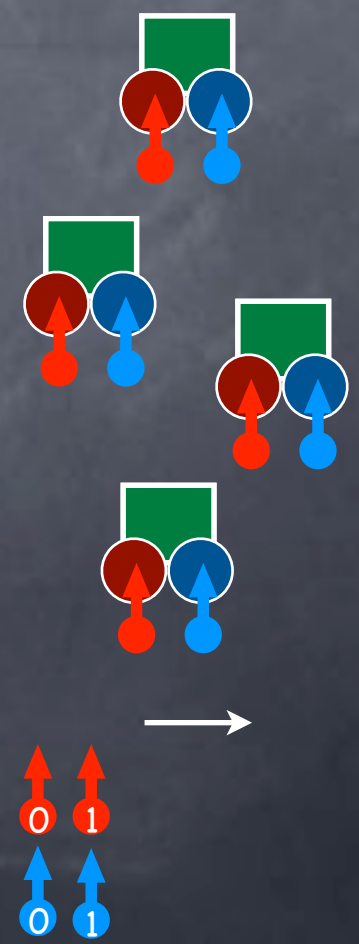

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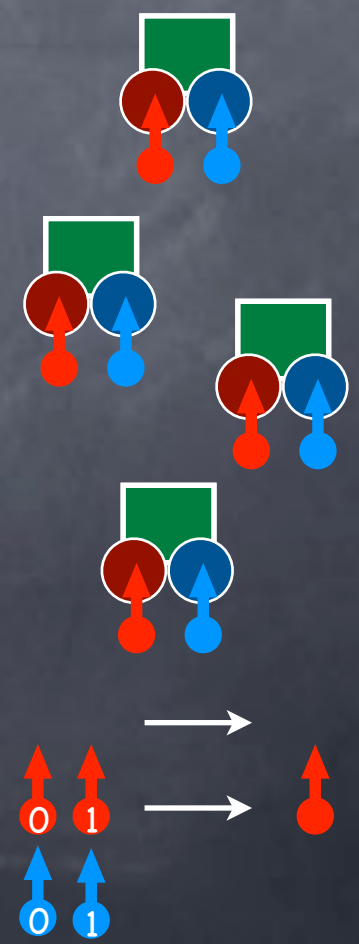

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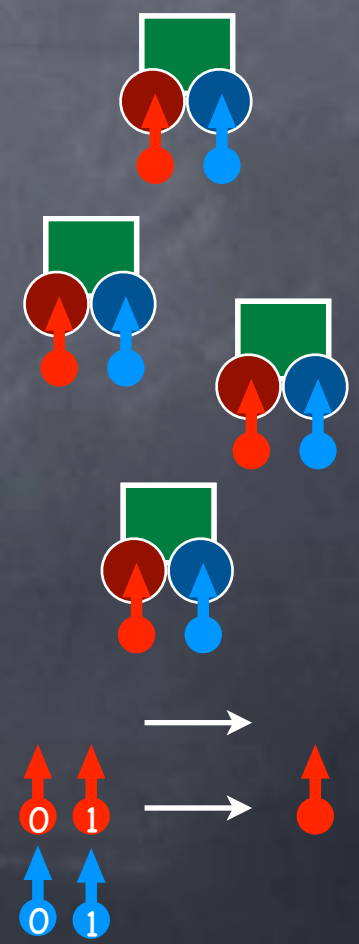
- Inside $B_{xy=ab}$ she places the bit OR(a,b) and locks it with two padlocks $K_{x=a}$ and $K_{y=b}$ (need to open both to open the box)
- She un-labels the four boxes and sends them in random order to Bob. Also sends the key $K_{x=a}$ (labeled only as K_x).
  - So far Bob gets no information

	0	1
0	0	1
1	1	1



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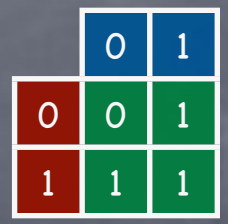

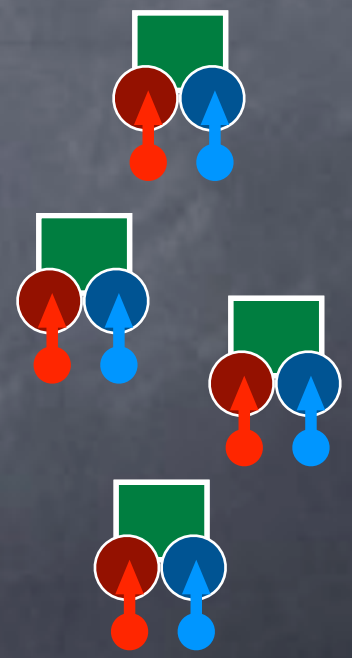

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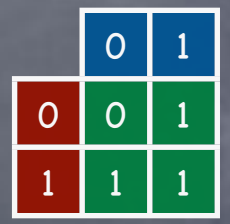

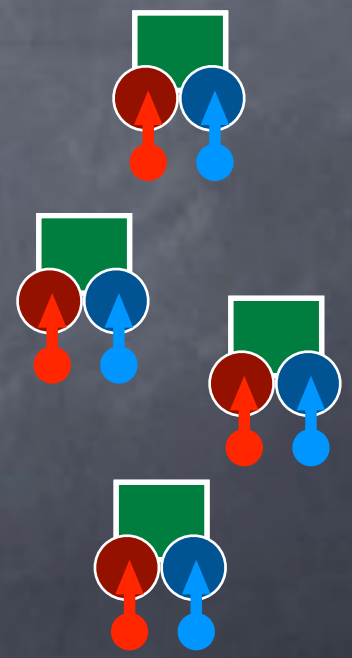

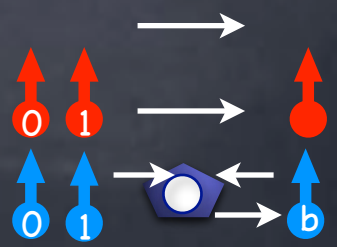

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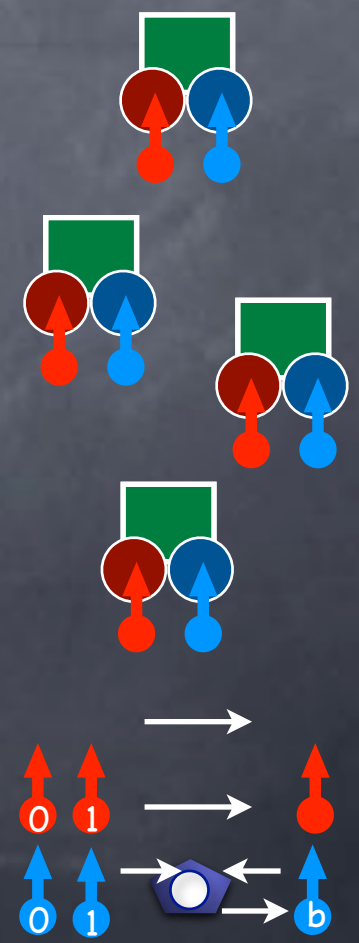

A Physical Protocol

- Secure?

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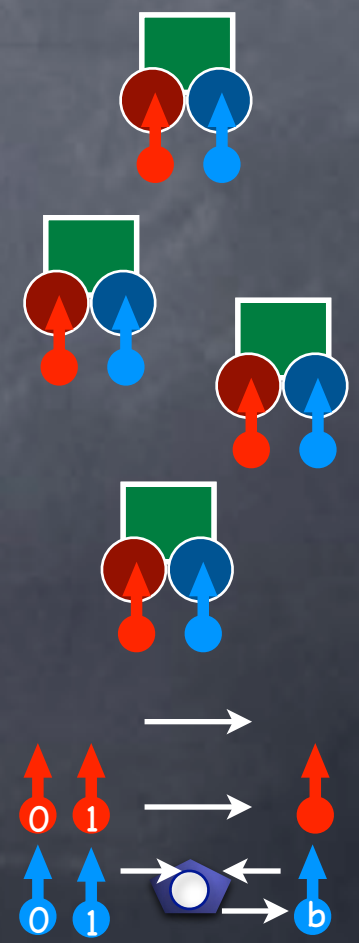
0	0	1
1	1	1



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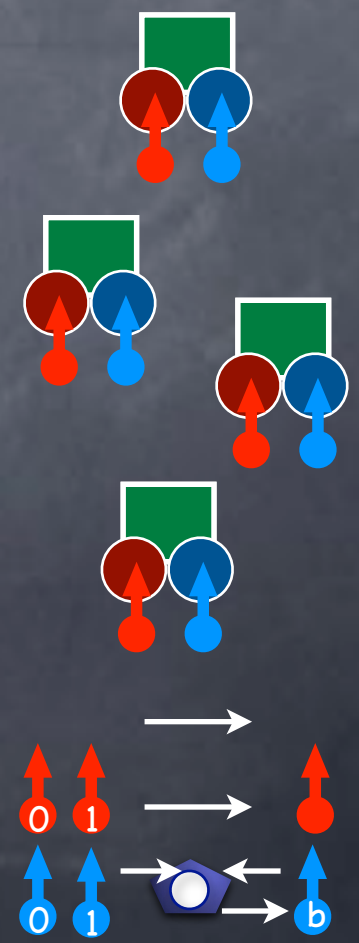

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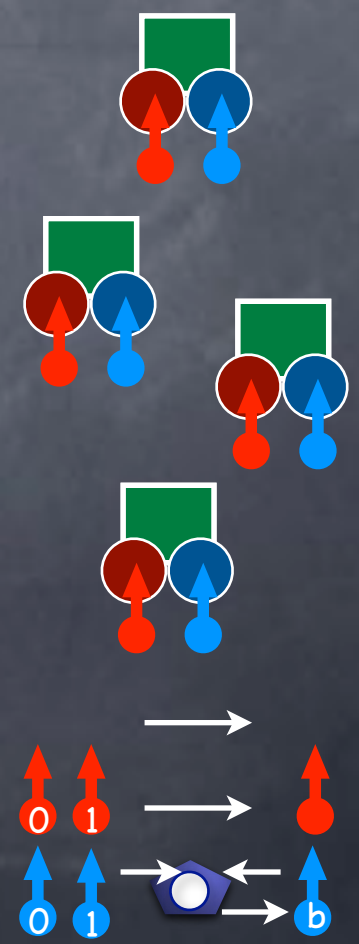

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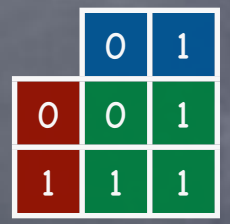

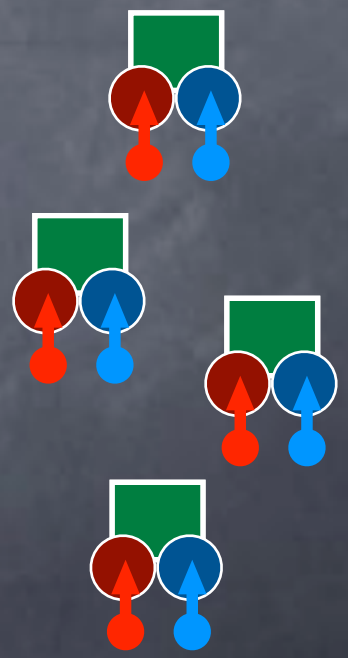

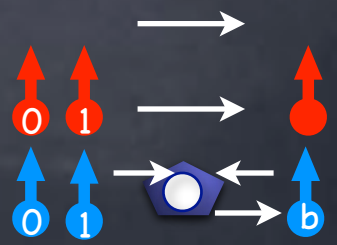

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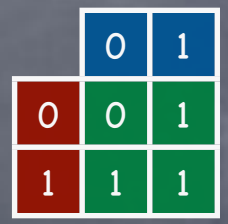

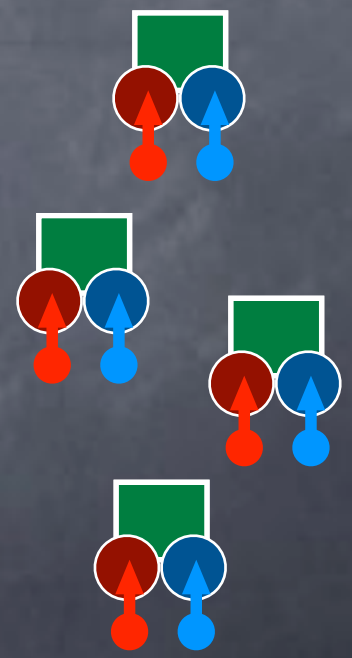

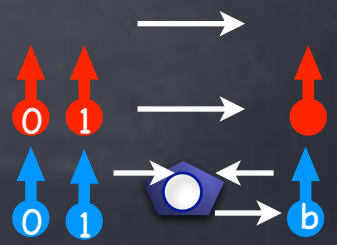

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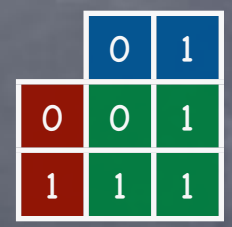

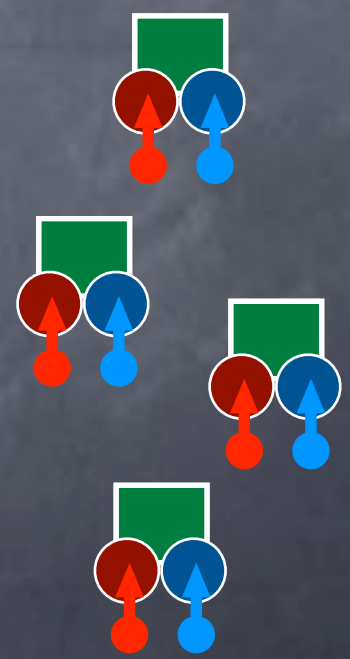

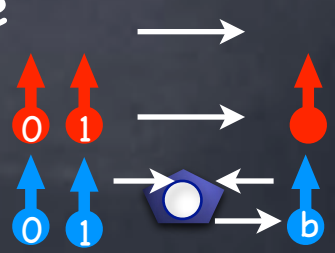

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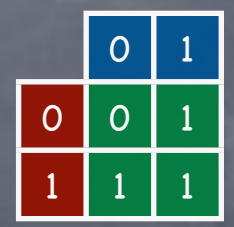
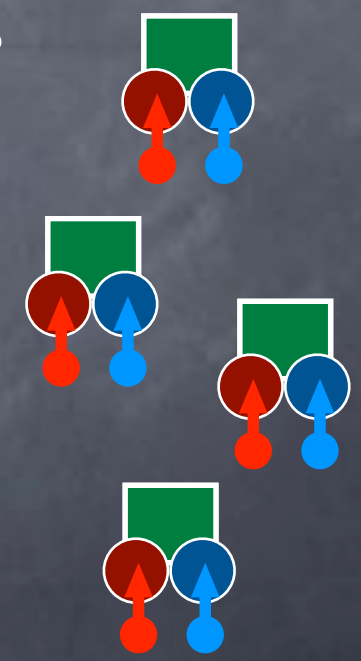
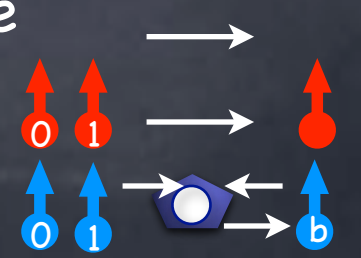
  - Note when y=1, cases x=0 and x=1 appear same
  - Formally, easy to simulate (can stuff unopenable boxes randomly)







Larger Circuits

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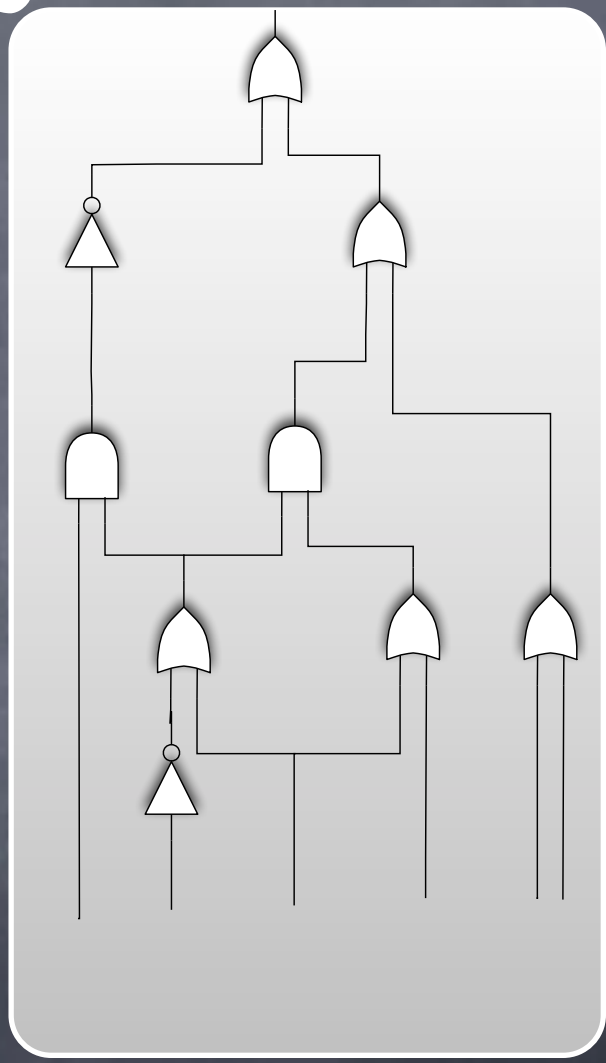

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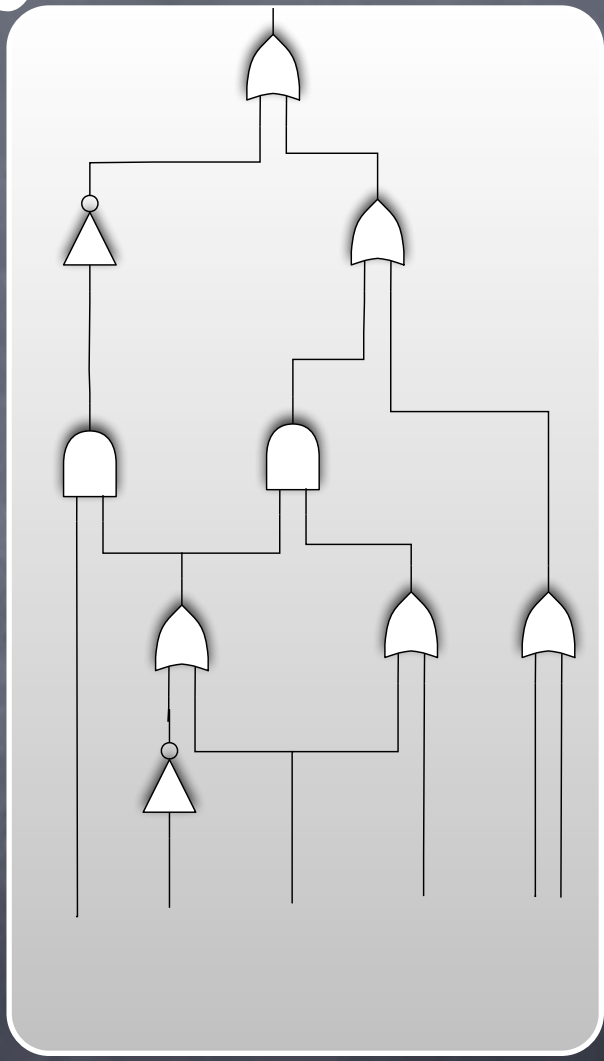
- Idea: For each gate in the circuit Alice will prepare locked boxes, but will use it to keep keys for the next gate



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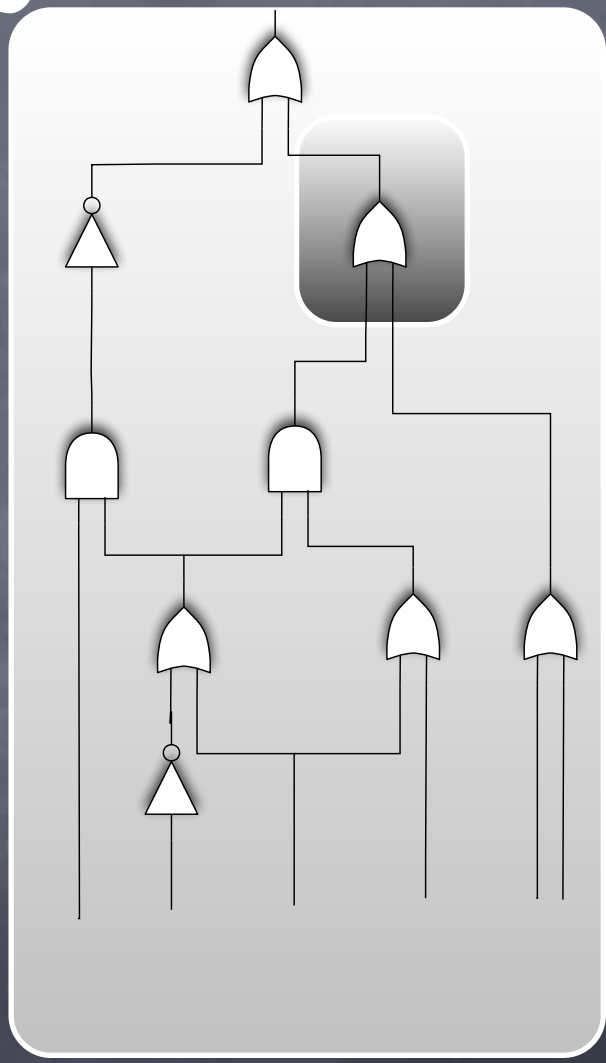

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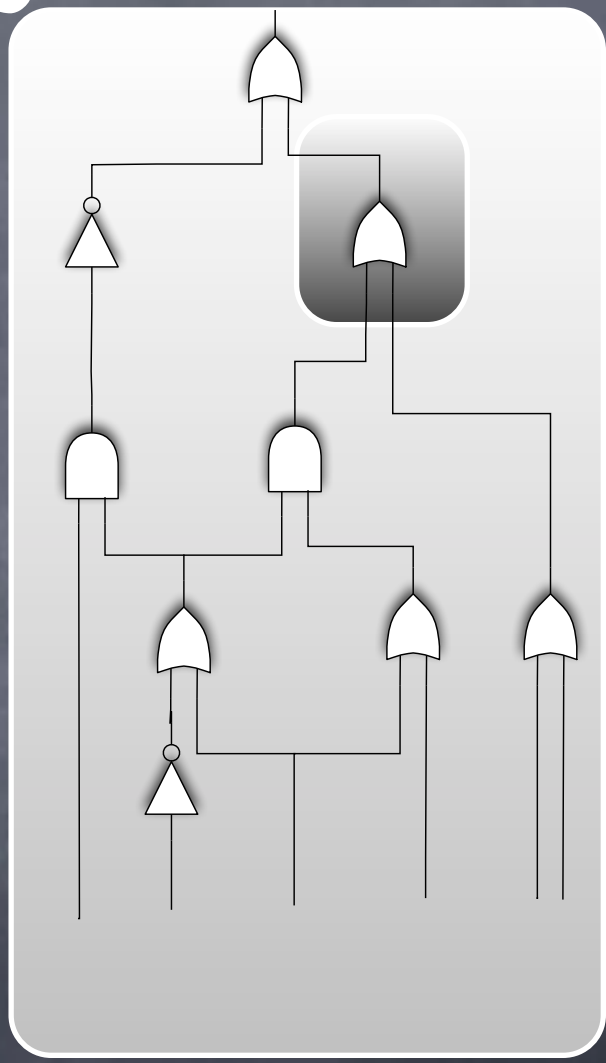

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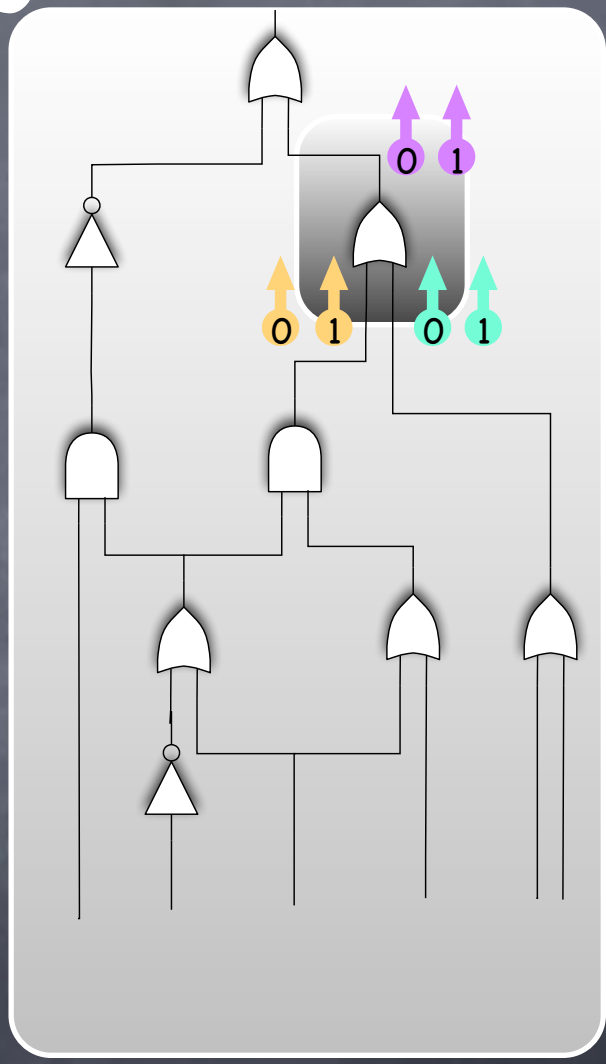

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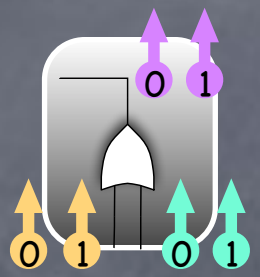

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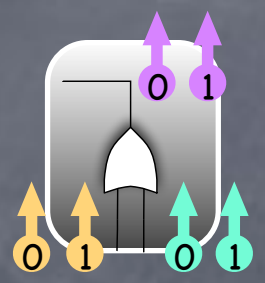

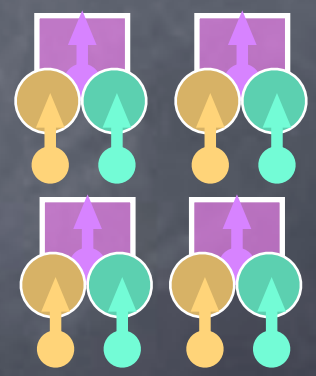

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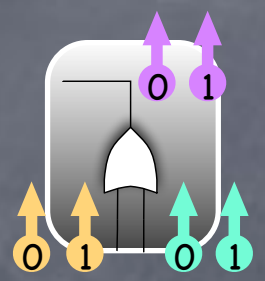

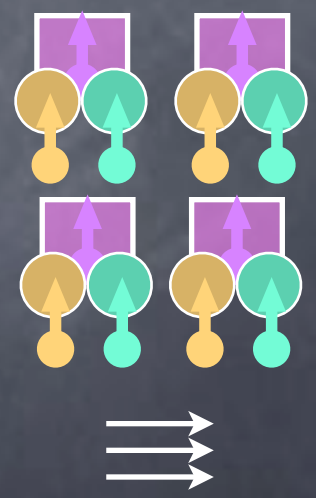

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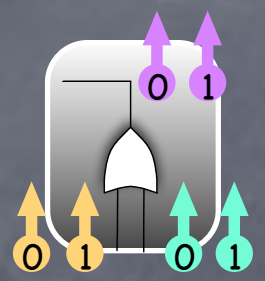

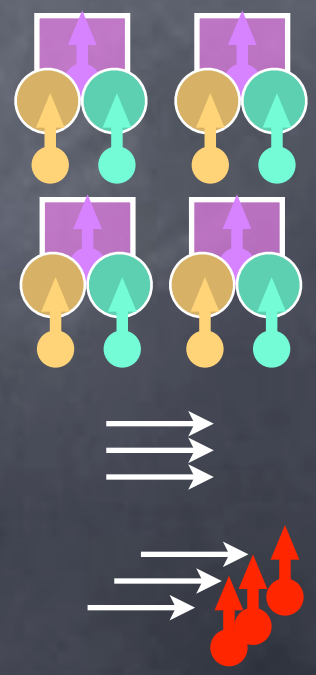

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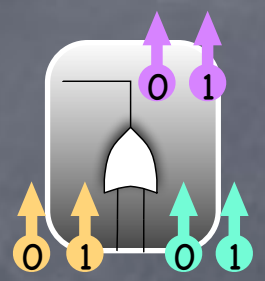

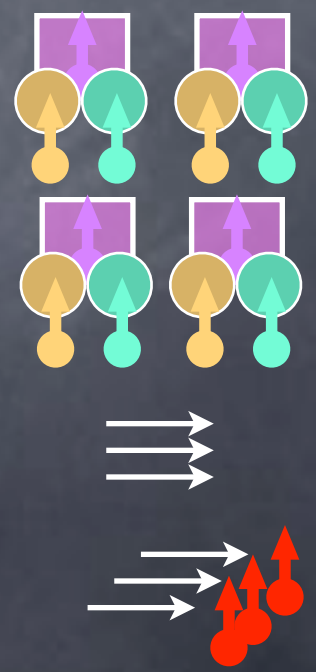

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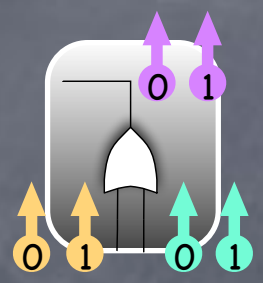

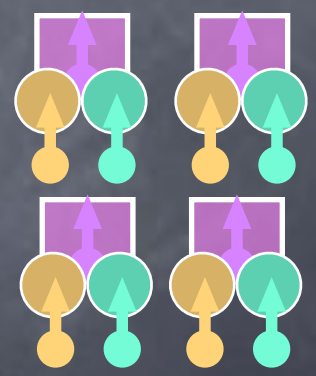

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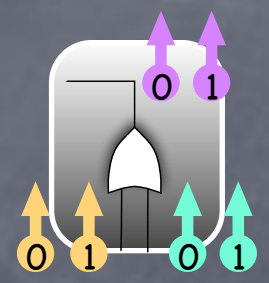

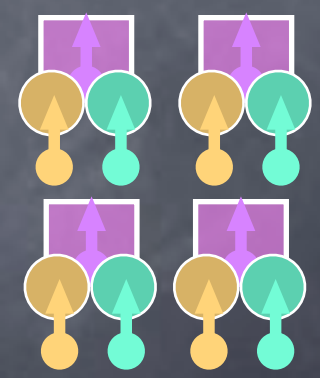

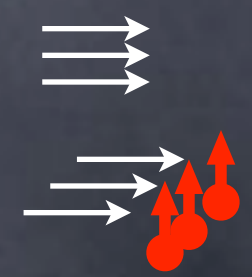

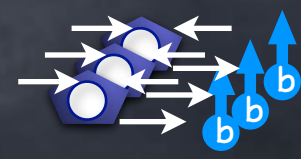

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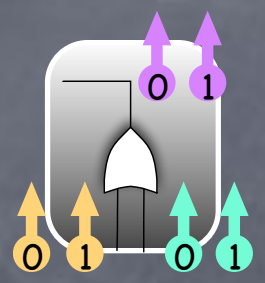

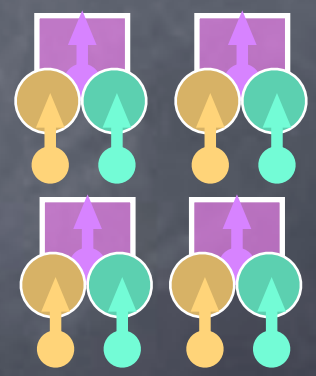

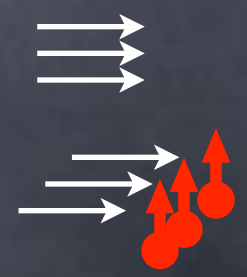

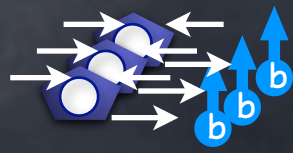

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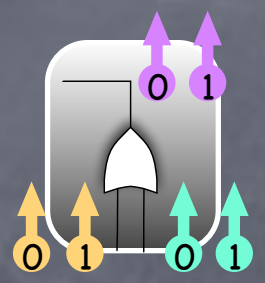

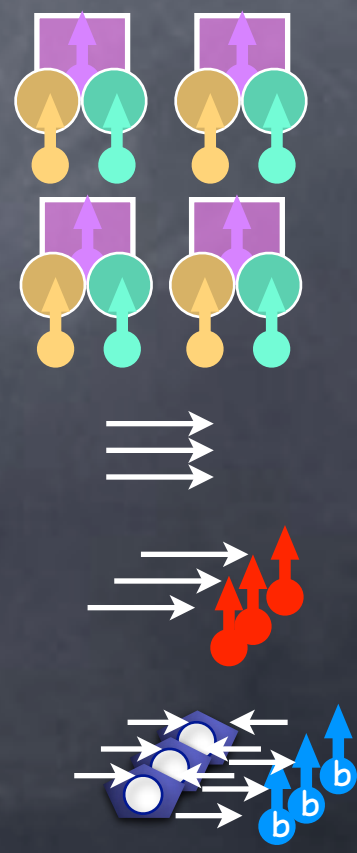

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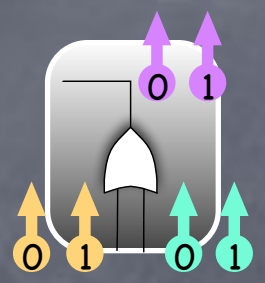

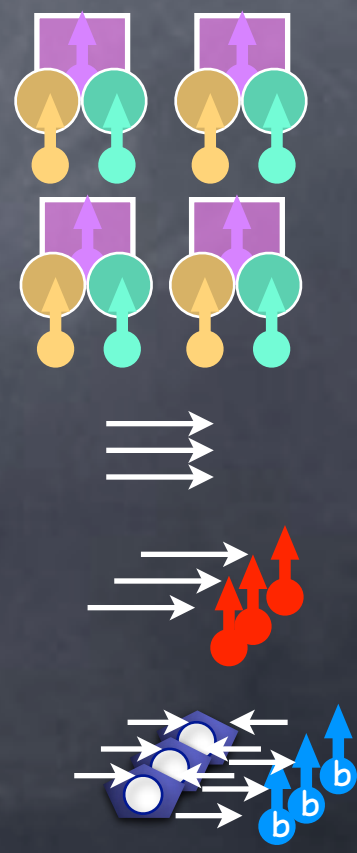

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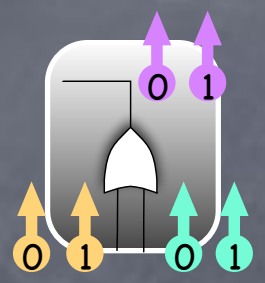

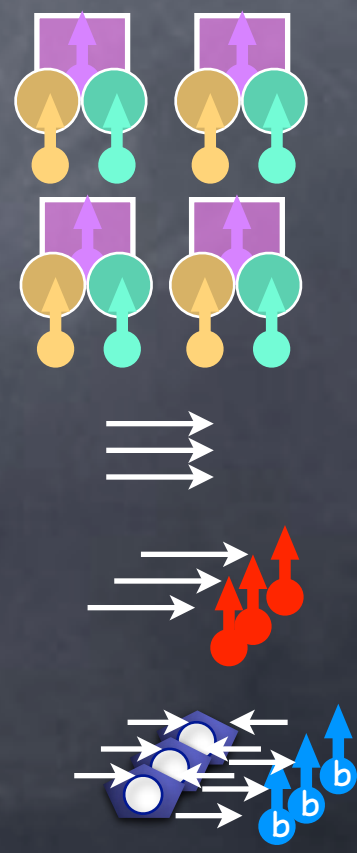

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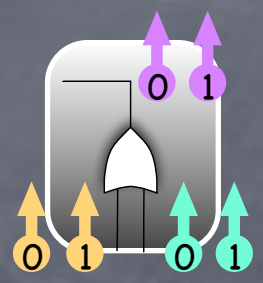

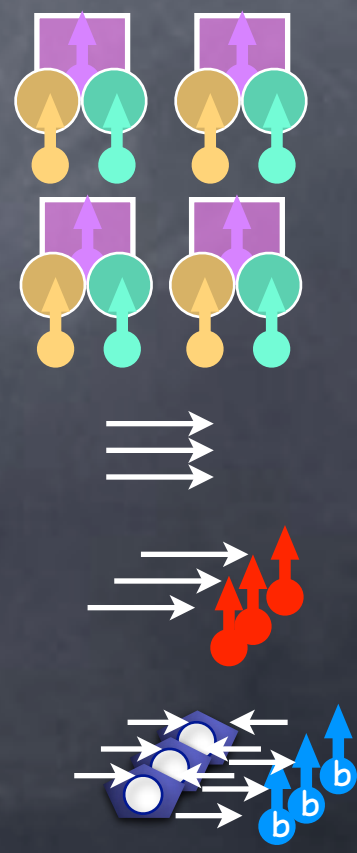

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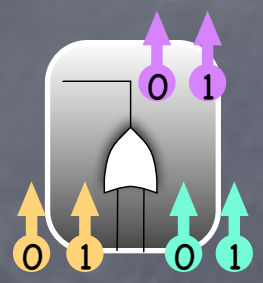
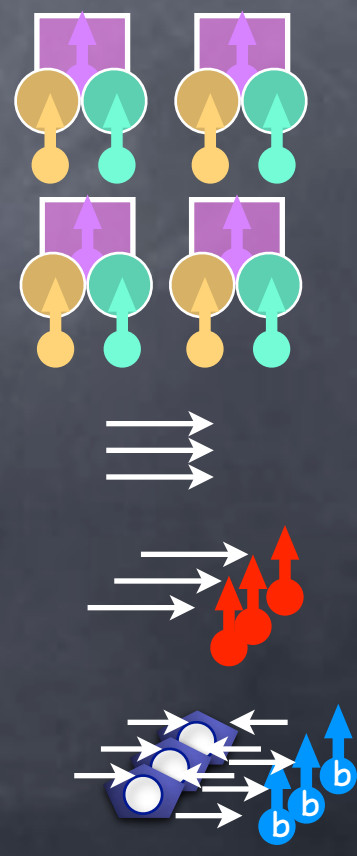
  - Everything is simulatable for curious Bob given final output: Bob could prepare boxes and keys (stuffing unopenable boxes arbitrarily); for an output gate, place the output bit in the box that opens





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  - Can we really compose? Yes, for passive security.

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- Coming up: Zero-Knowledge proofs and general multi-party computation, more protocols (for different settings). Universal Composition