Public-Key Cryptography

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Lecture 8

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Public-Key Encryption from Trapdoor OWP

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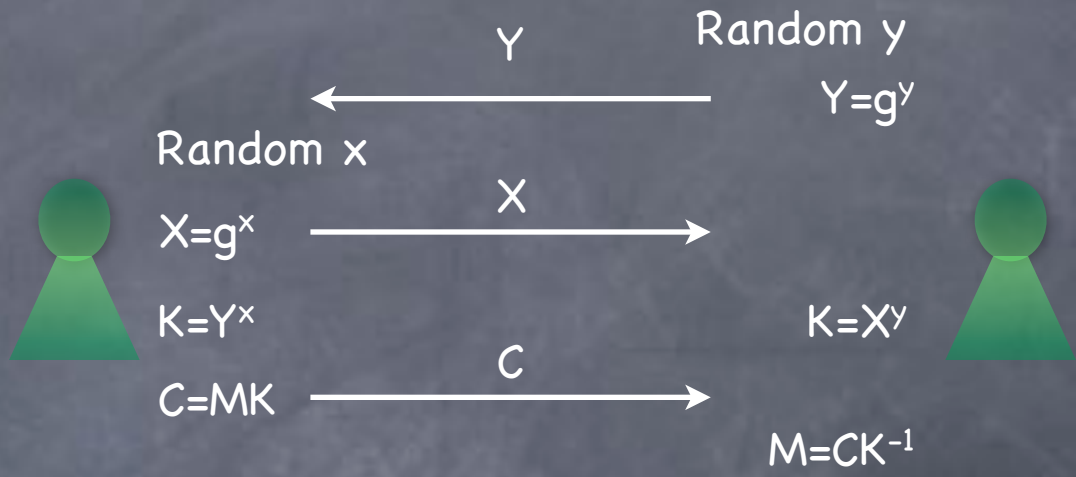
CCA Security

Abstracting El Gamal



KeyGen: PK=(G,g,Y), SK=(G,g,y)

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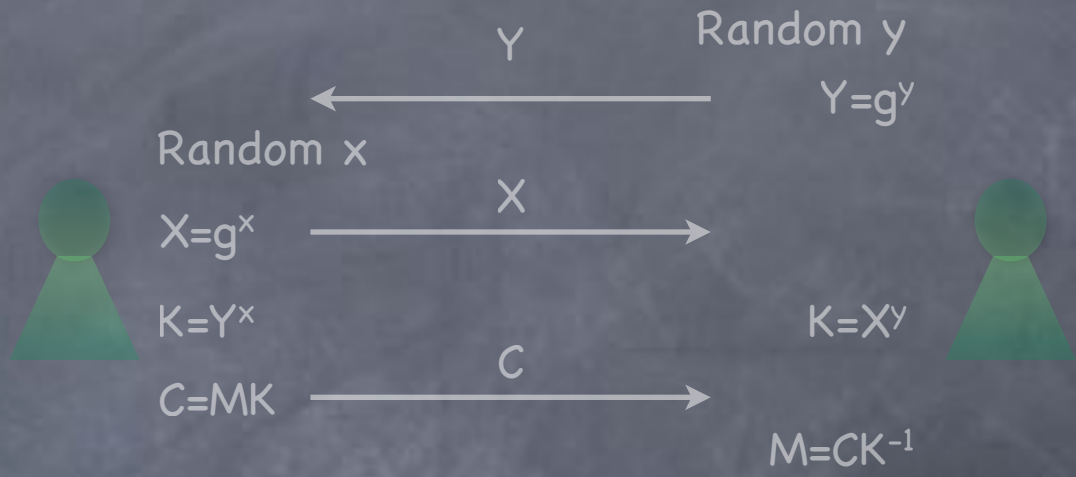

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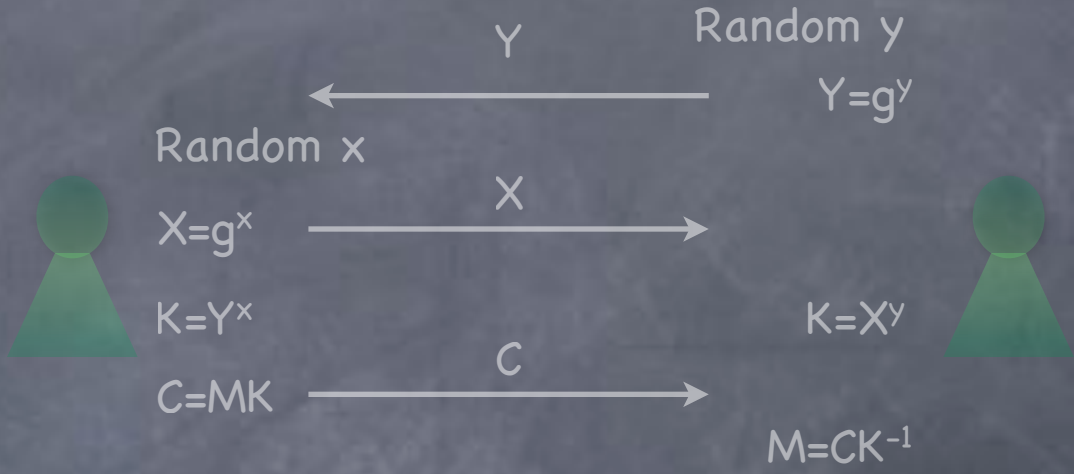

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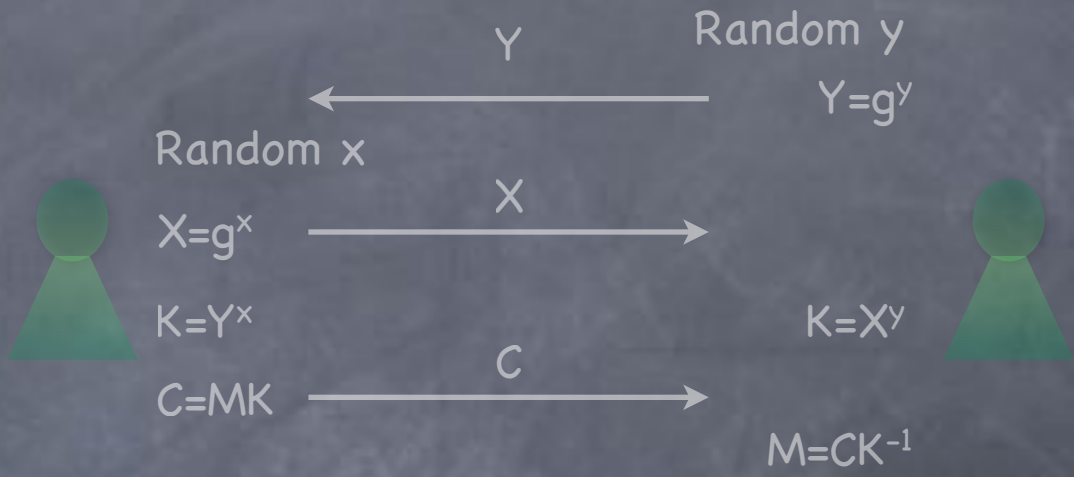

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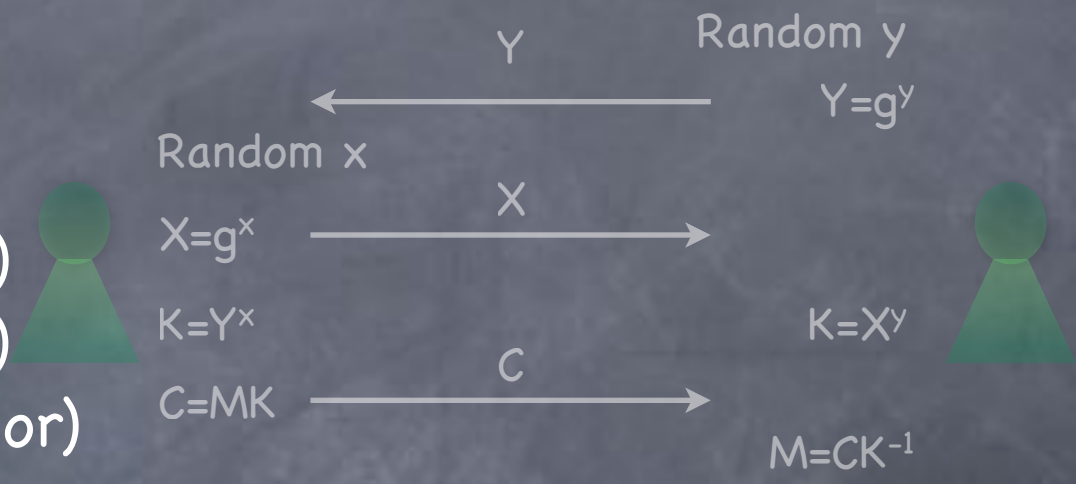

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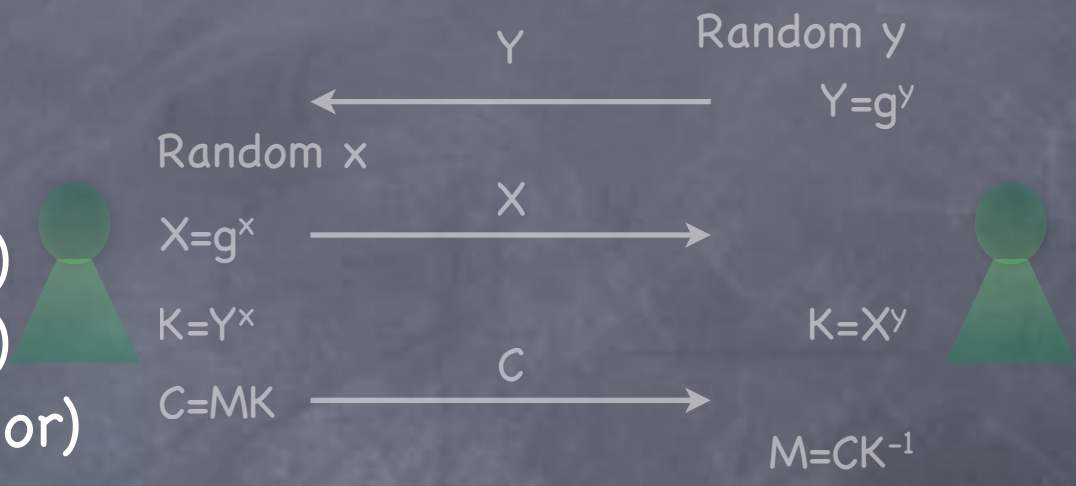

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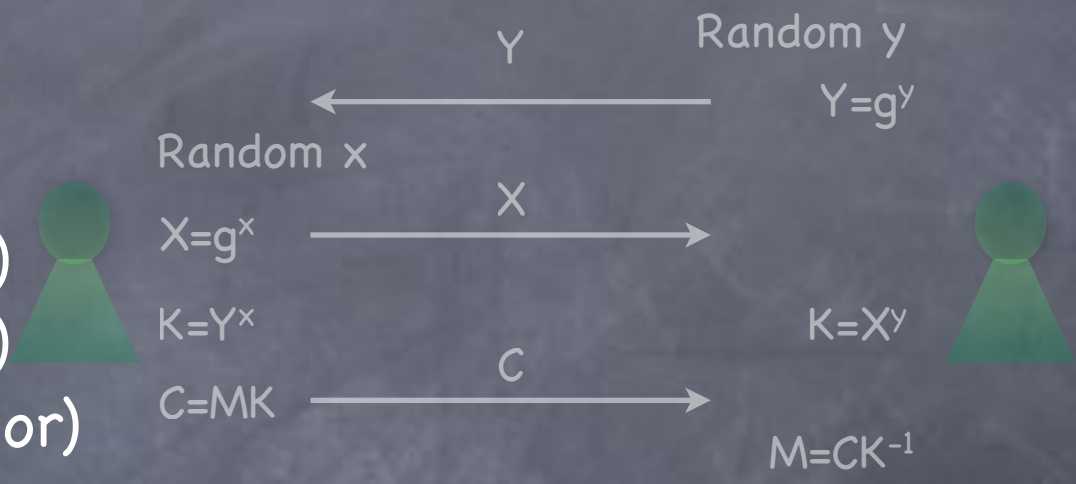

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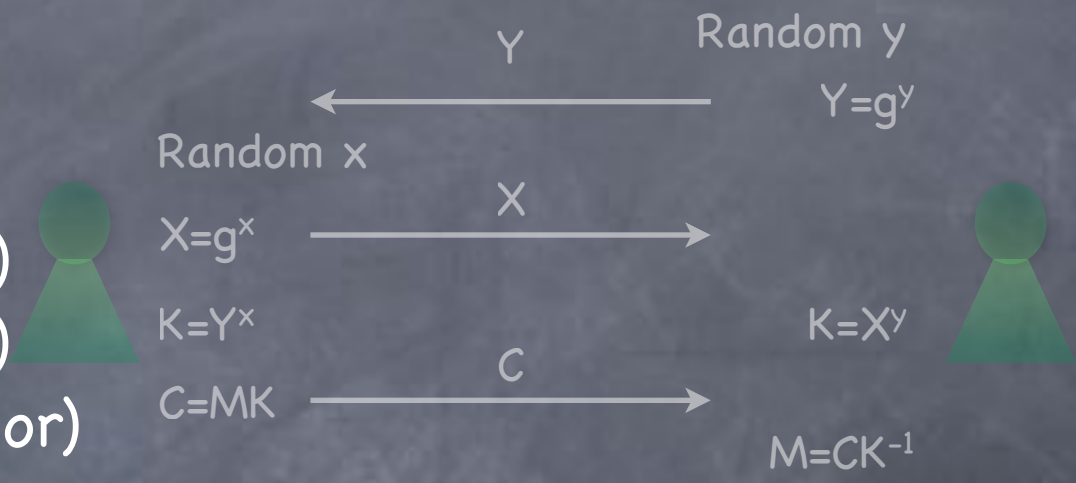

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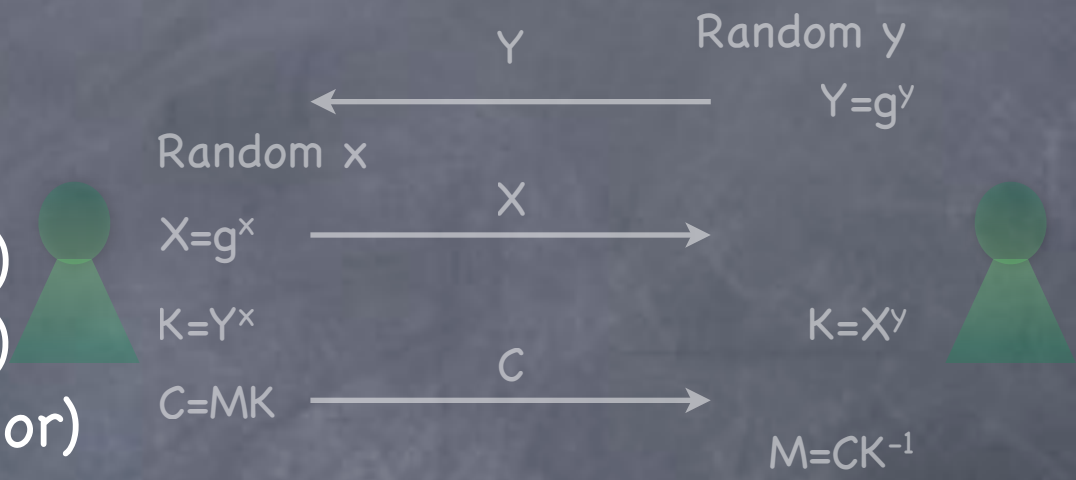

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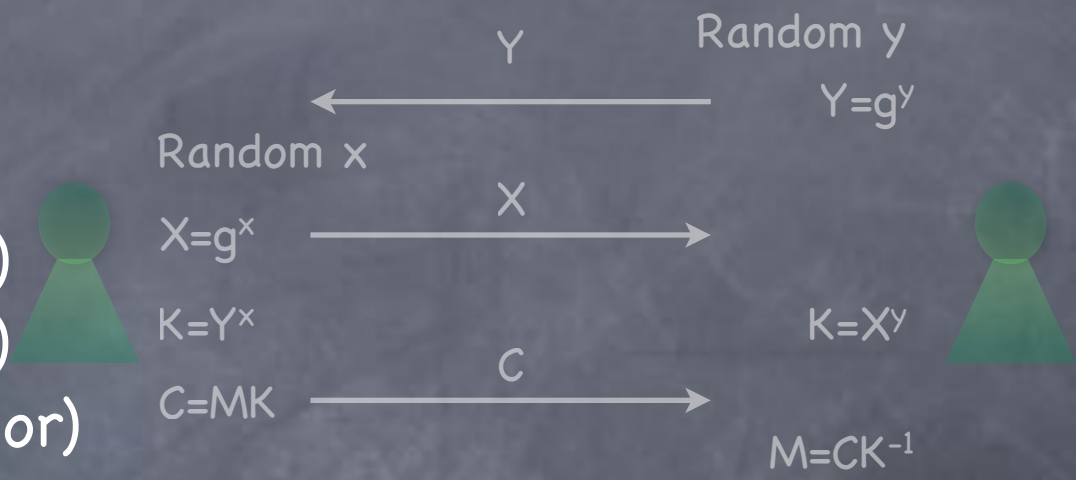

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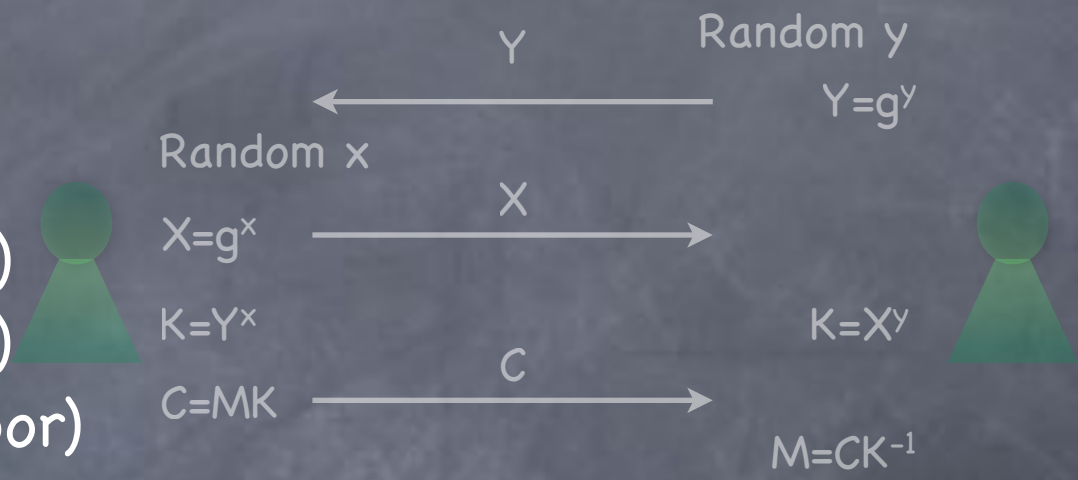

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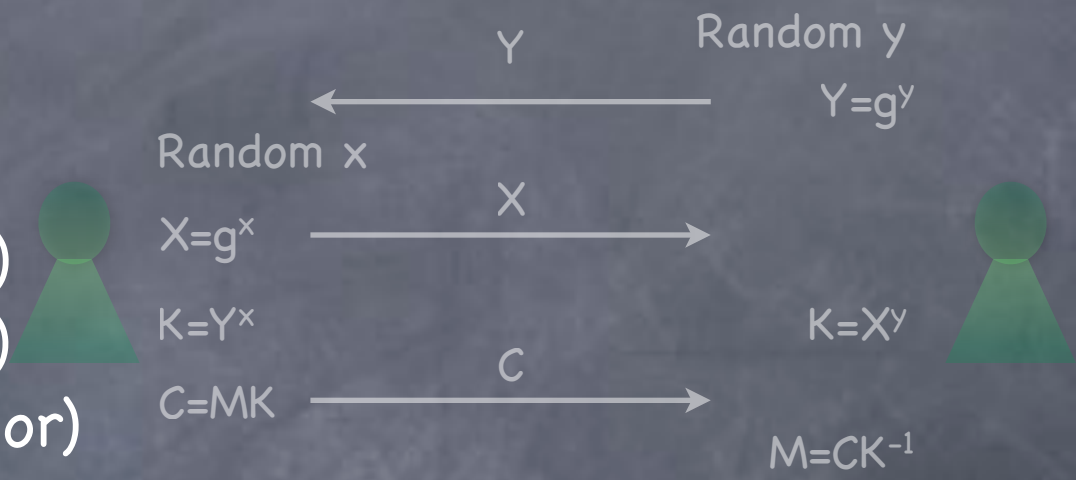

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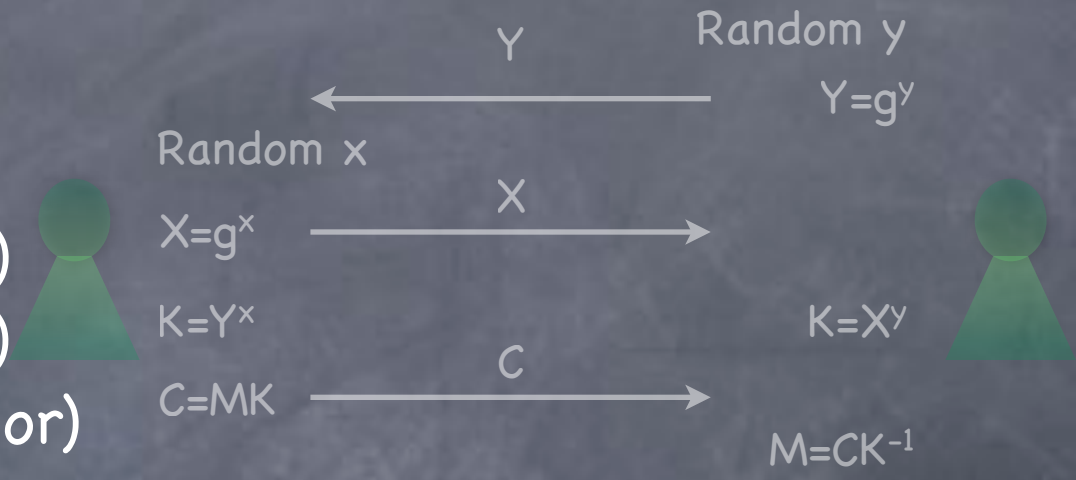

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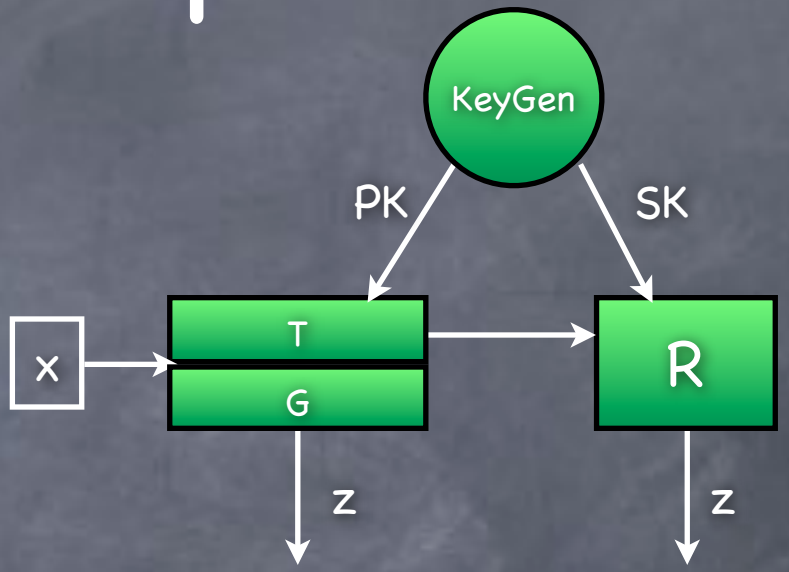
Trapdoor PRG from Generic Assumption?



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Trapdoor PRG from Generic Assumption?

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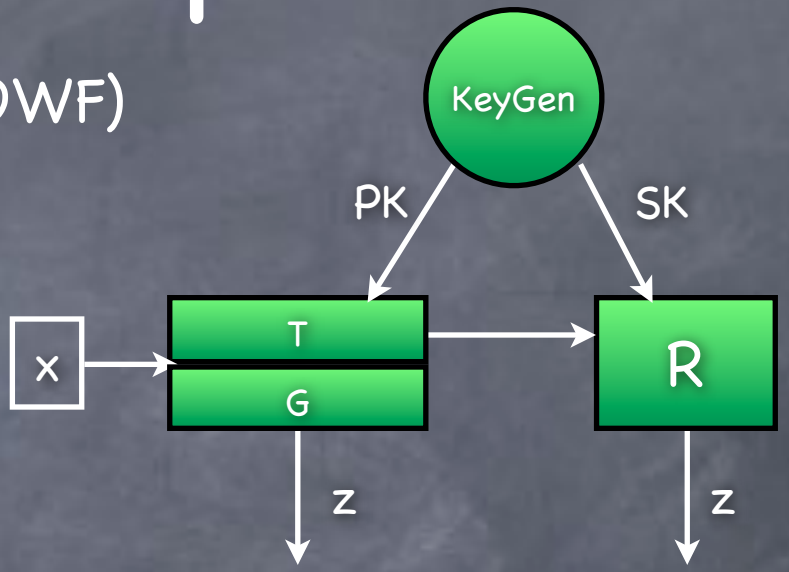

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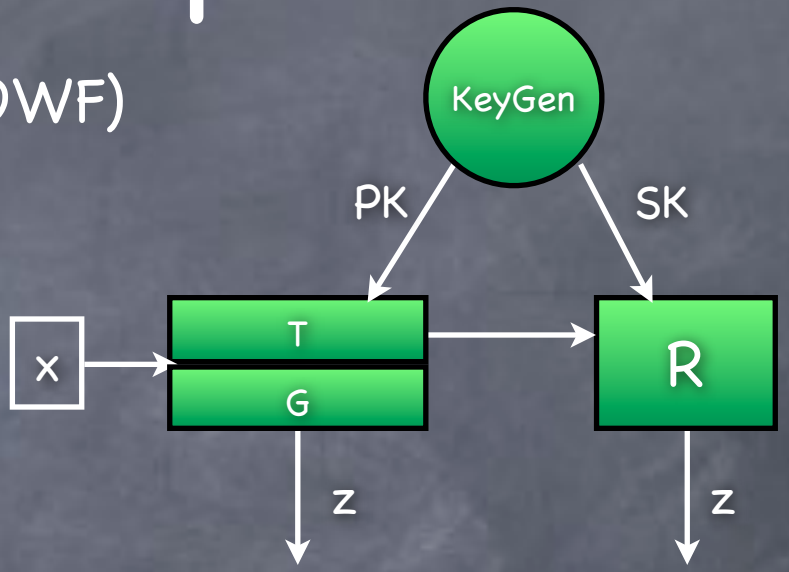

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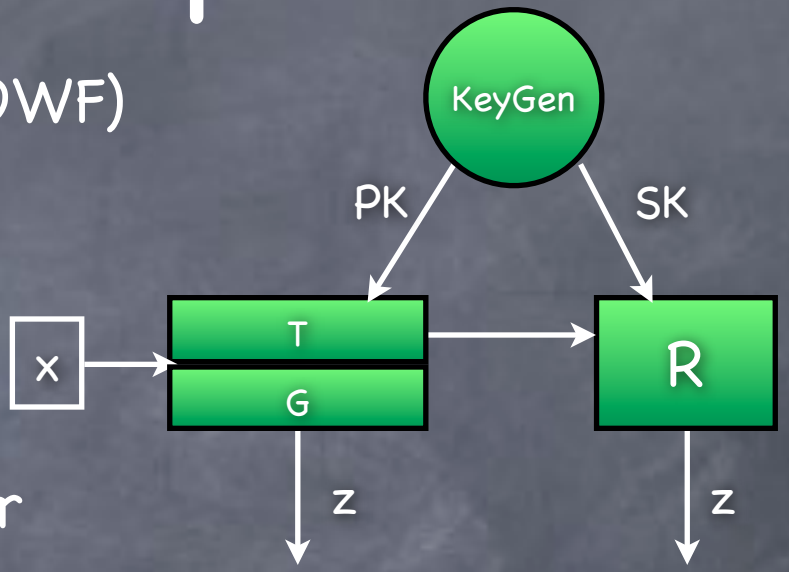

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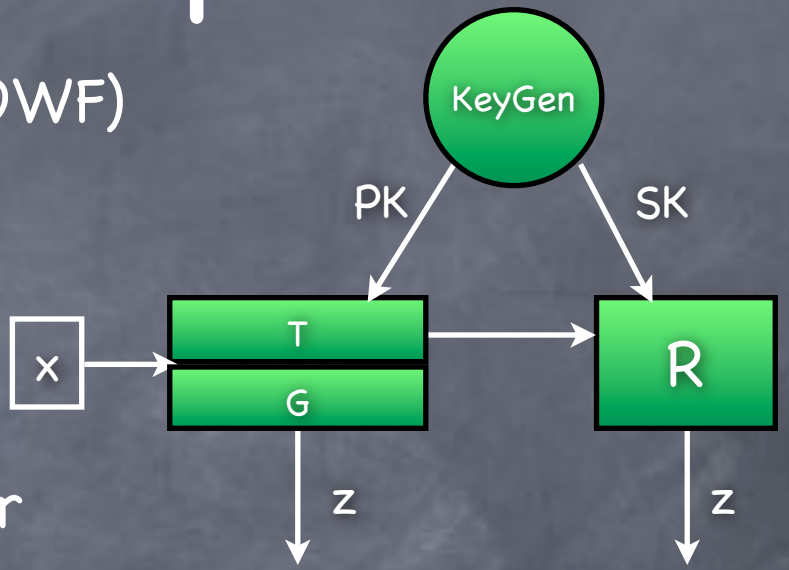

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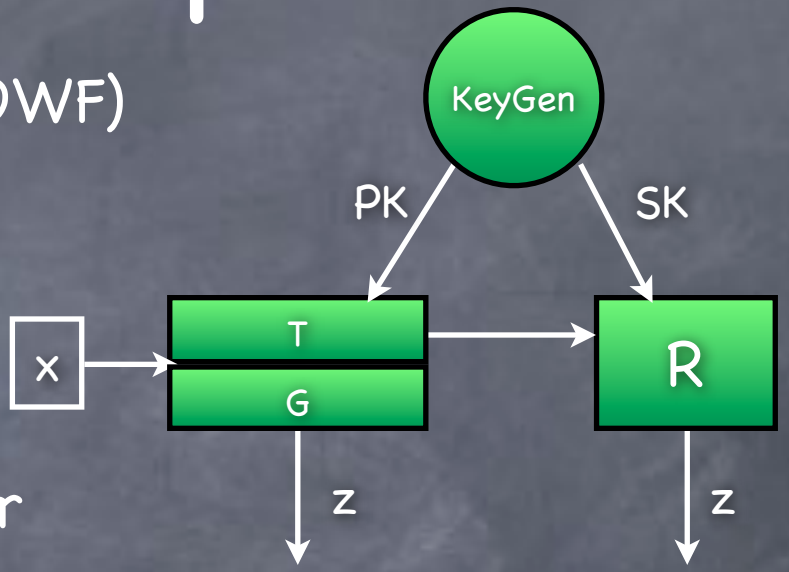
  - Will start with "Trapdoor OWP"



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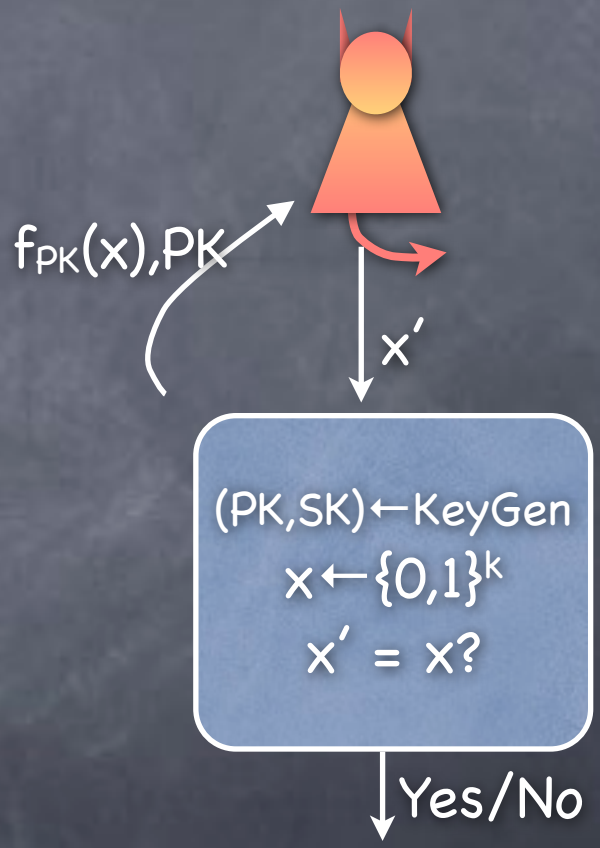

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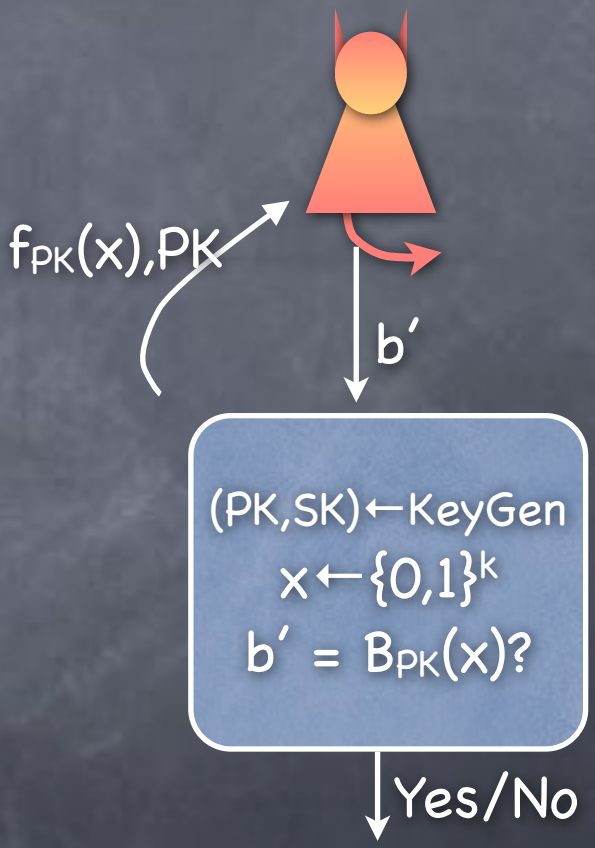
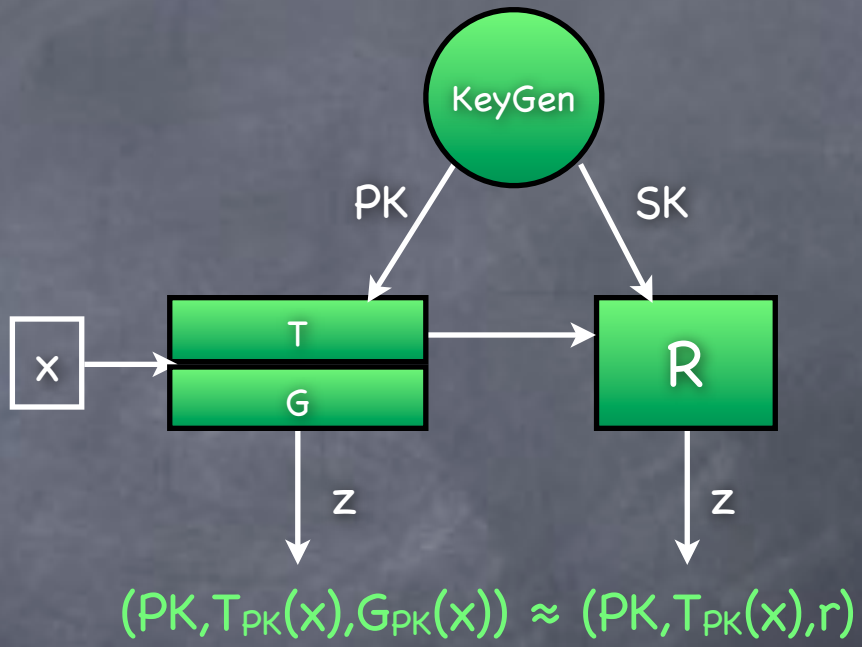
  - For all PPT adversary, probability of success in the TOWP experiment is negligible
  - Hardcore predicate:
    - B_{PK} s.t. $(PK, f_{PK}(x), B_{PK}(x)) \approx (PK, f_{PK}(x), r)$

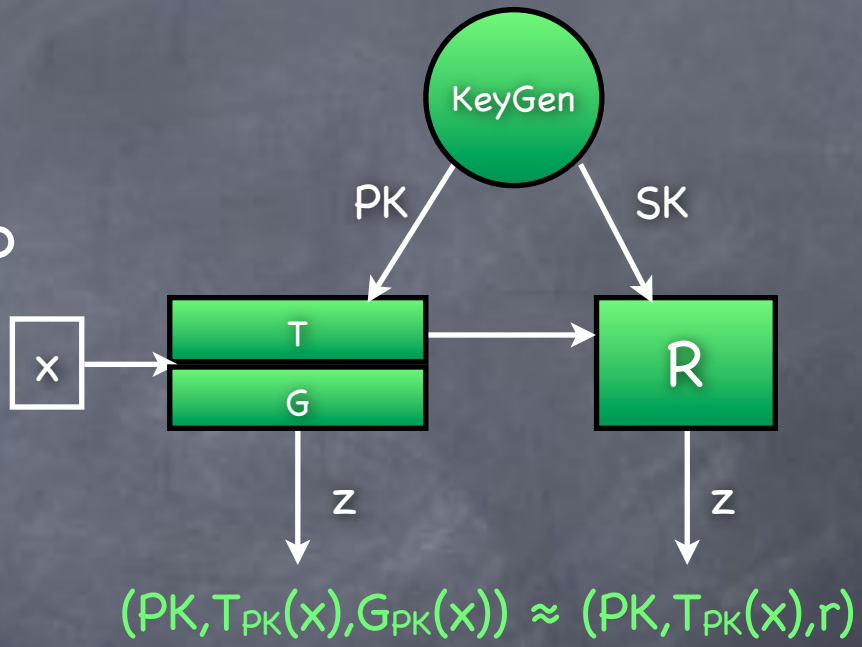


Trapdoor PRG from Trapdoor OWP



Trapdoor PRG from Trapdoor OWP

- Same construction as PRG from OWP



Trapdoor PRG from Trapdoor OWP

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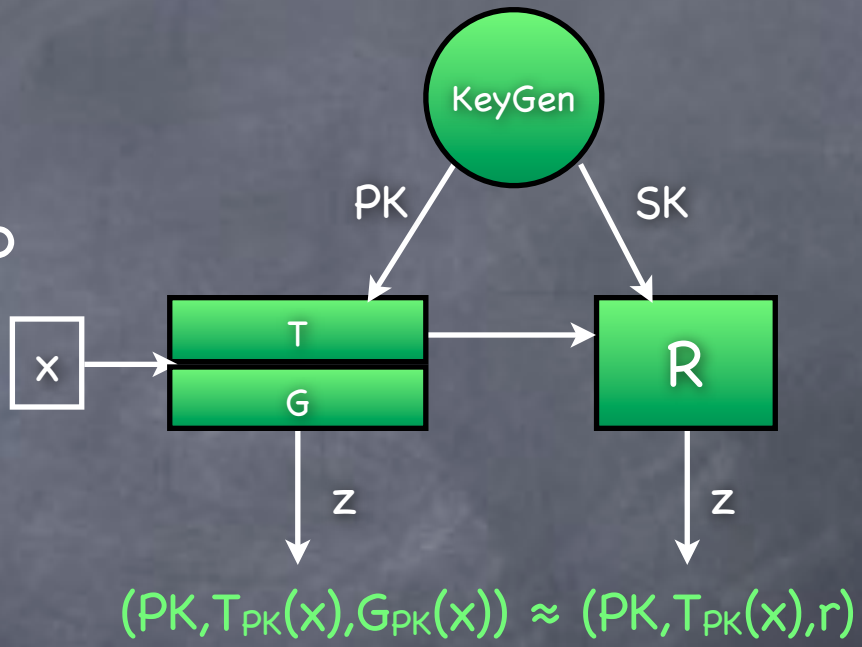

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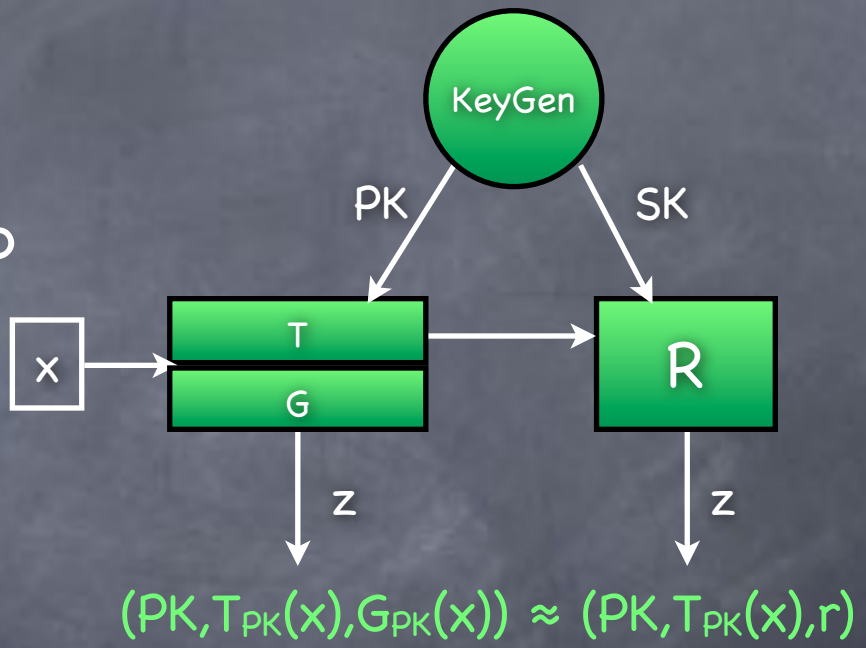

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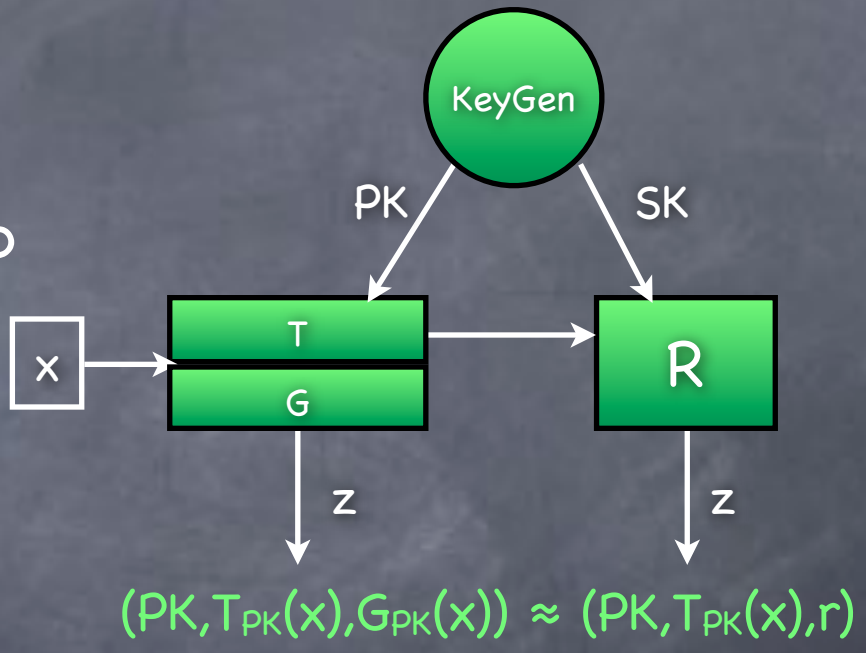

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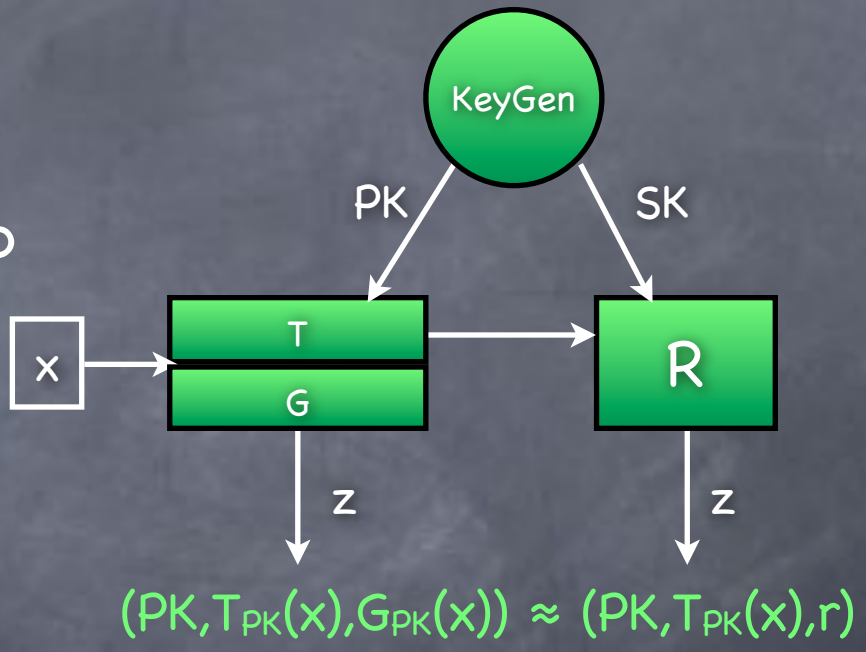

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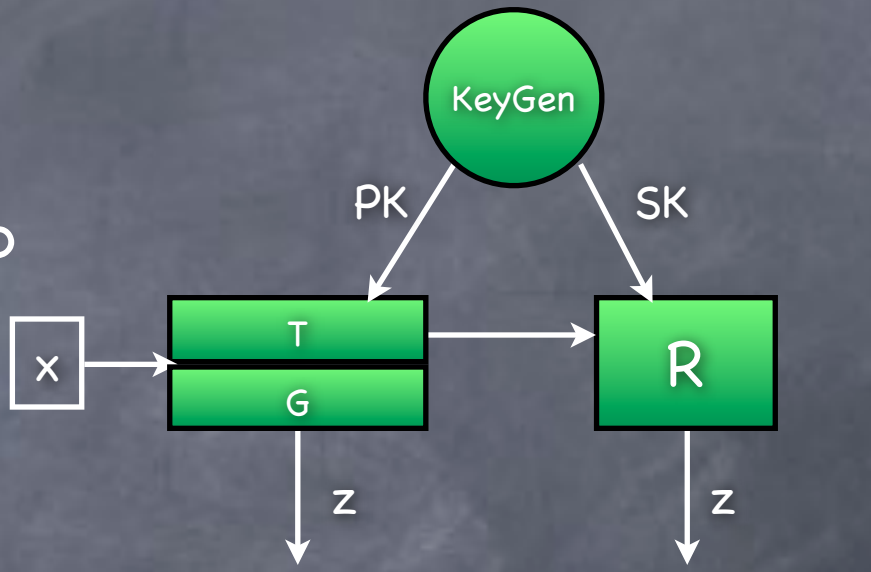

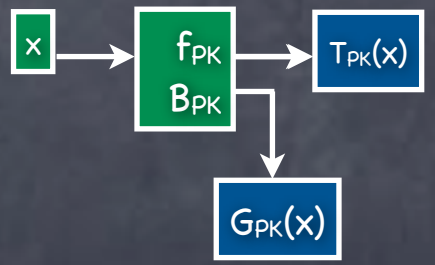

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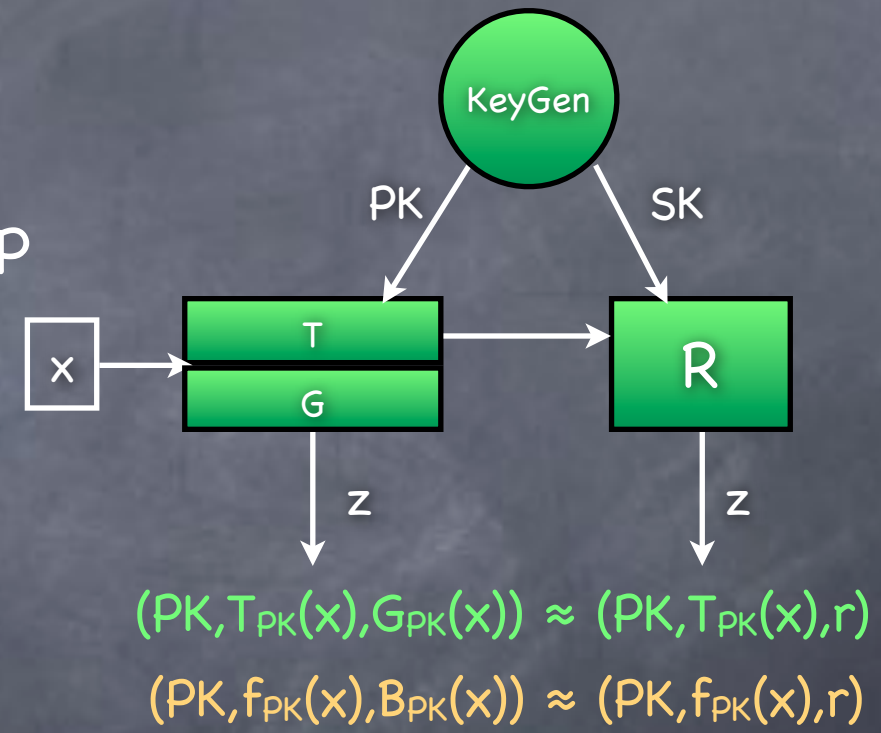

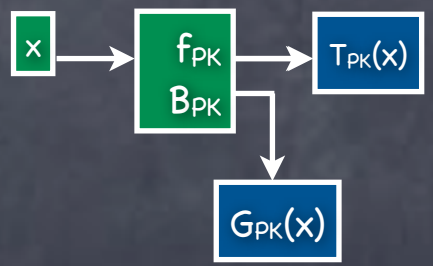

Trapdoor PRG from Trapdoor OWP

- Same construction as PRG from OWP
- One bit TPRG
  - KeyGen same as TOWP's KeyGen
  - $G_{PK}(x) := B_{PK}(x)$. $T_{PK}(x) := f_{PK}(x)$. $R_{SK}(y) := G_{PK}(f'_{SK}(y))$
    - (SK assumed to contain PK)
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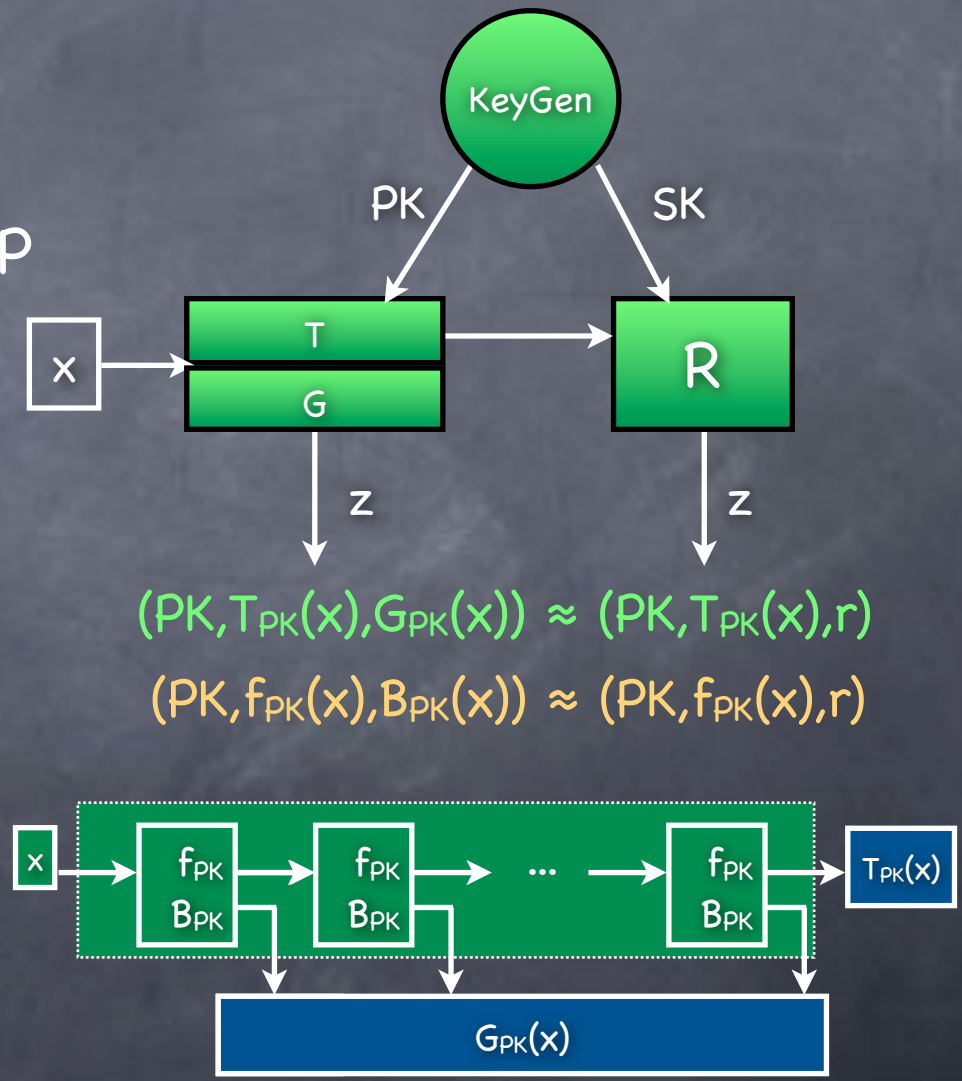

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see handout

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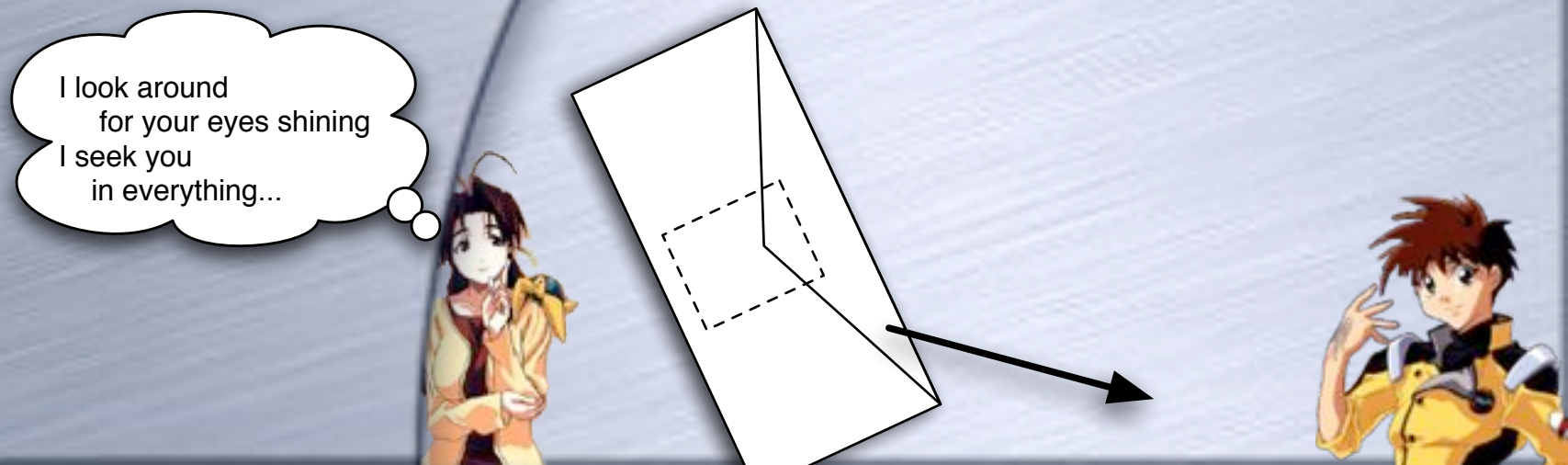

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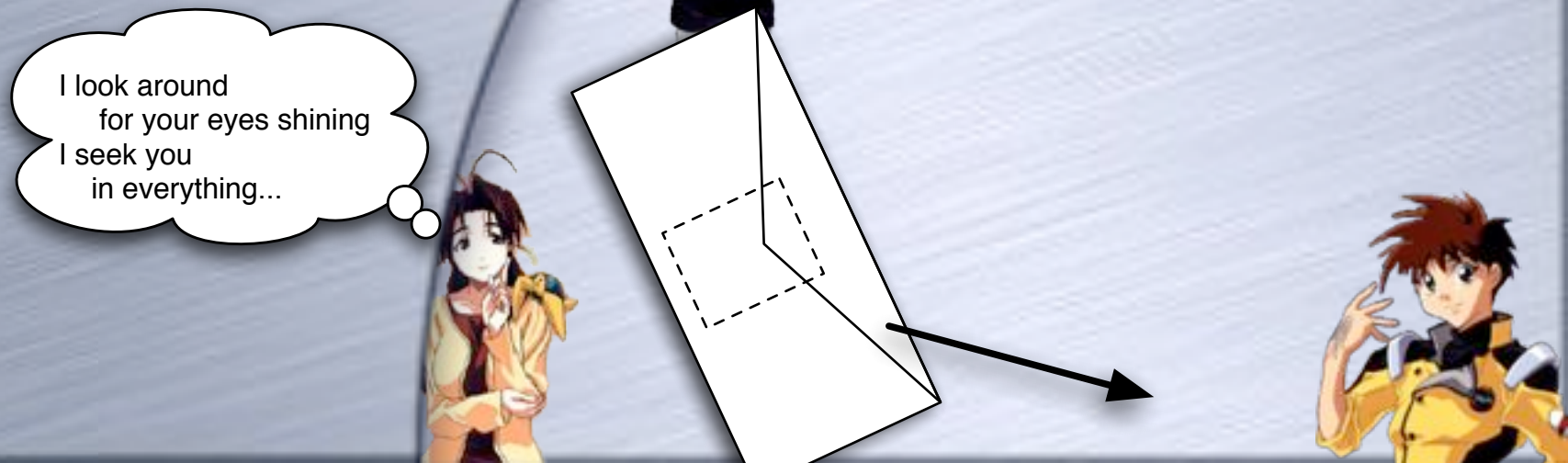

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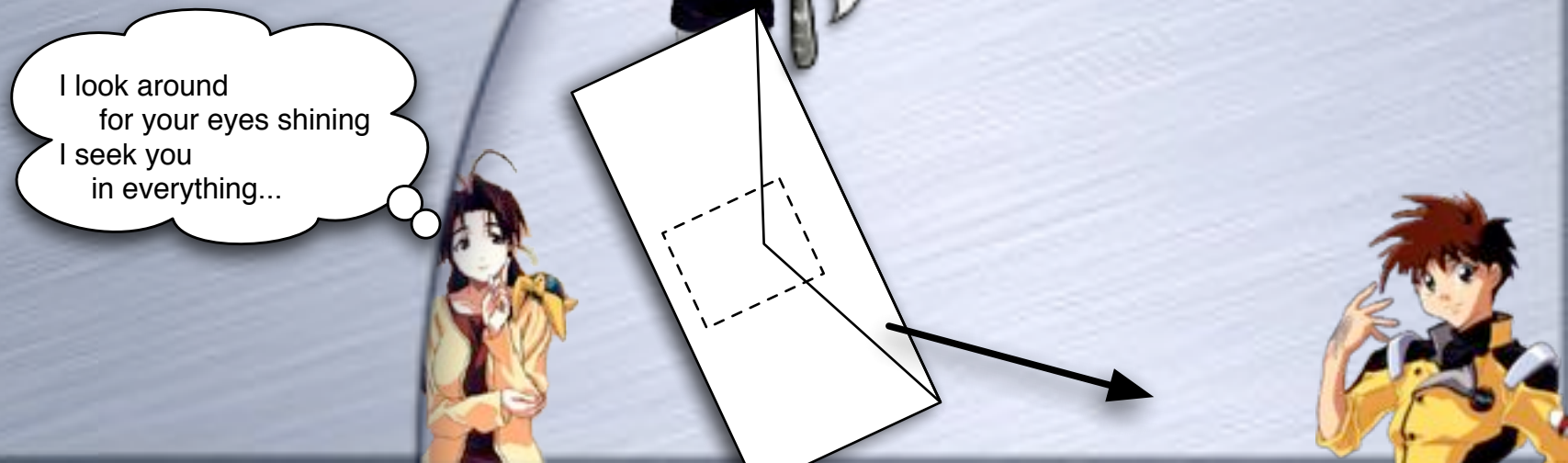

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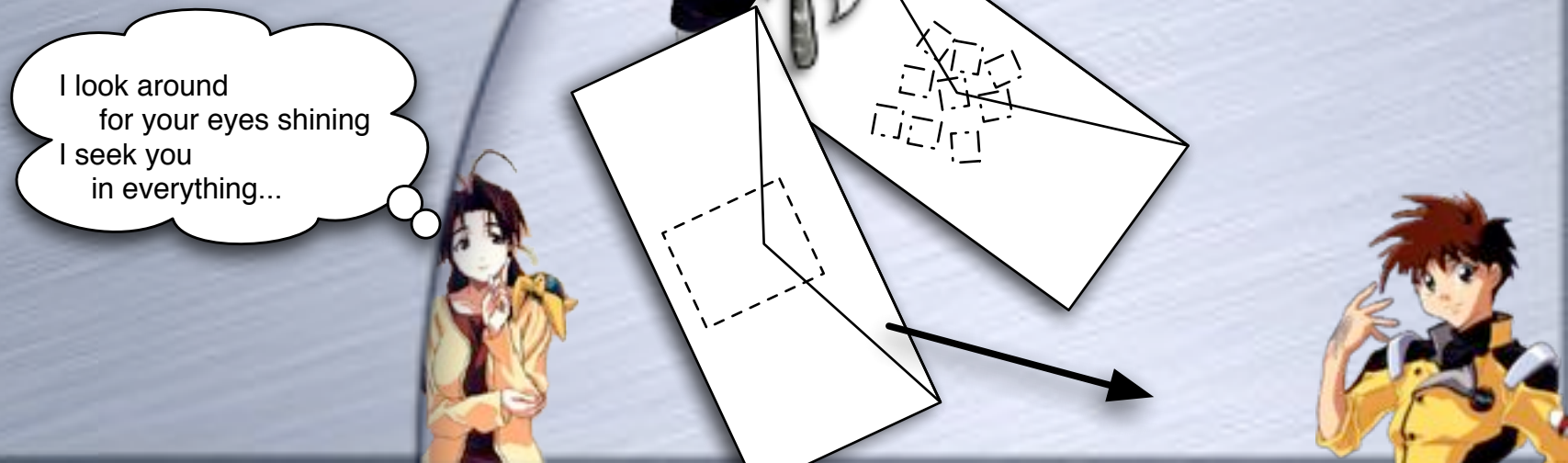

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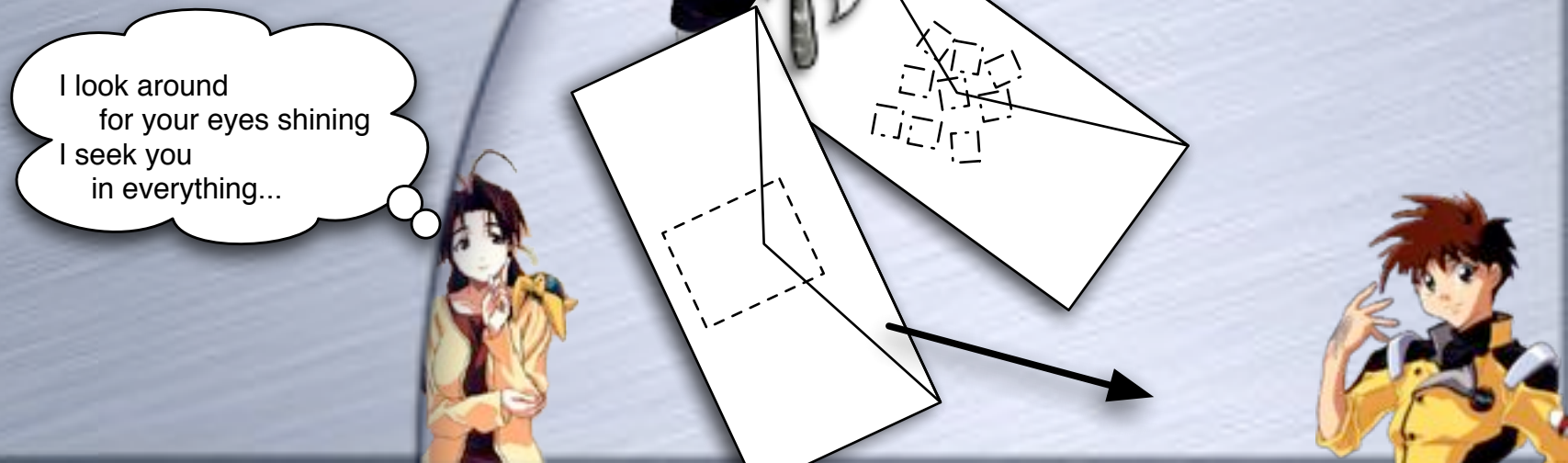

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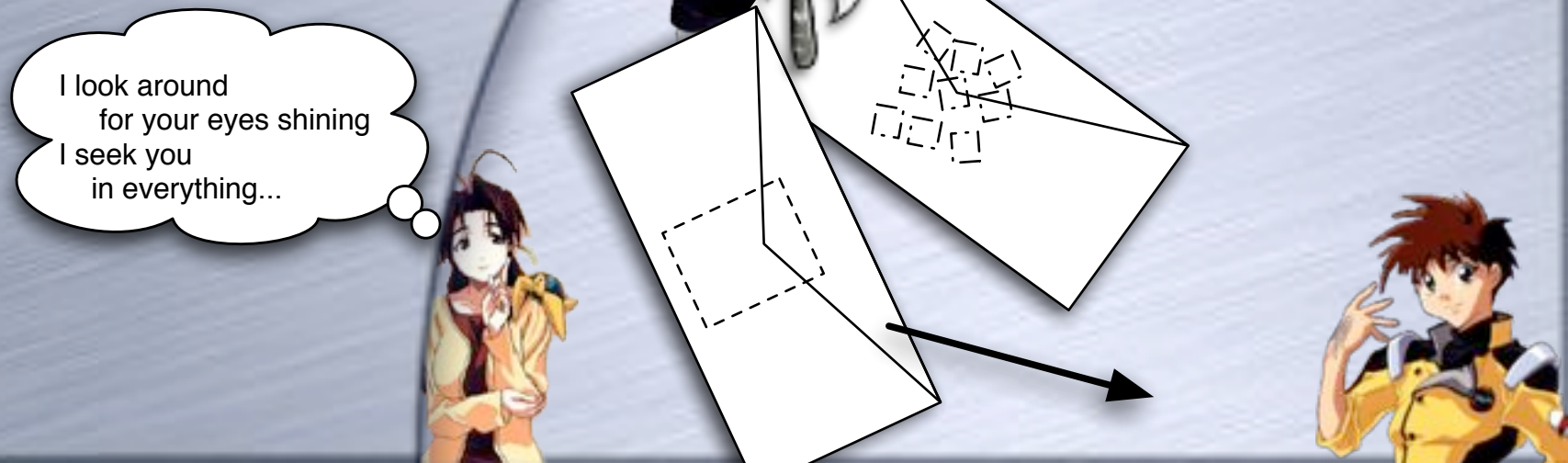

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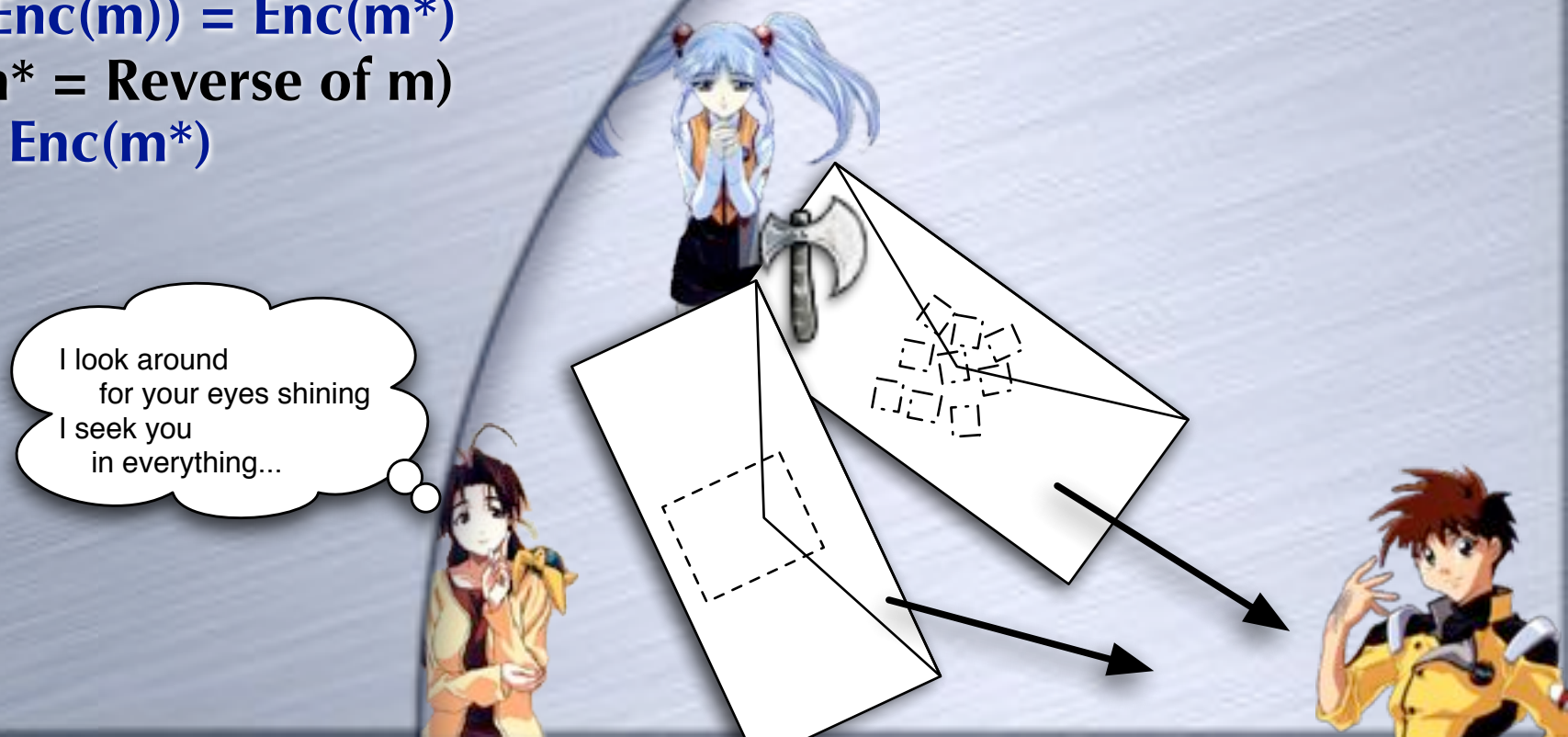

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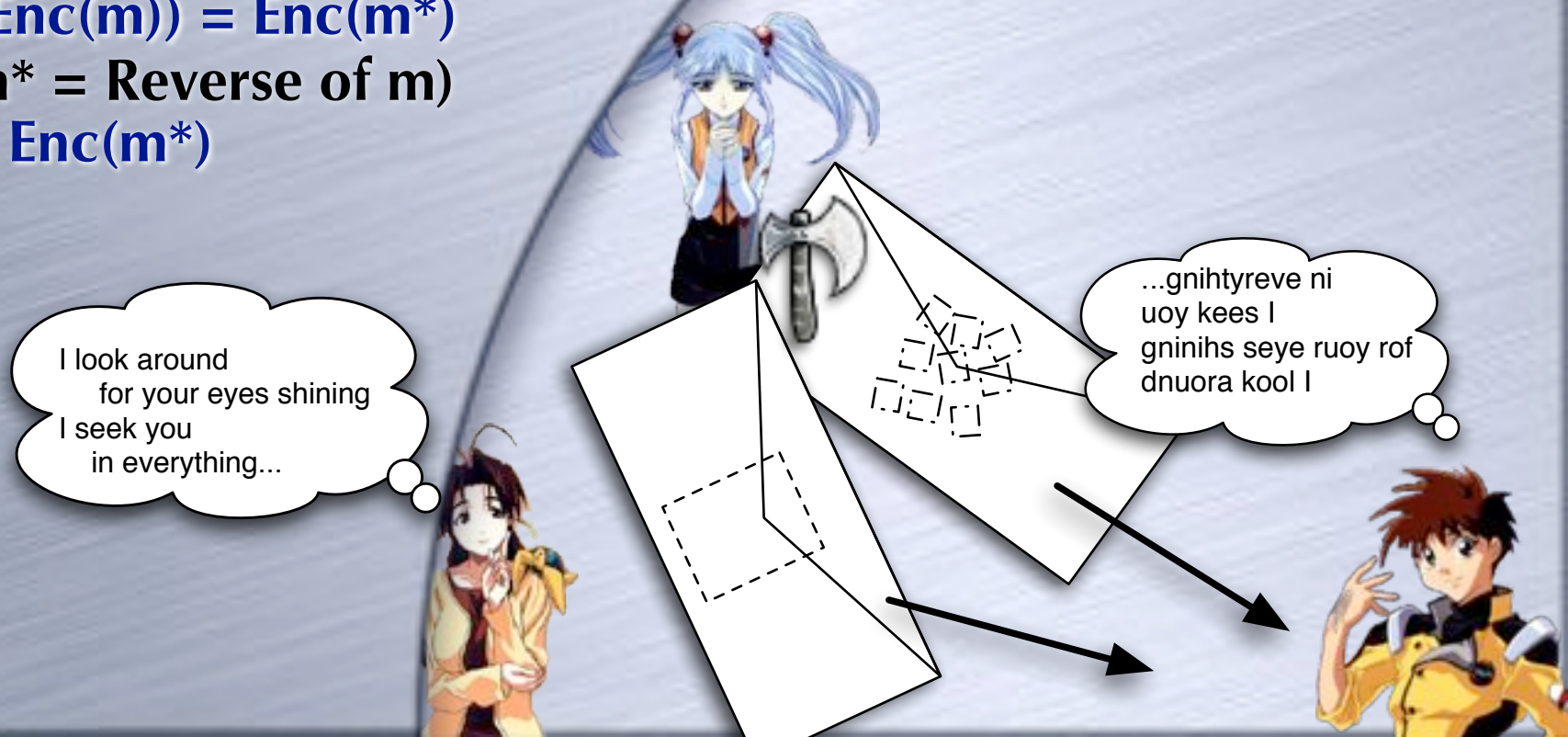

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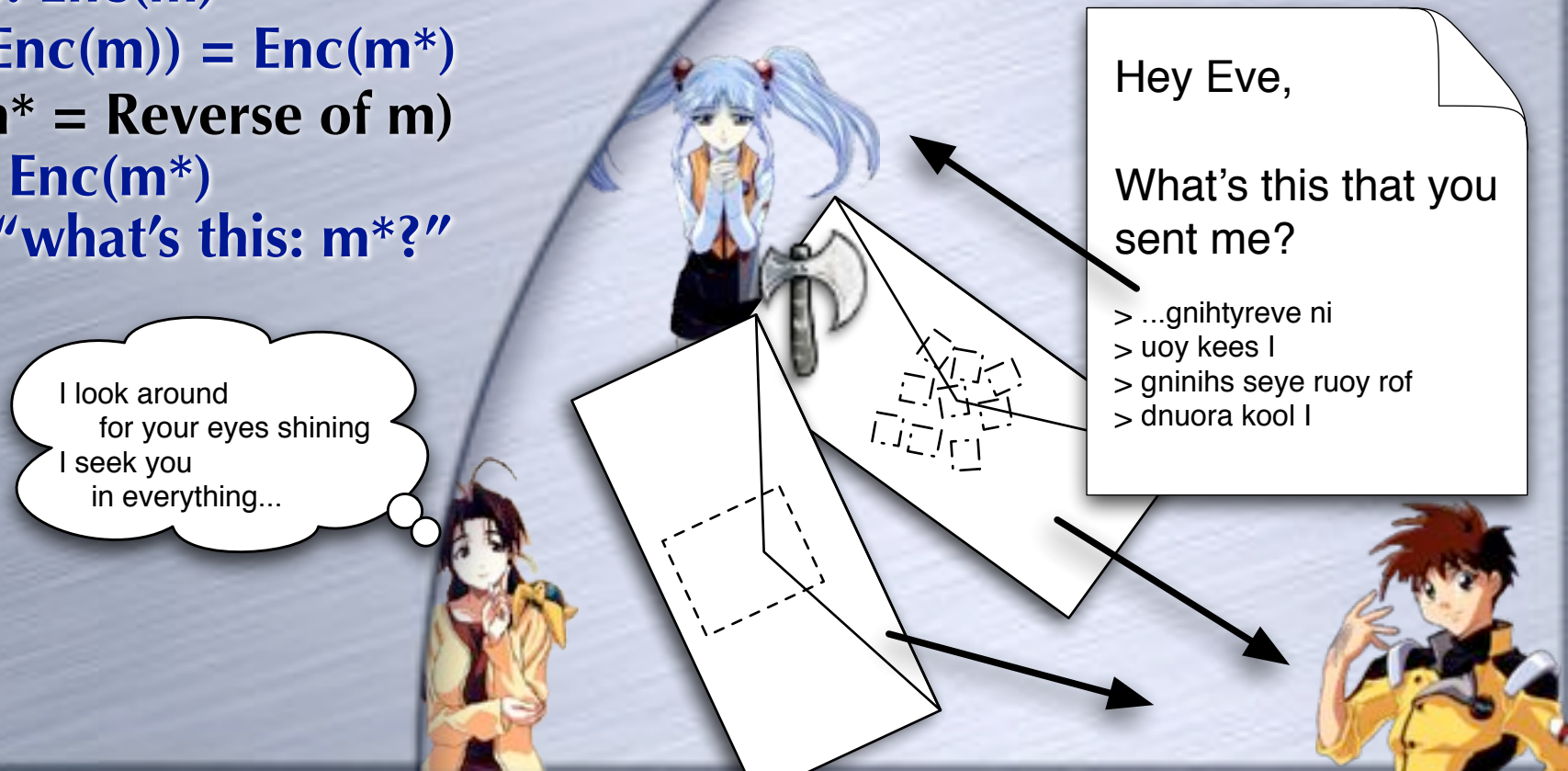

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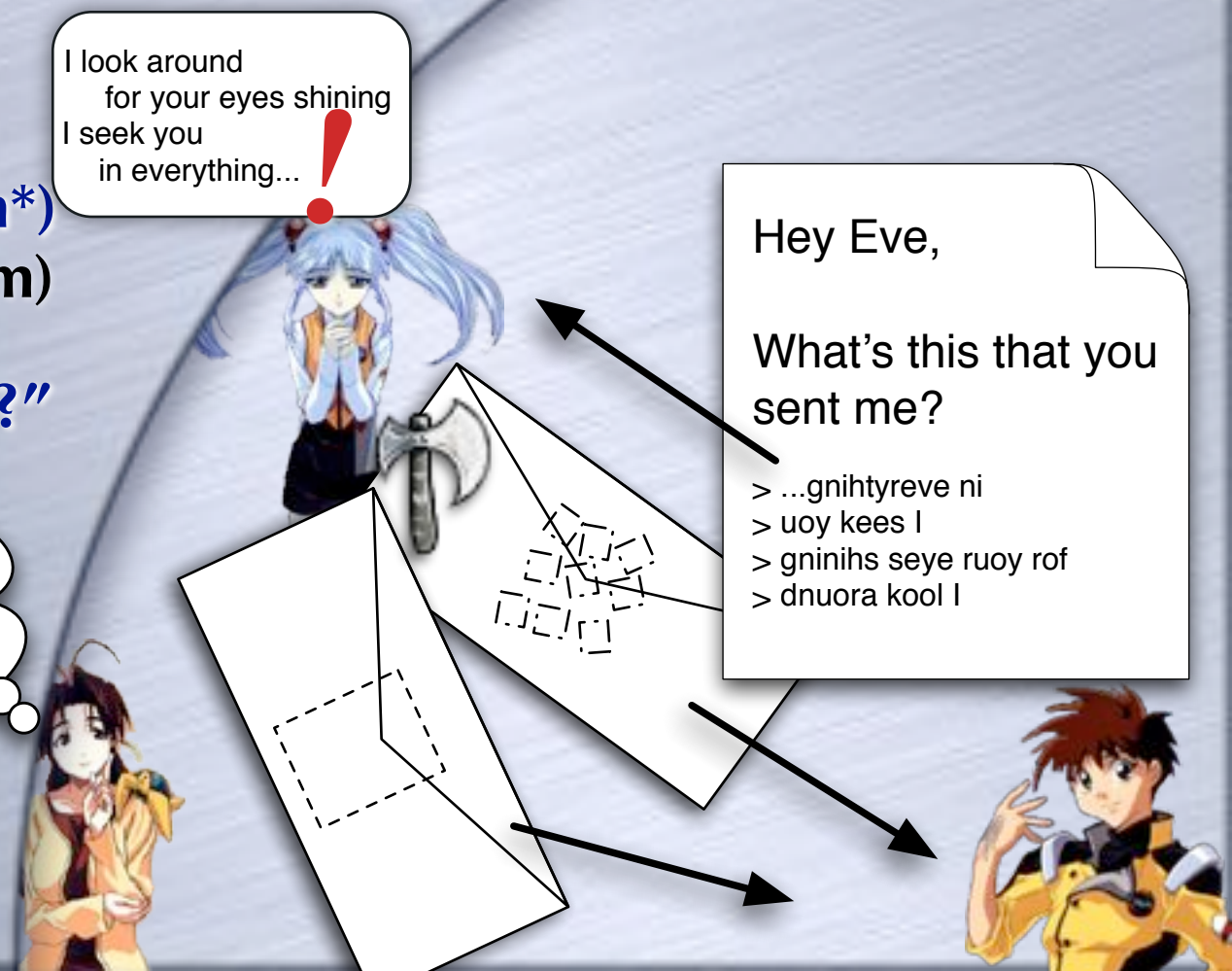

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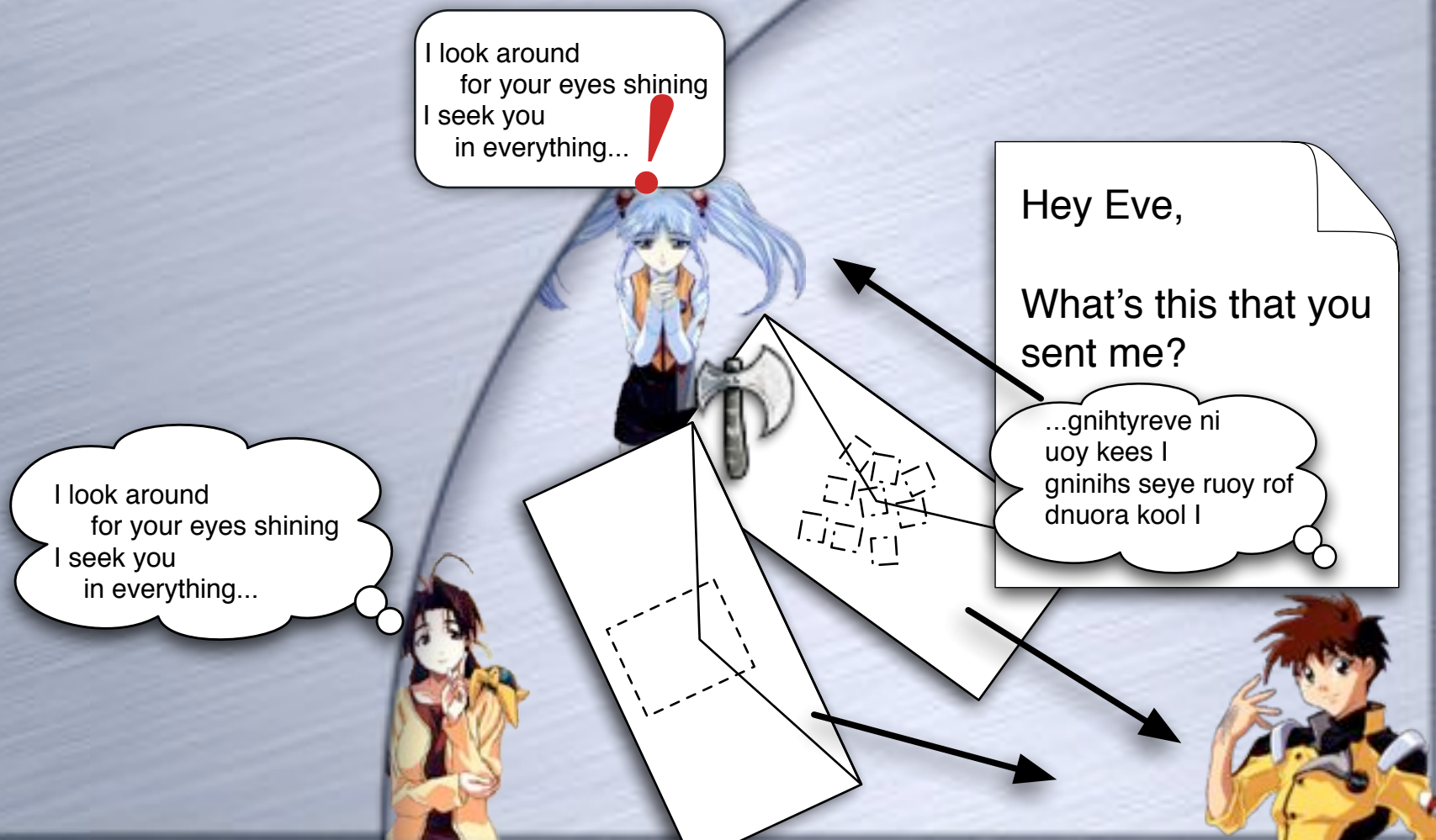

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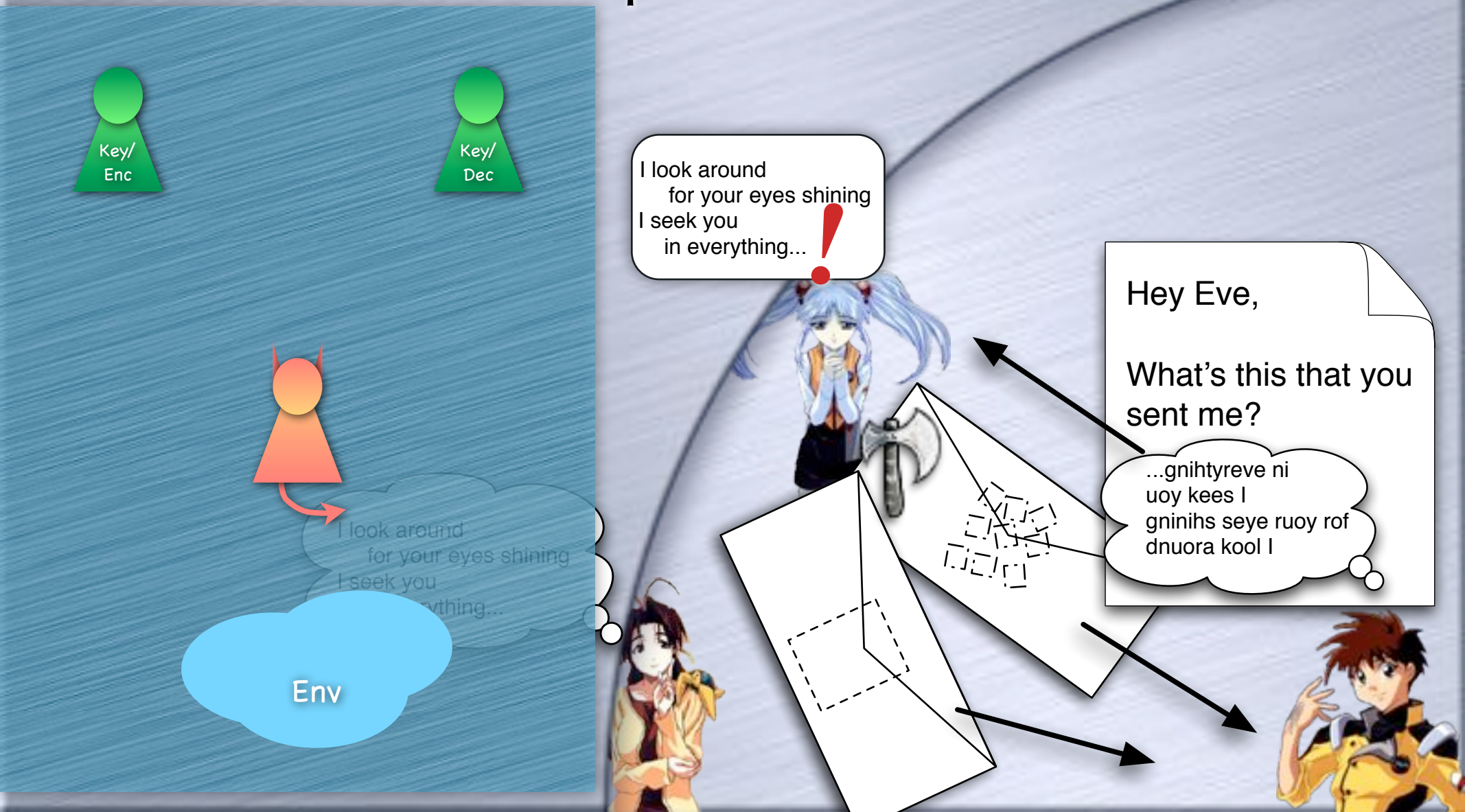

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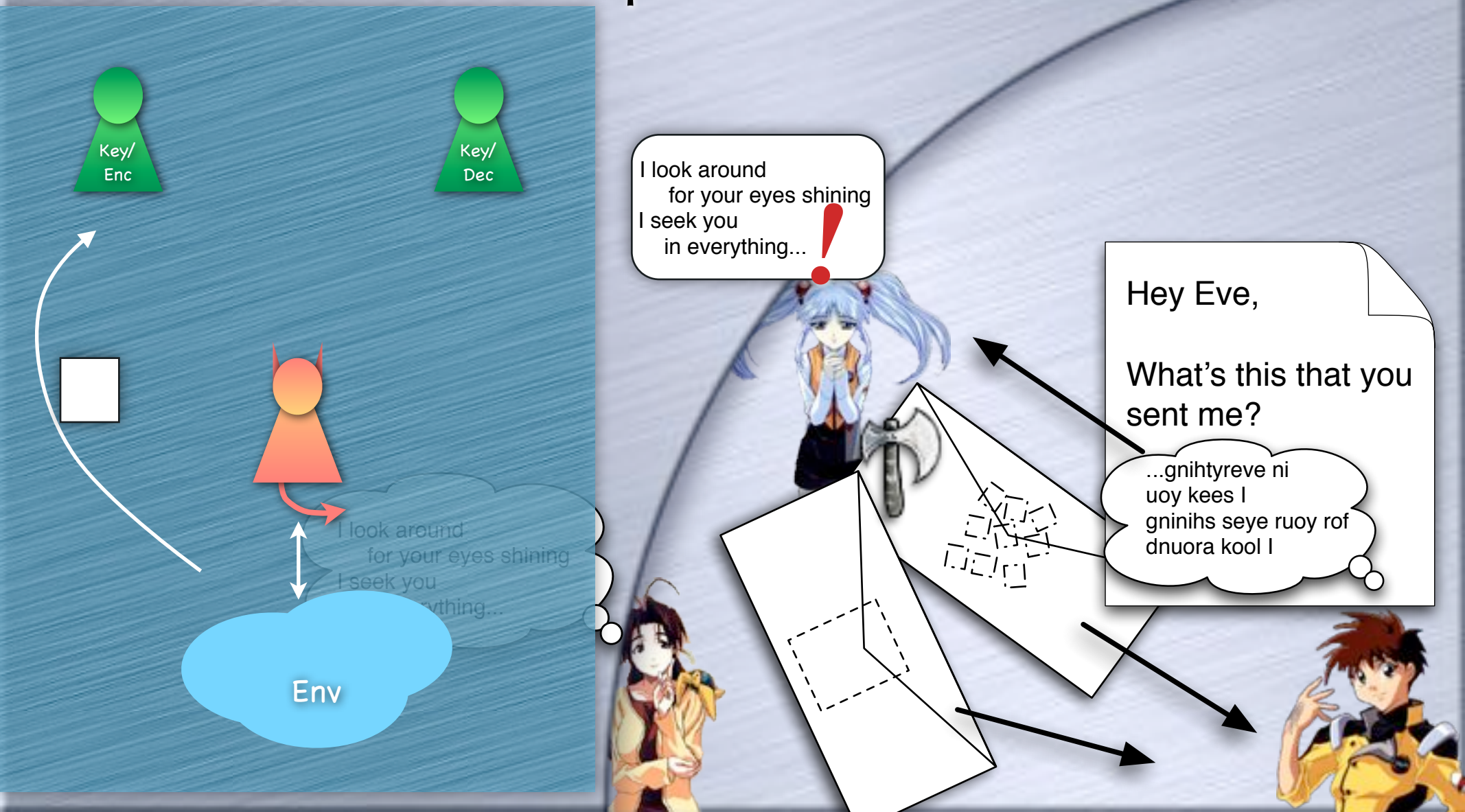

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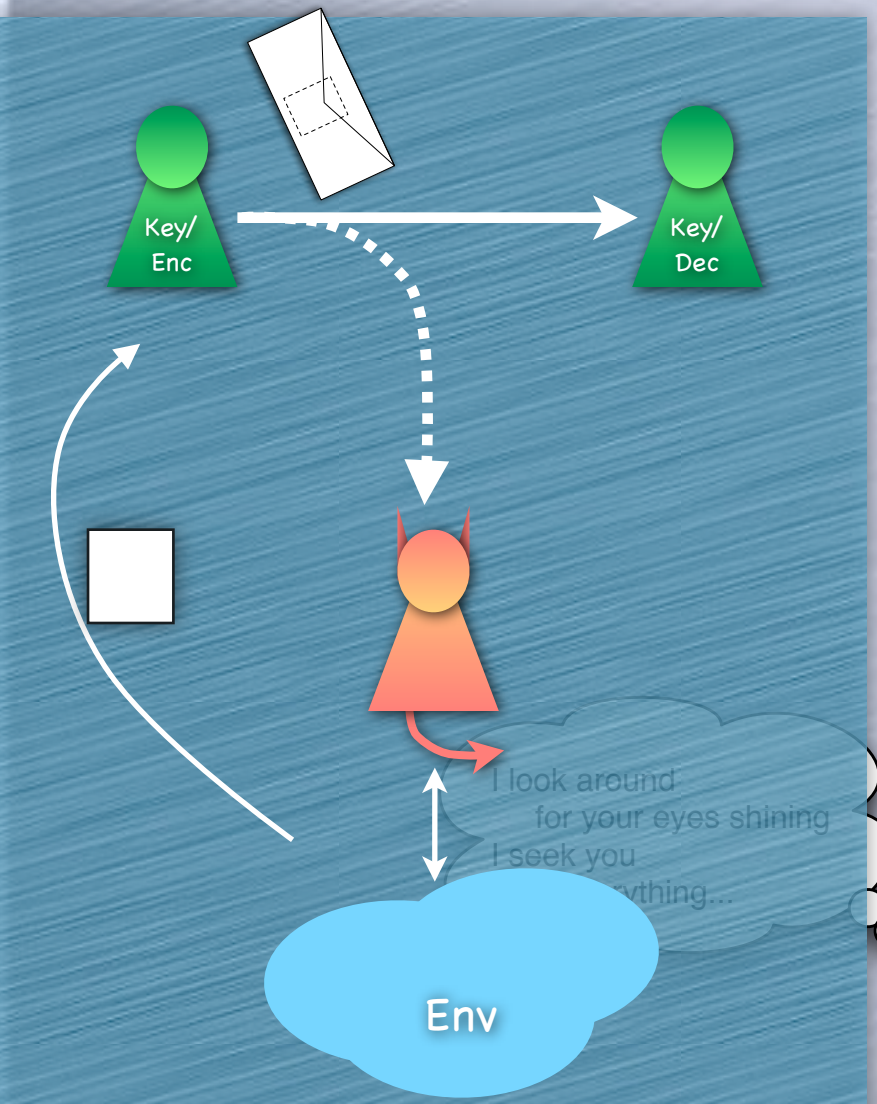

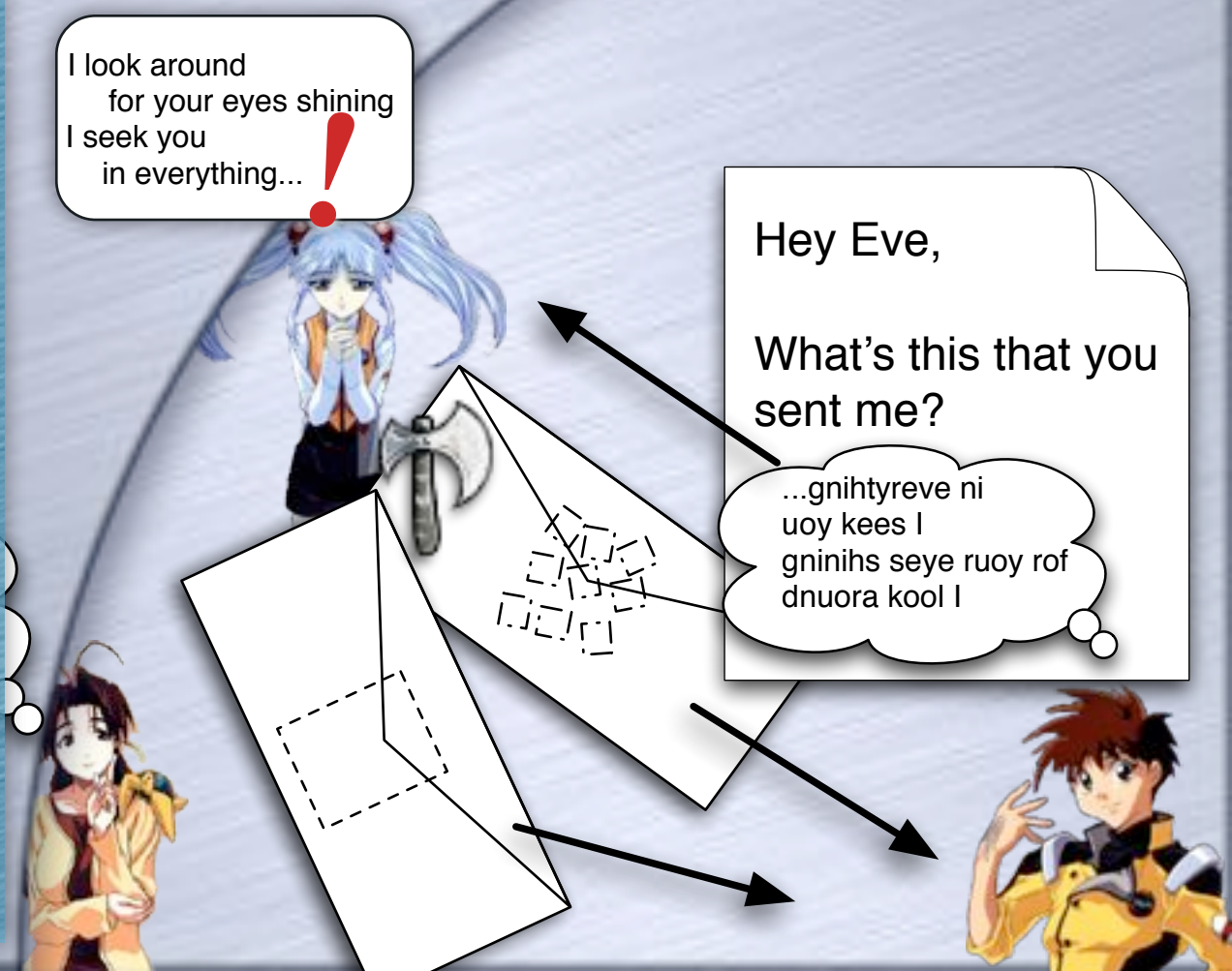

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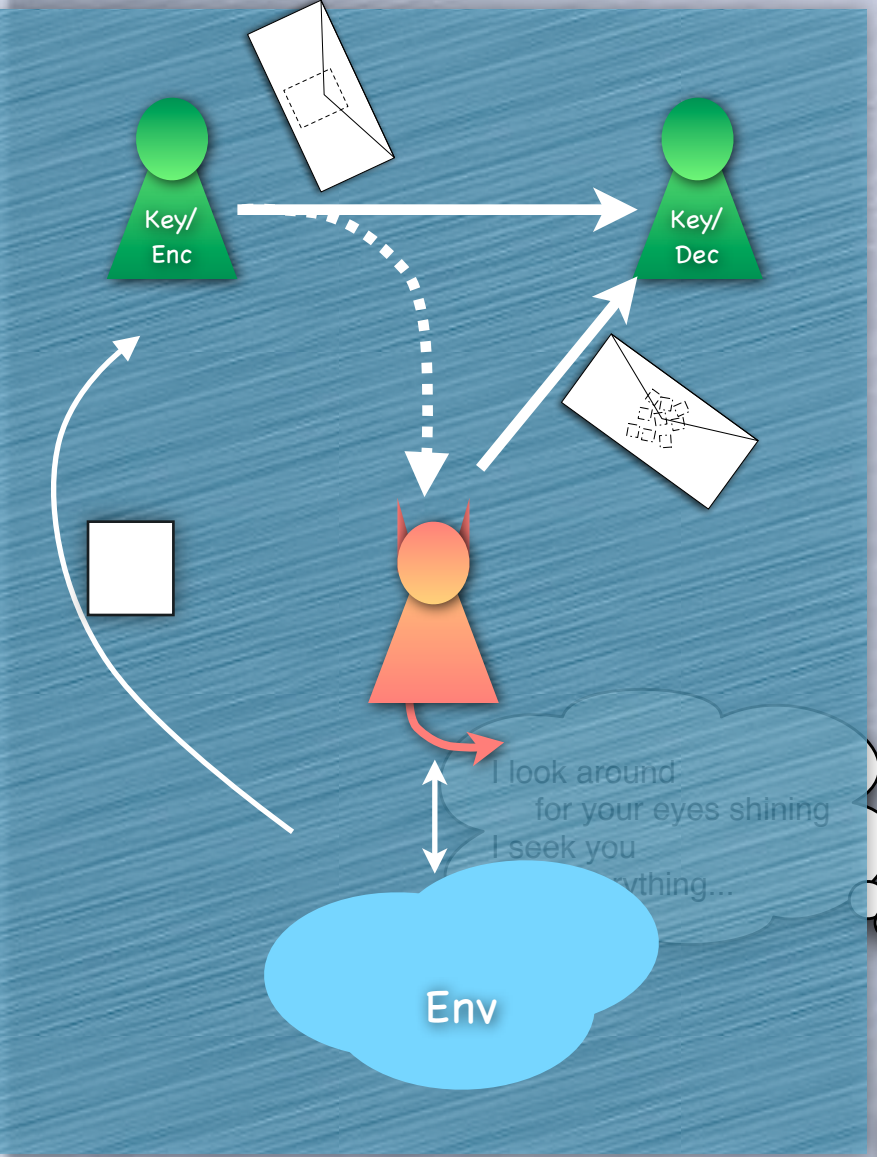

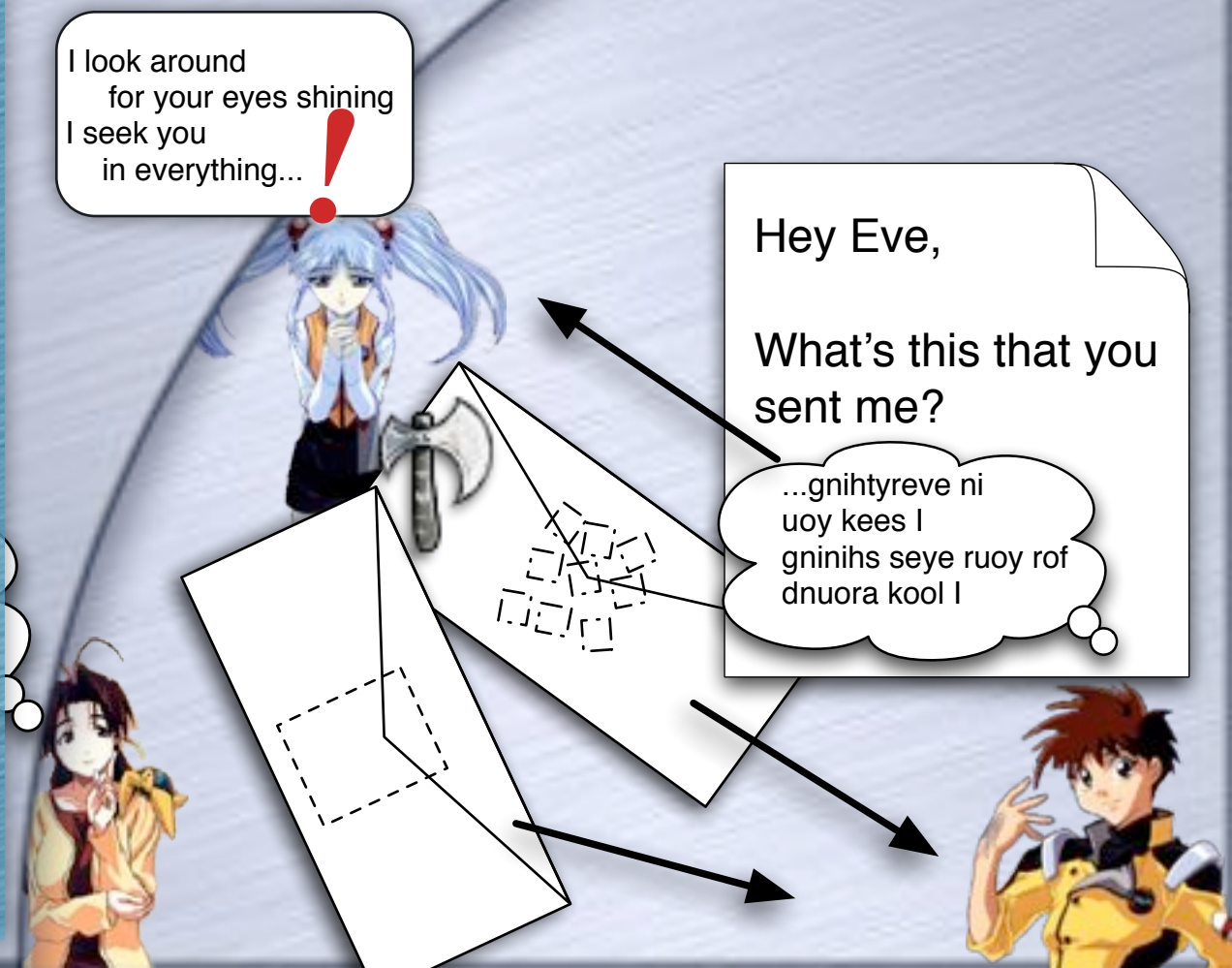

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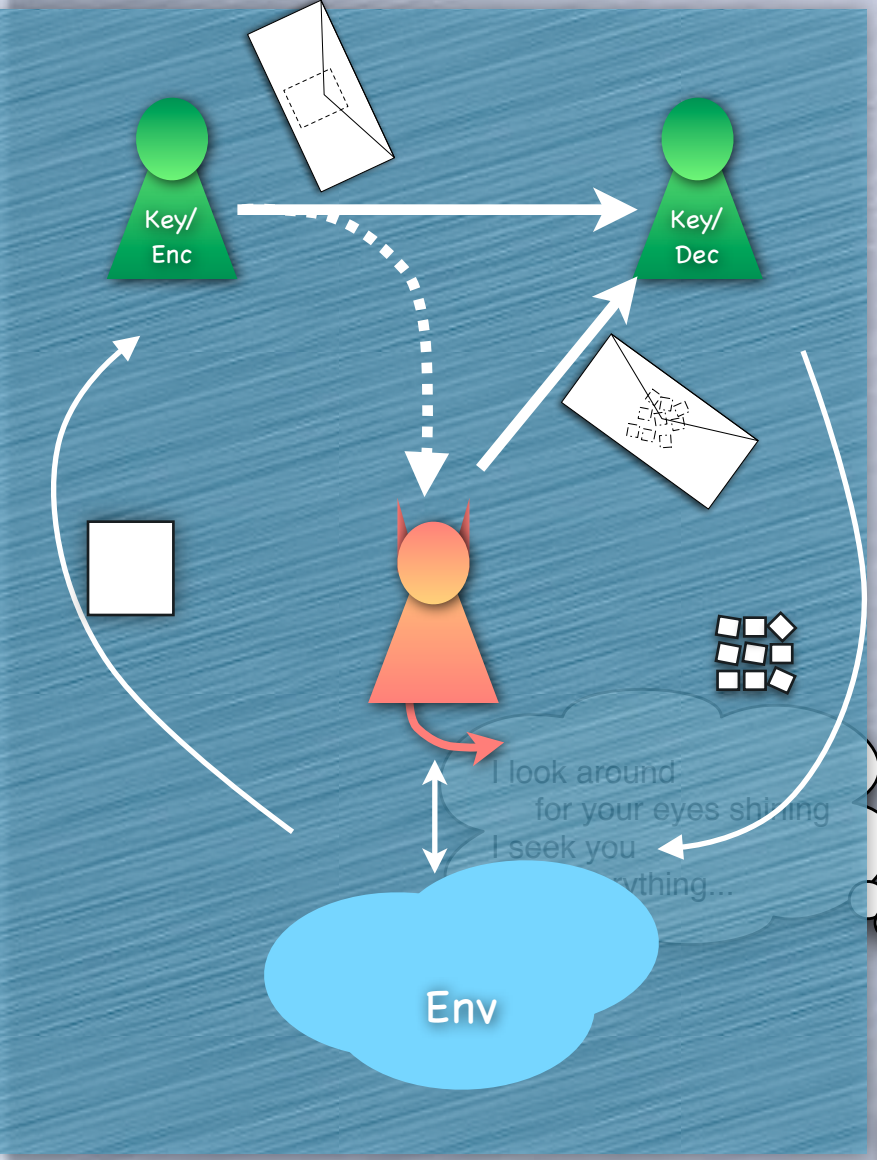

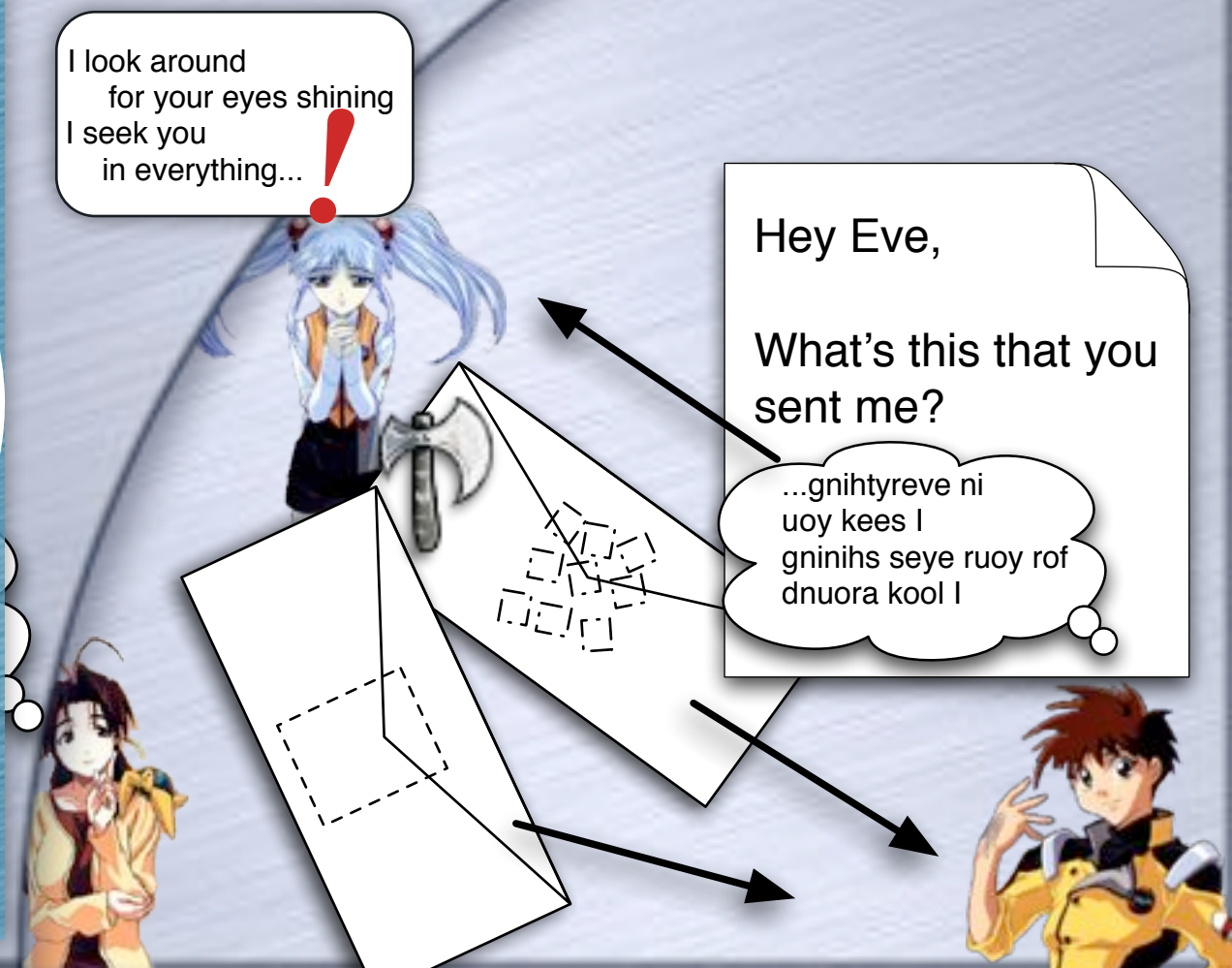

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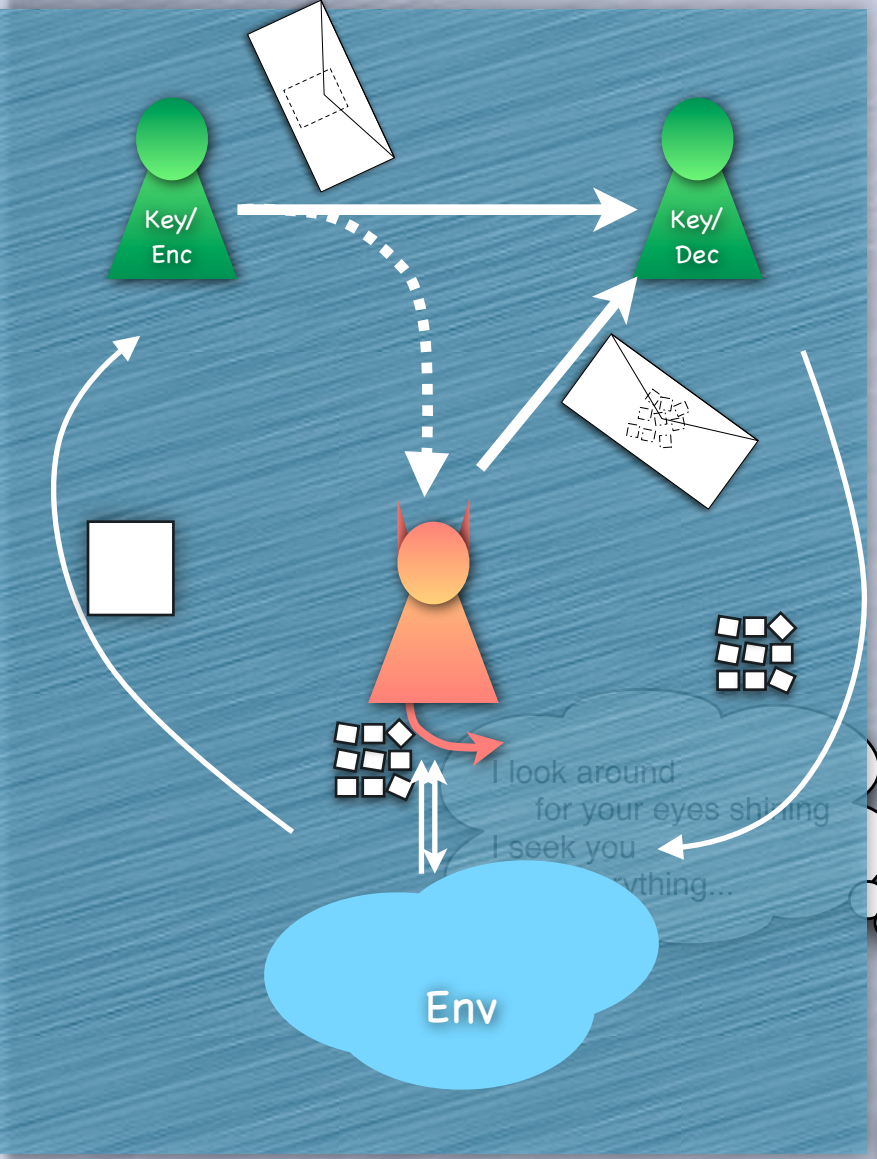

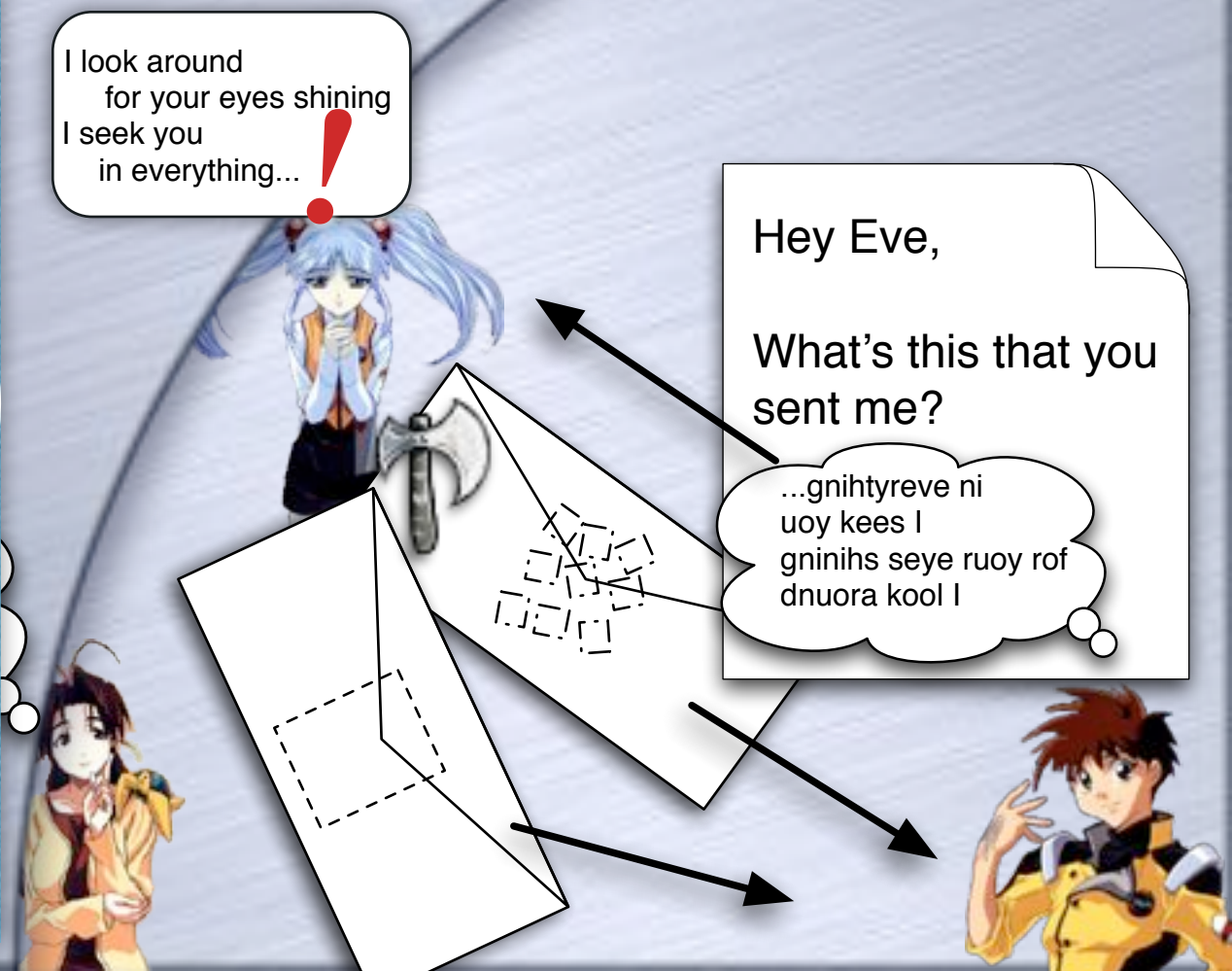

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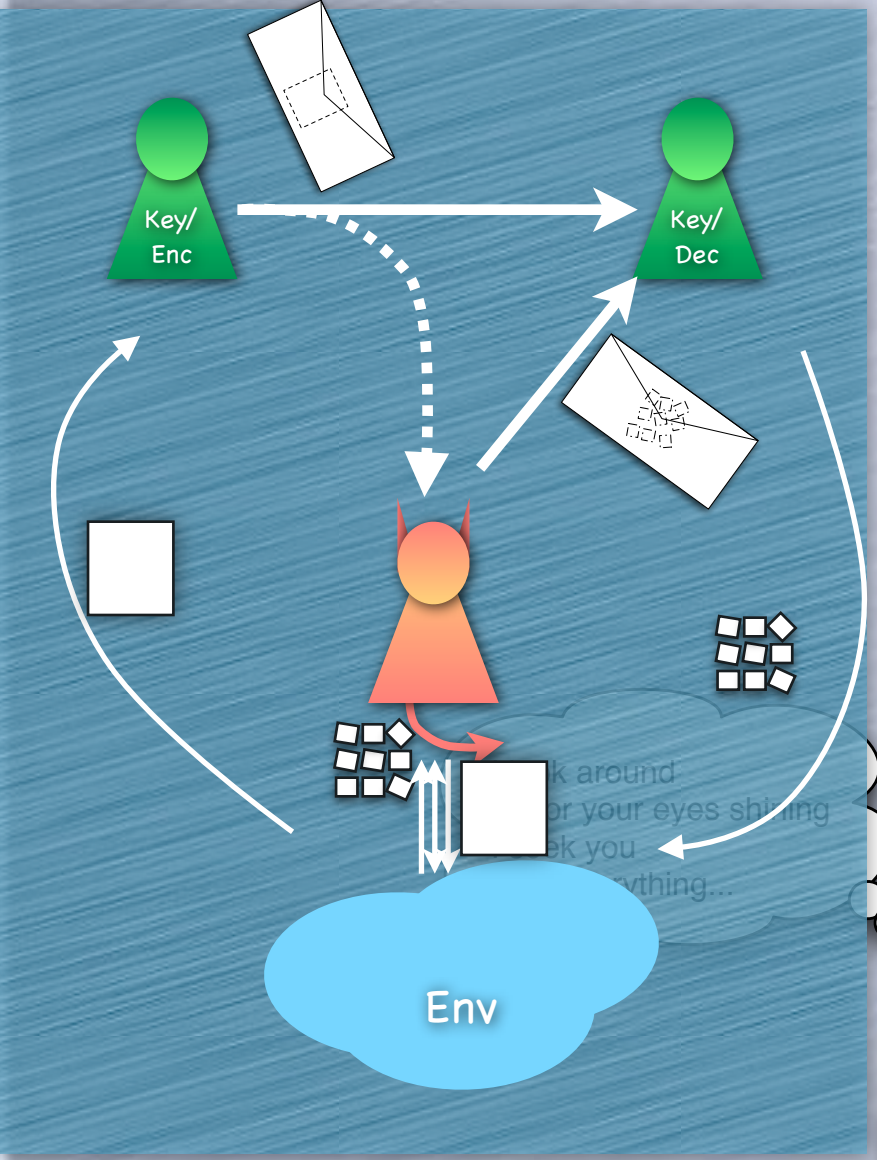

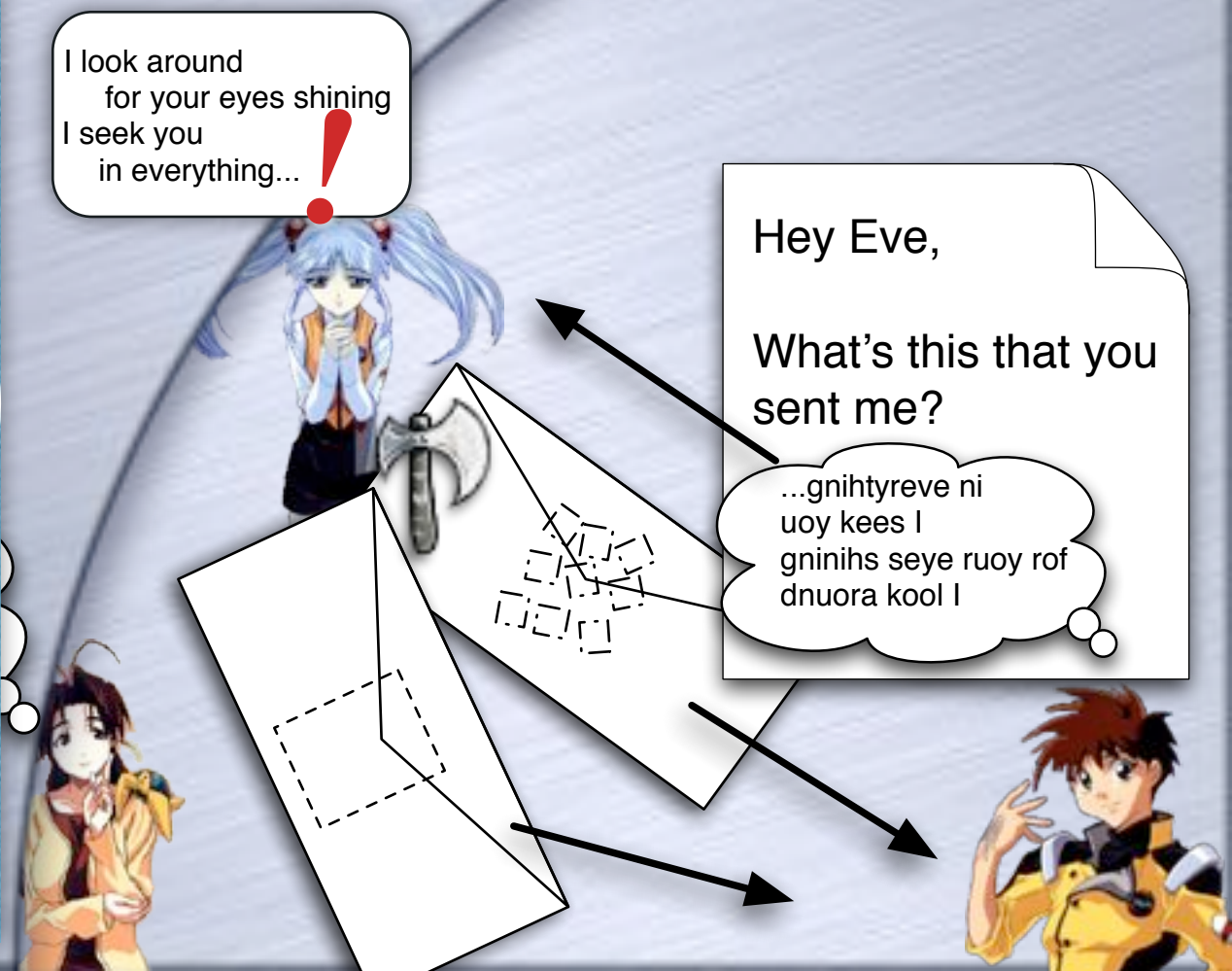

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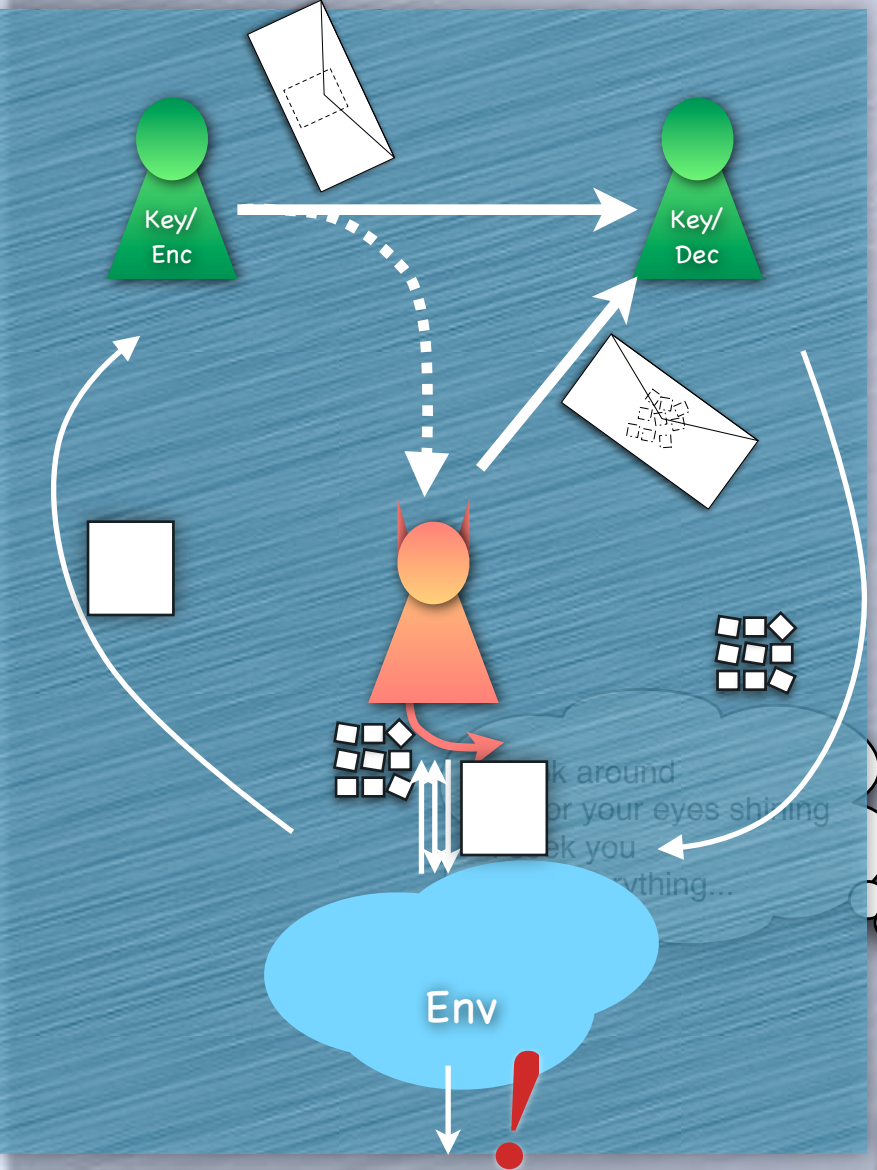
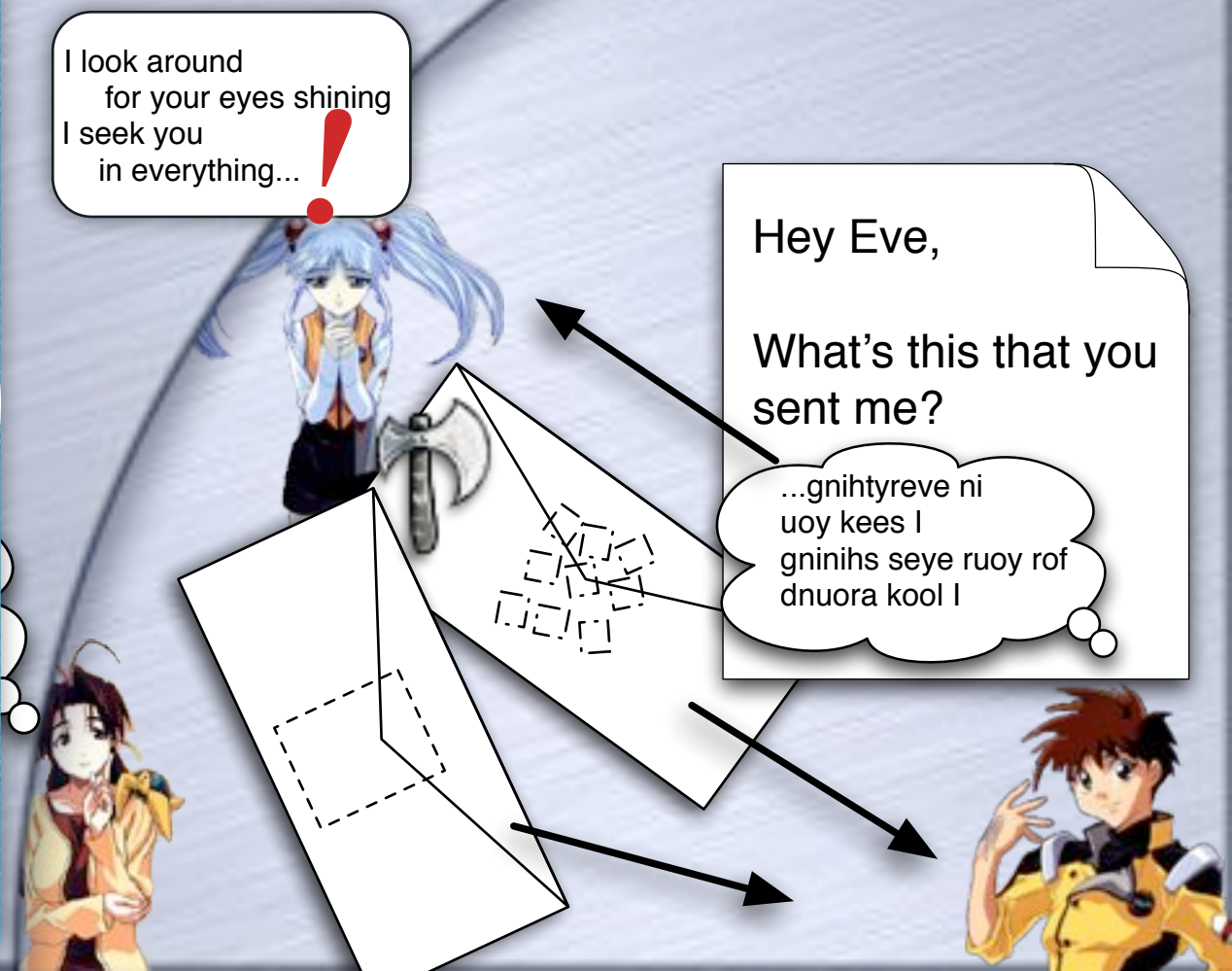
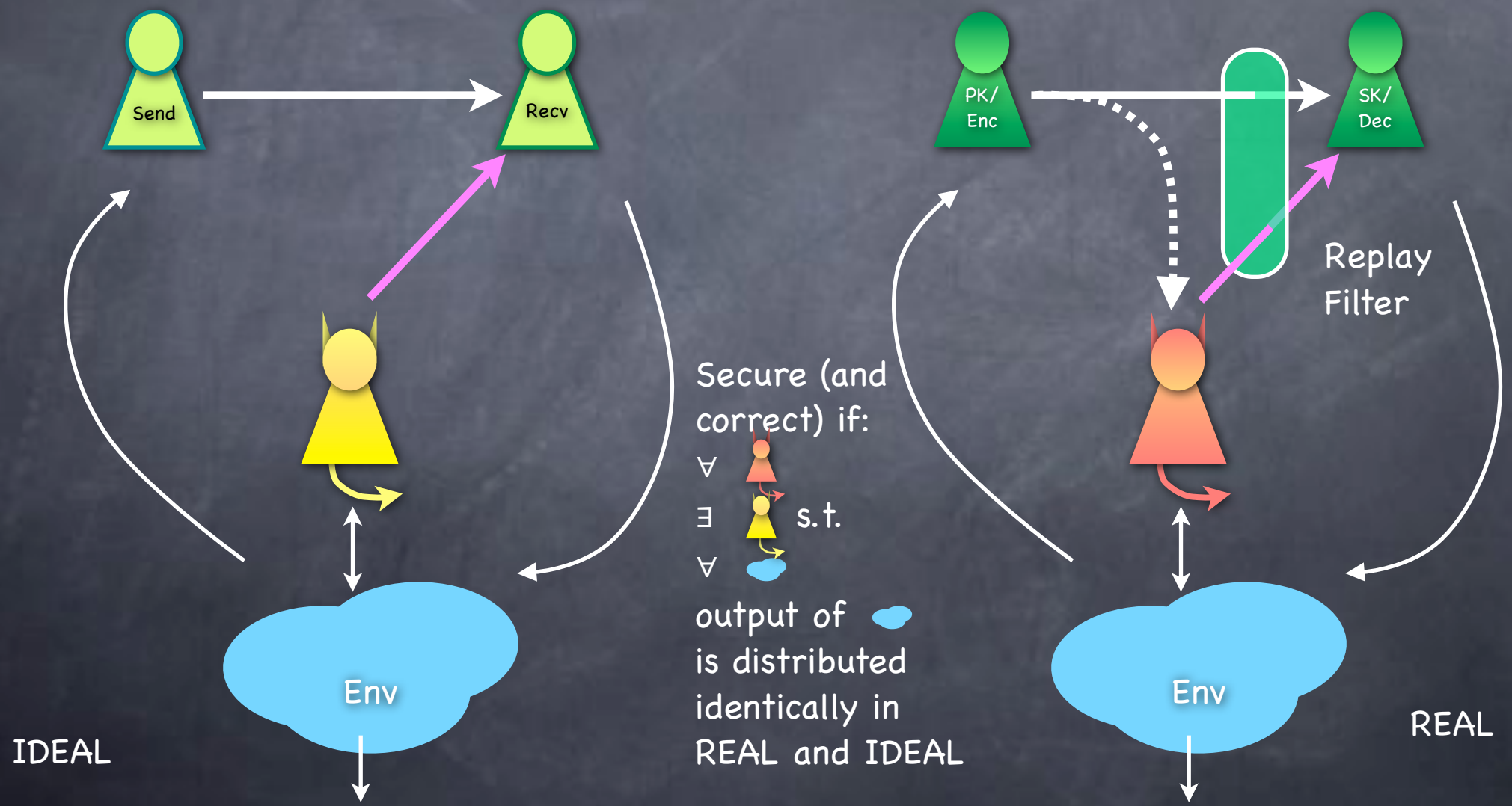
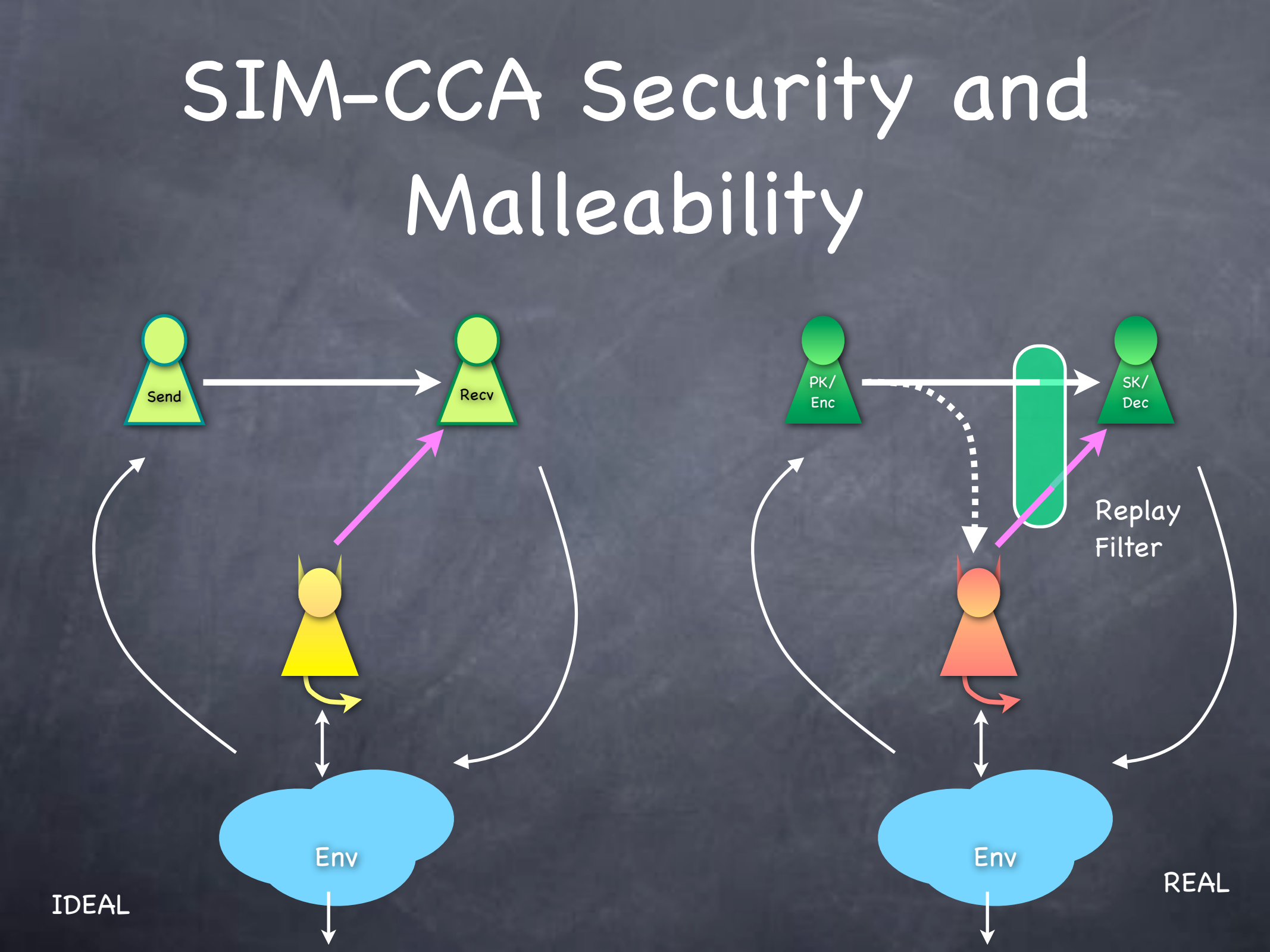




SIM-CCA Security (PKE)



SIM-CCA Security and Malleability



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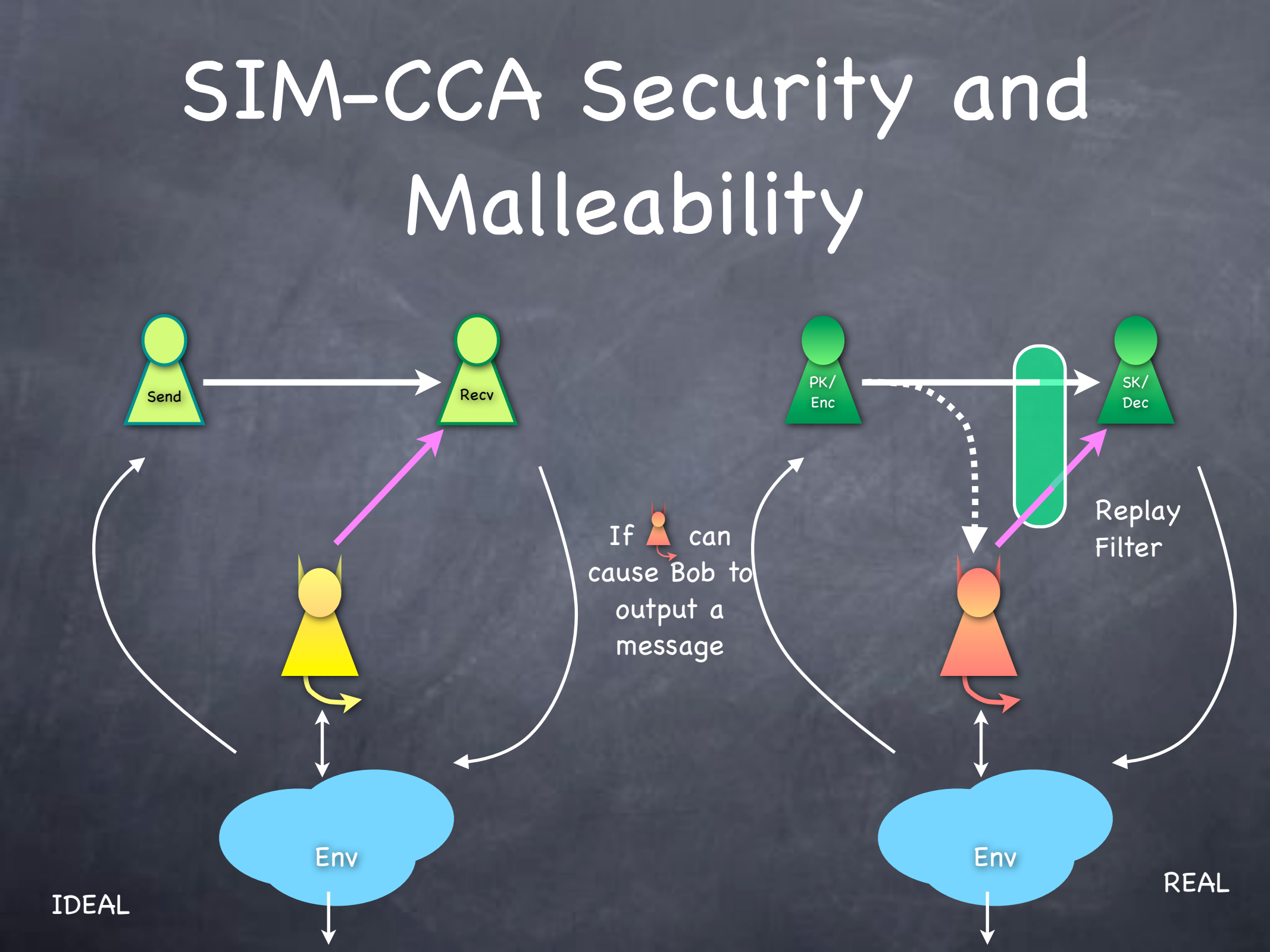

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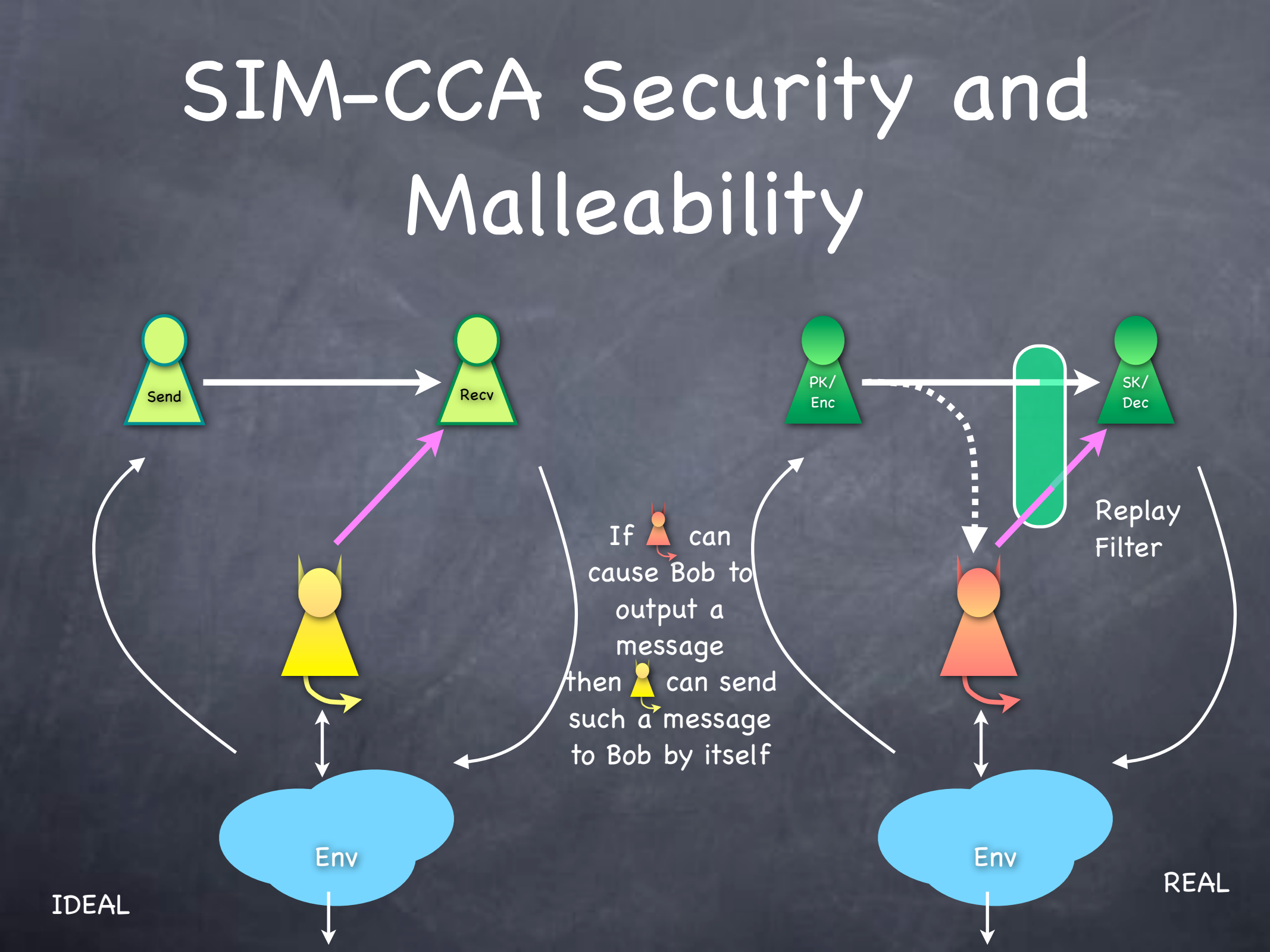

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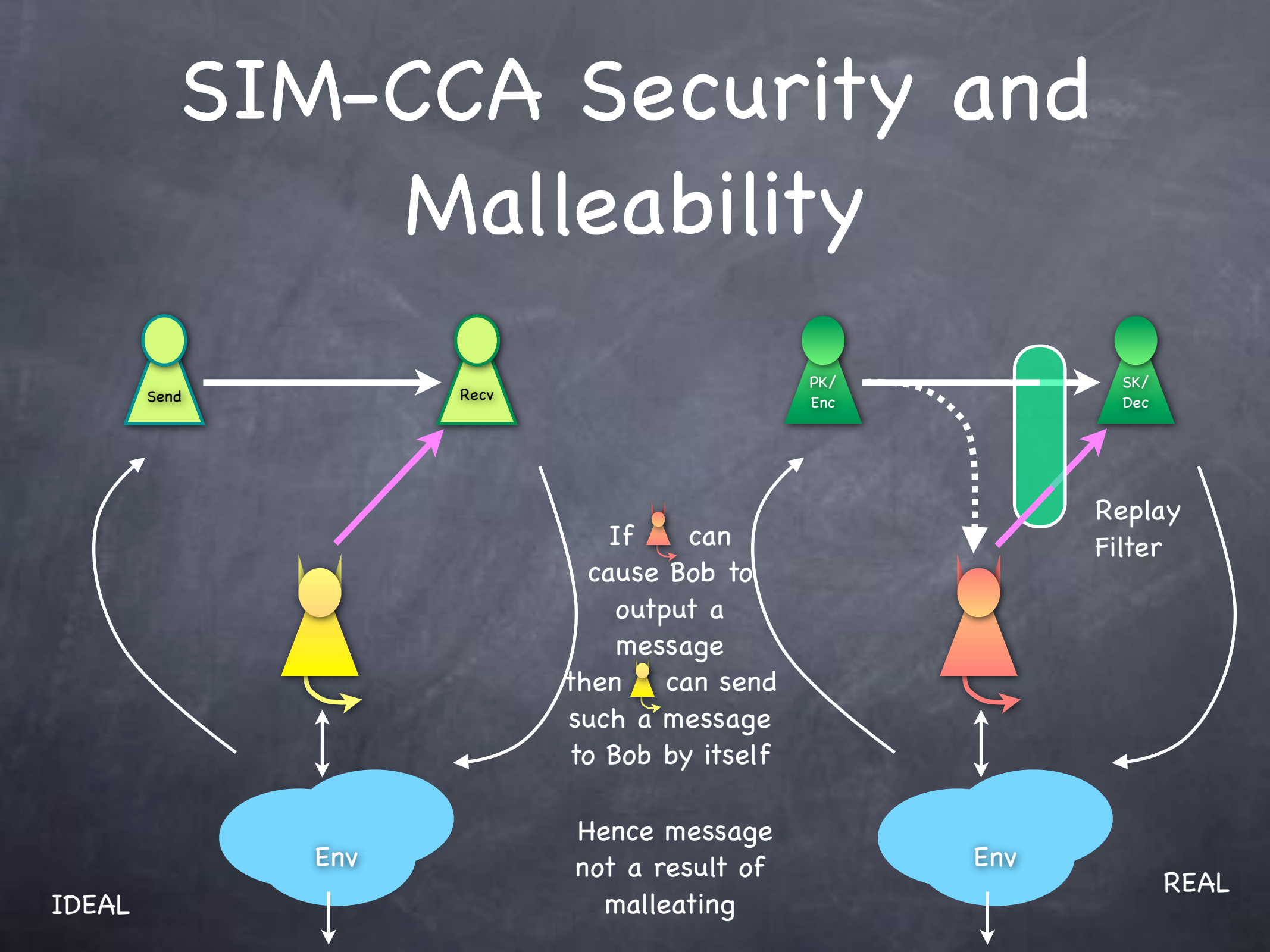

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  - Decryption: Check S (assuming $x_1=x_2$) and extract M

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