

# Zero-Knowledge Proofs

Lecture 15

# Interactive Proofs

# Interactive Proofs



# Interactive Proofs

- *Prover* wants to convince *verifier* that  $x$  has some property



# Interactive Proofs

- **Prover** wants to convince **verifier** that  $x$  has some property
  - i.e.  $x$  is in language  $L$



# Interactive Proofs

- **Prover** wants to convince **verifier** that  $x$  has some property
- i.e.  $x$  is in language  $L$



$x \in L$

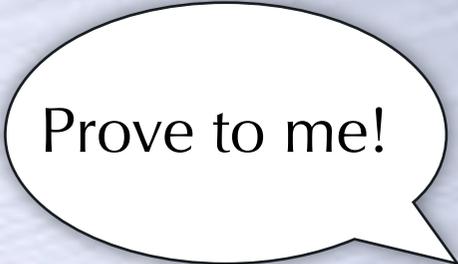


# Interactive Proofs

- *Prover* wants to convince *verifier* that  $x$  has some property
- i.e.  $x$  is in language  $L$



$x \in L$

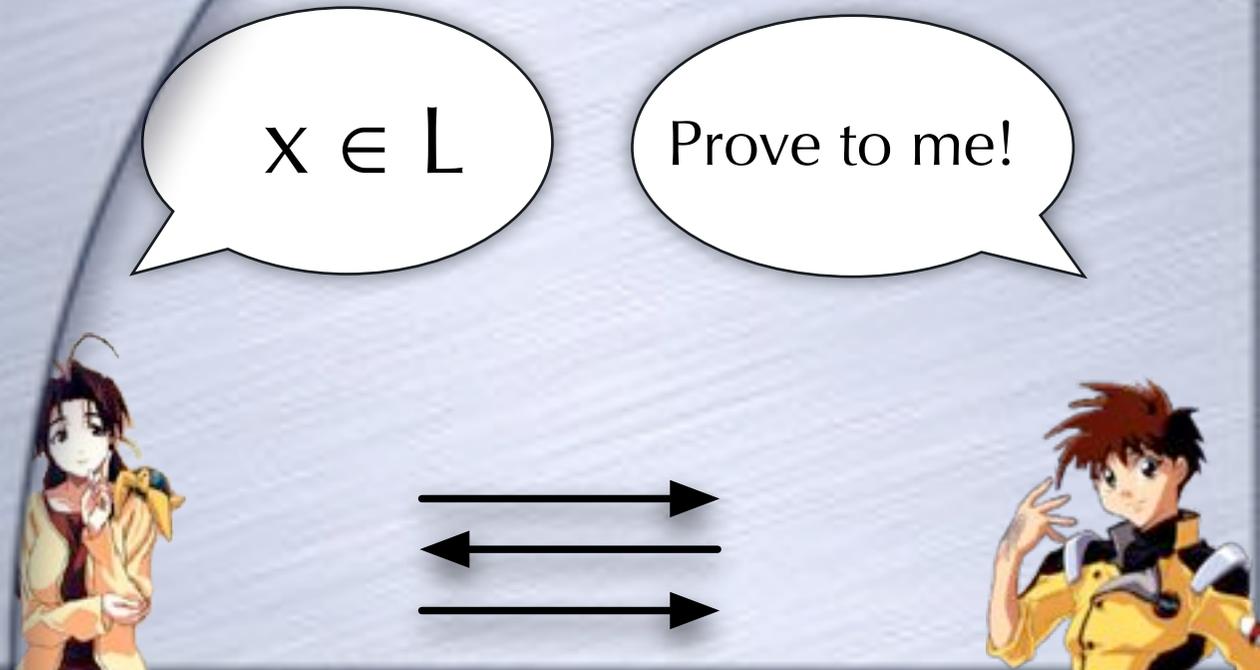


Prove to me!



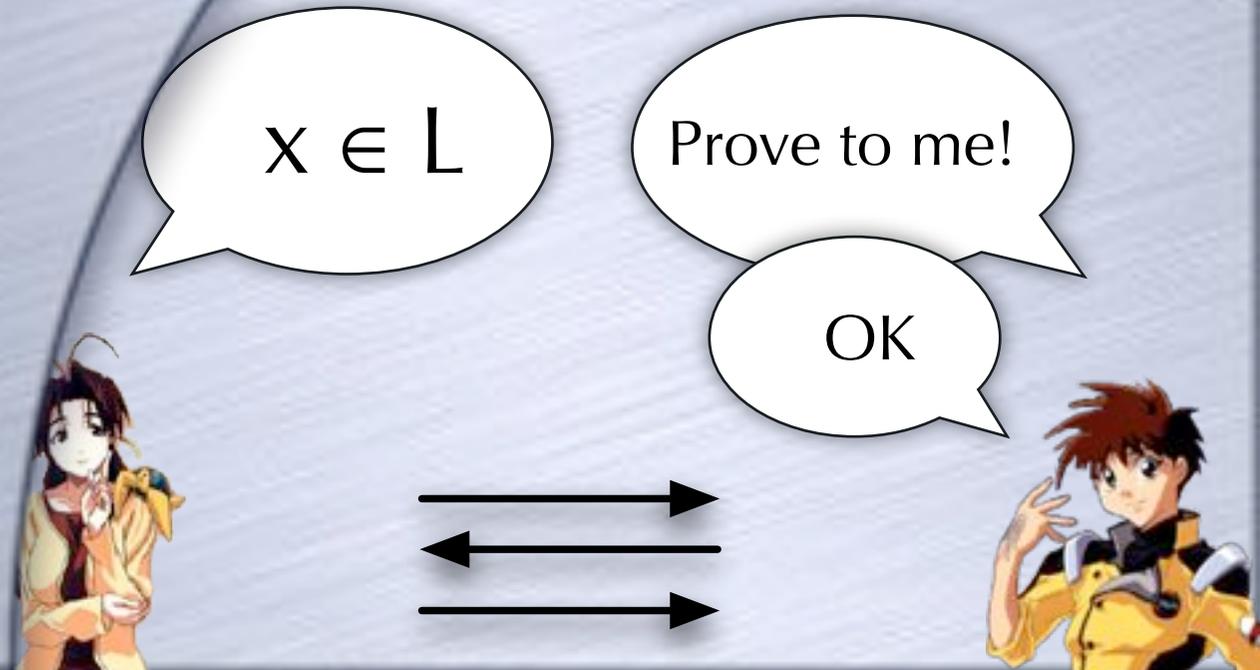
# Interactive Proofs

- **Prover** wants to convince **verifier** that  $x$  has some property
- i.e.  $x$  is in language  $L$



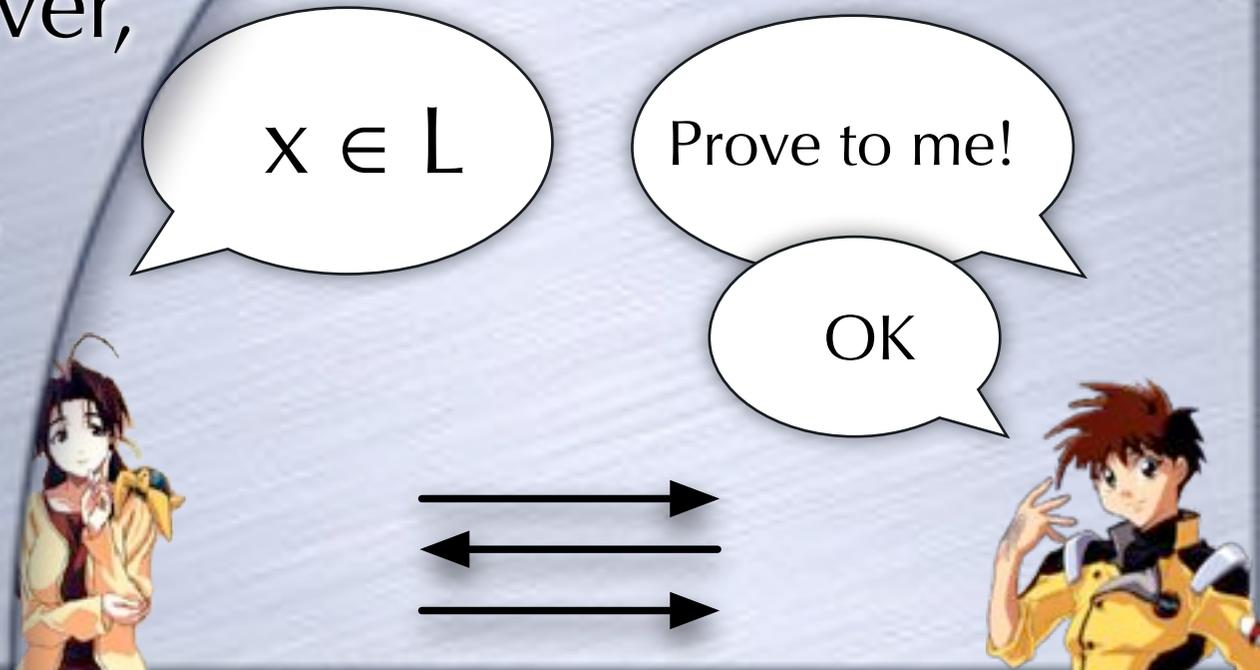
# Interactive Proofs

- **Prover** wants to convince **verifier** that  $x$  has some property
- i.e.  $x$  is in language  $L$



# Interactive Proofs

- **Prover** wants to convince **verifier** that  $x$  has some property
  - i.e.  $x$  is in language  $L$
- All powerful prover, computationally bounded verifier (for now)



# Interactive Proofs



# Interactive Proofs

- **Completeness**



# Interactive Proofs

- **Completeness**
  - If  $x$  in  $L$ , honest Prover will convince honest Verifier



# Interactive Proofs

- **Completeness**
  - If  $x$  in  $L$ , honest Prover will convince honest Verifier
- **Soundness**



# Interactive Proofs

- **Completeness**

- If  $x$  in  $L$ , honest Prover will convince honest Verifier

- **Soundness**

- If  $x$  not in  $L$ , honest Verifier won't accept any purported proof



# Interactive Proofs

- **Completeness**

- If  $x$  in  $L$ , honest Prover will convince honest Verifier

- **Soundness**

- If  $x$  not in  $L$ , honest Verifier won't accept any purported proof



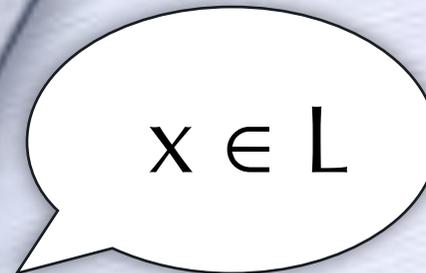
# Interactive Proofs

- **Completeness**

- If  $x$  in  $L$ , honest Prover will convince honest Verifier

- **Soundness**

- If  $x$  not in  $L$ , honest Verifier won't accept any purported proof



# Interactive Proofs

- **Completeness**

- If  $x$  in  $L$ , honest Prover will convince honest Verifier

- **Soundness**

- If  $x$  not in  $L$ , honest Verifier won't accept any purported proof



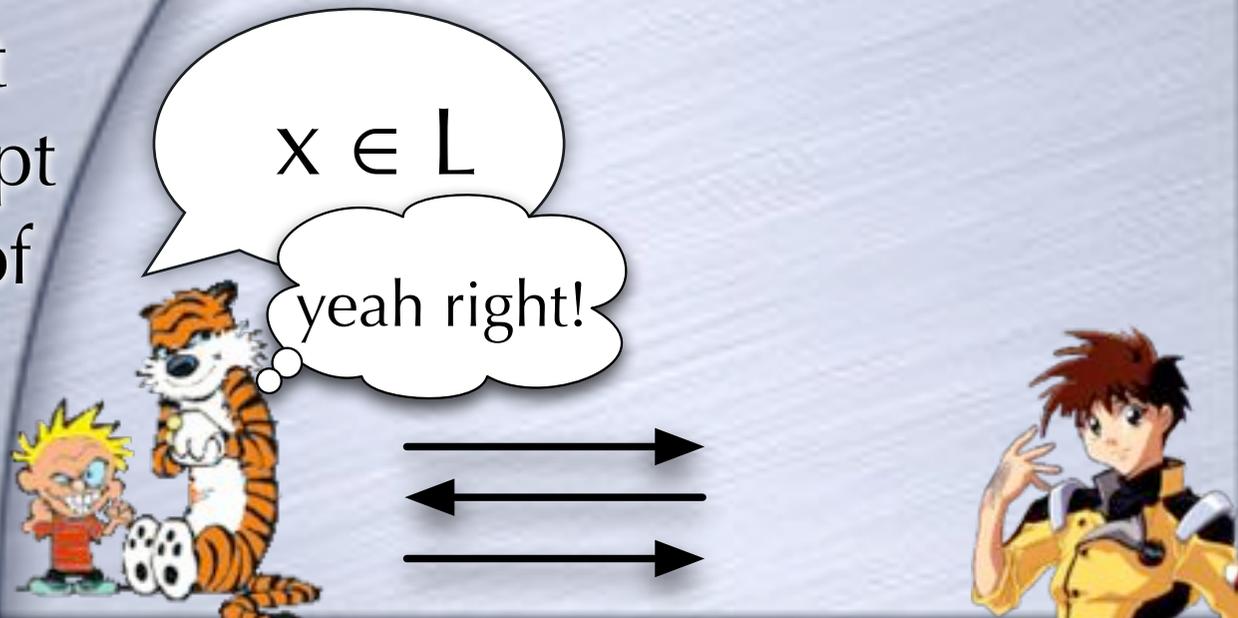
# Interactive Proofs

- **Completeness**

- If  $x$  in  $L$ , honest Prover will convince honest Verifier

- **Soundness**

- If  $x$  not in  $L$ , honest Verifier won't accept any purported proof



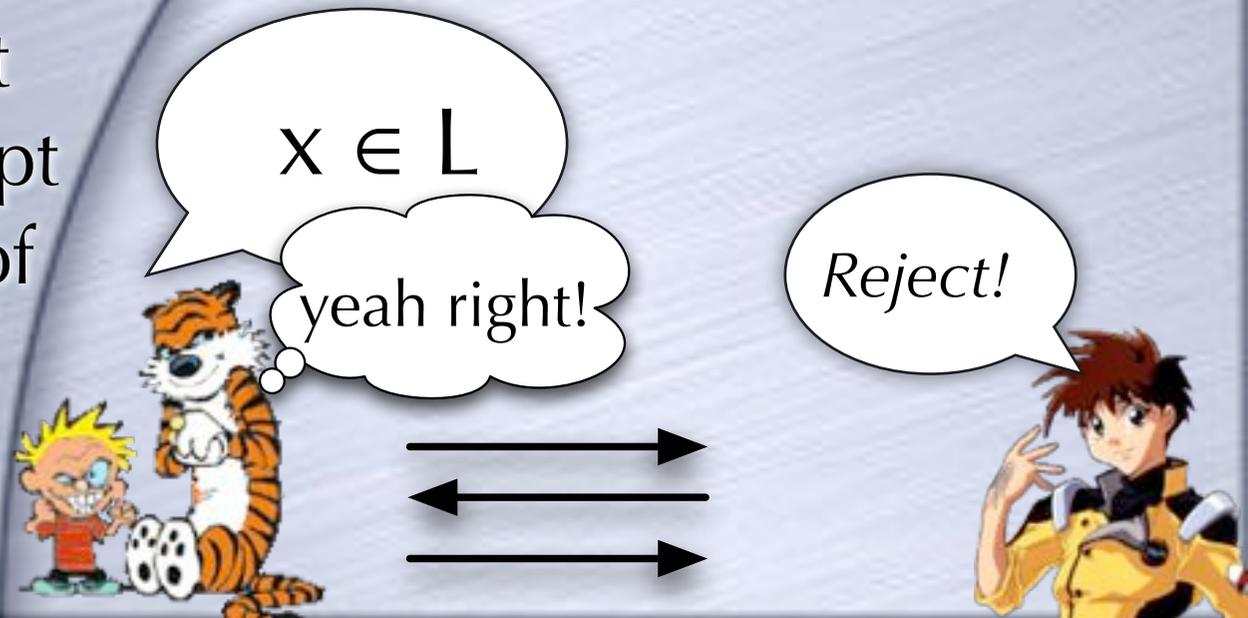
# Interactive Proofs

- **Completeness**

- If  $x$  in  $L$ , honest Prover will convince honest Verifier

- **Soundness**

- If  $x$  not in  $L$ , honest Verifier won't accept any purported proof



# An Example



# An Example

- **Coke in bottle or can**



# An Example

- **Coke in bottle or can**
  - Prover claims: coke in bottle and coke in can are different



# An Example

- **Coke in bottle or can**
  - Prover claims: coke in bottle and coke in can are different
- IP protocol:



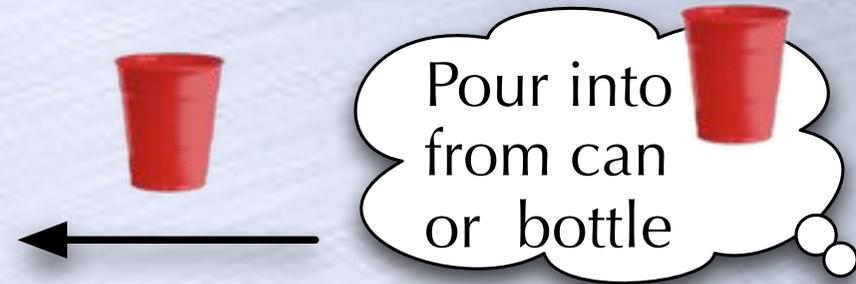
# An Example

- **Coke in bottle or can**
  - Prover claims: coke in bottle and coke in can are different
- IP protocol:



# An Example

- **Coke in bottle or can**
  - Prover claims: coke in bottle and coke in can are different
- IP protocol:



# An Example

- **Coke in bottle or can**

- Prover claims: coke in bottle and coke in can are different

- **IP protocol:**

- prover tells whether cup was filled from can or bottle



# An Example

- **Coke in bottle or can**

- Prover claims: coke in bottle and coke in can are different

- **IP protocol:**

- prover tells whether cup was filled from can or bottle



# An Example

- **Coke in bottle or can**

- Prover claims: coke in bottle and coke in can are different

- **IP protocol:**

- prover tells whether cup was filled from can or bottle

- repeat till verifier is convinced



# An Example

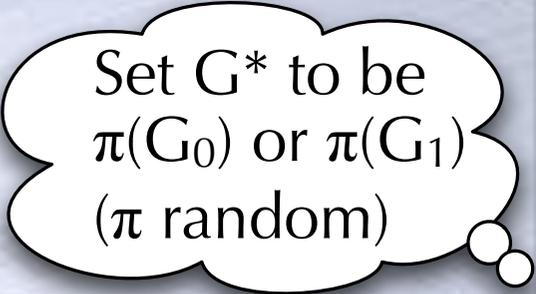
- **Graph Non-Isomorphism**

- Prover claims:  $G_0$  *not* isomorphic to  $G_1$

- IP protocol:

- prover tells whether  $G^*$  is an isomorphism of  $G_0$  or  $G_1$

- repeat till verifier is convinced



Set  $G^*$  to be  $\pi(G_0)$  or  $\pi(G_1)$   
( $\pi$  random)



# An Example

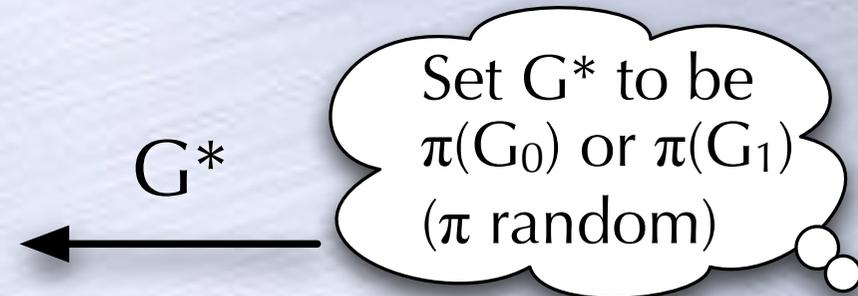
- **Graph Non-Isomorphism**

- Prover claims:  $G_0$  *not* isomorphic to  $G_1$

- IP protocol:

- prover tells whether  $G^*$  is an isomorphism of  $G_0$  or  $G_1$

- repeat till verifier is convinced



# An Example

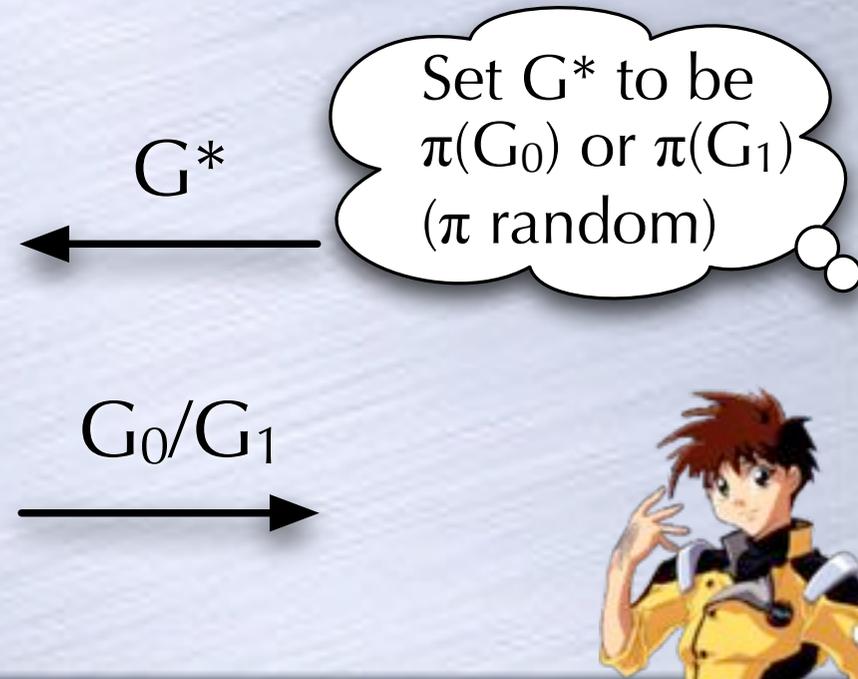
- **Graph Non-Isomorphism**

- Prover claims:  $G_0$  *not* isomorphic to  $G_1$

- IP protocol:

- prover tells whether  $G^*$  is an isomorphism of  $G_0$  or  $G_1$

- repeat till verifier is convinced



# Proofs for NP languages

$x \in L$

Prove to me!



# Proofs for NP languages

- Proving membership in an NP language L



$x \in L$



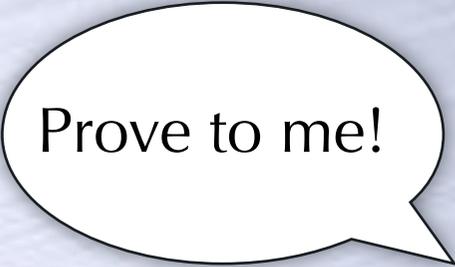
Prove to me!

# Proofs for NP languages

- Proving membership in an NP language L
- $x \in L$  iff  $\exists w R(x,w)=1$  (for R in P)



$x \in L$

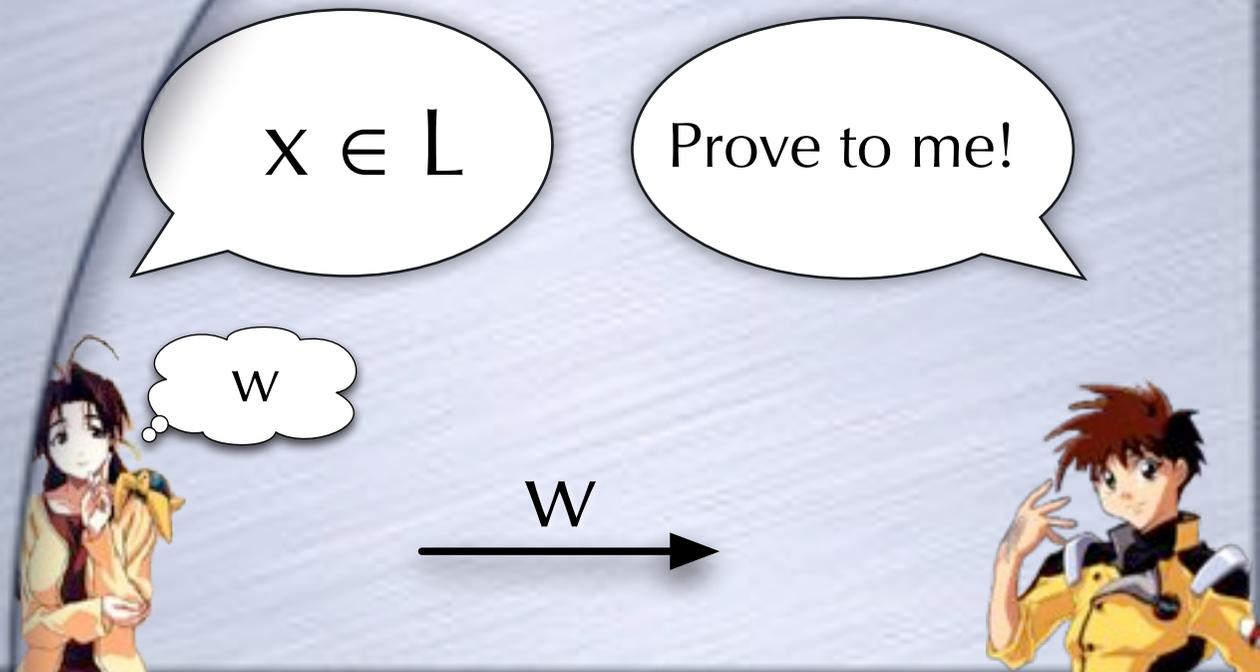


Prove to me!



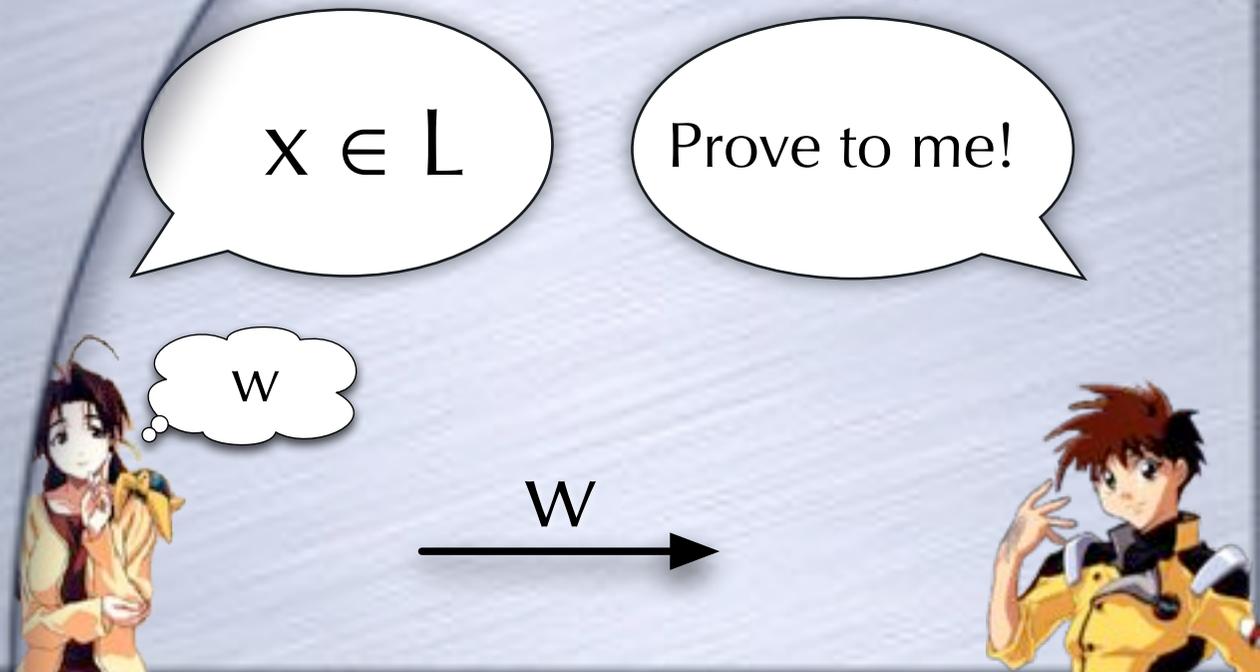
# Proofs for NP languages

- Proving membership in an NP language  $L$ 
  - $x \in L$  iff  $\exists w R(x,w)=1$  (for  $R$  in  $\mathbf{P}$ )
    - e.g. Graph Isomorphism



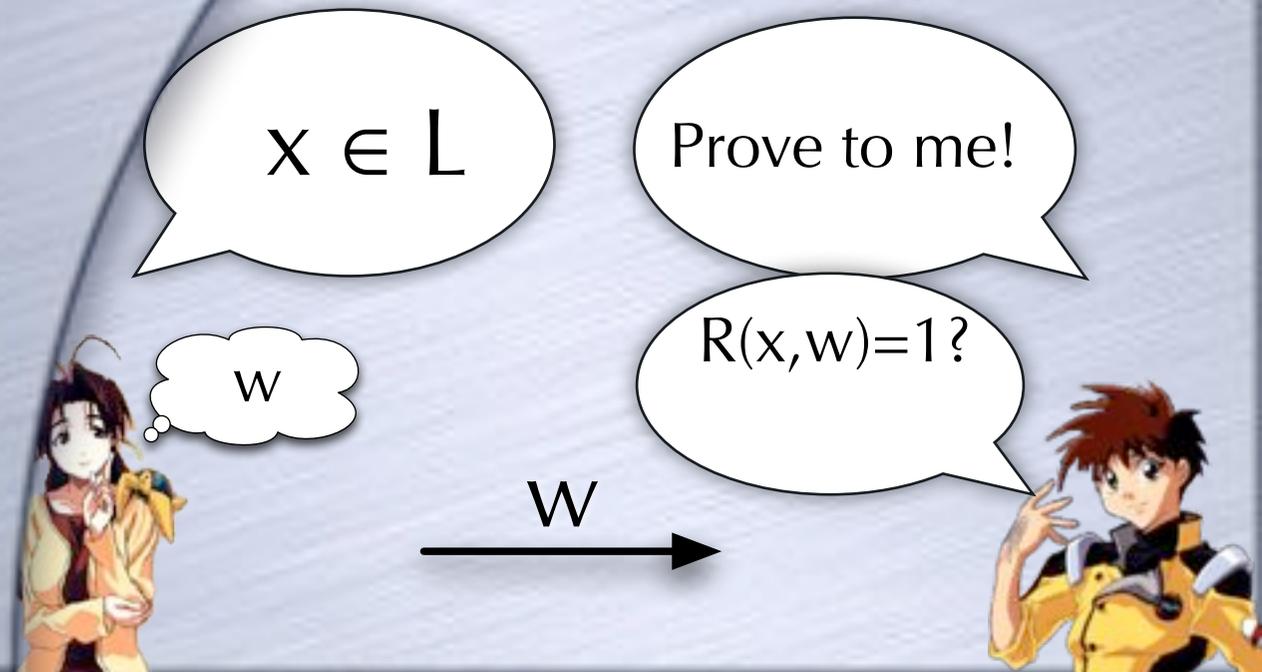
# Proofs for NP languages

- Proving membership in an NP language  $L$ 
  - $x \in L$  iff  $\exists w R(x,w)=1$  (for  $R$  in  $\mathbf{P}$ )
    - e.g. Graph Isomorphism
- IP protocol:



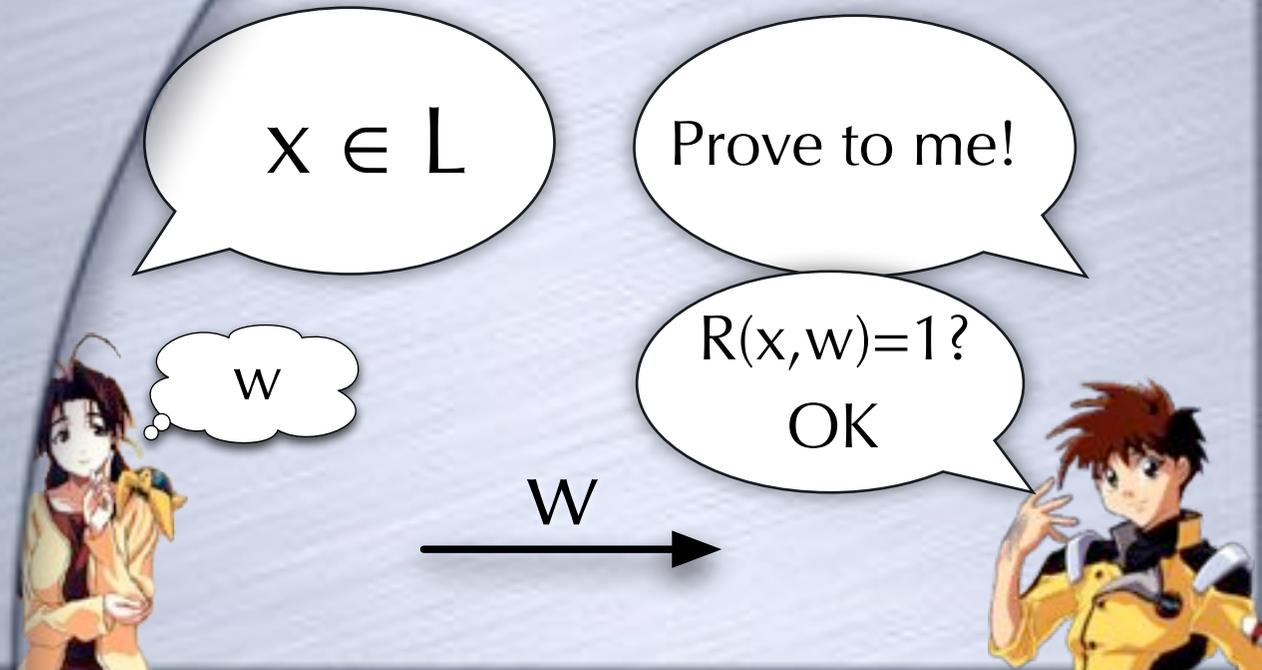
# Proofs for NP languages

- Proving membership in an NP language L
  - $x \in L$  iff  $\exists w R(x,w)=1$  (for R in P)
    - e.g. Graph Isomorphism
- IP protocol:



# Proofs for NP languages

- Proving membership in an NP language  $L$ 
  - $x \in L$  iff  $\exists w R(x,w)=1$  (for  $R$  in  $\mathbf{P}$ )
    - e.g. Graph Isomorphism
- IP protocol:



# Proofs for NP languages

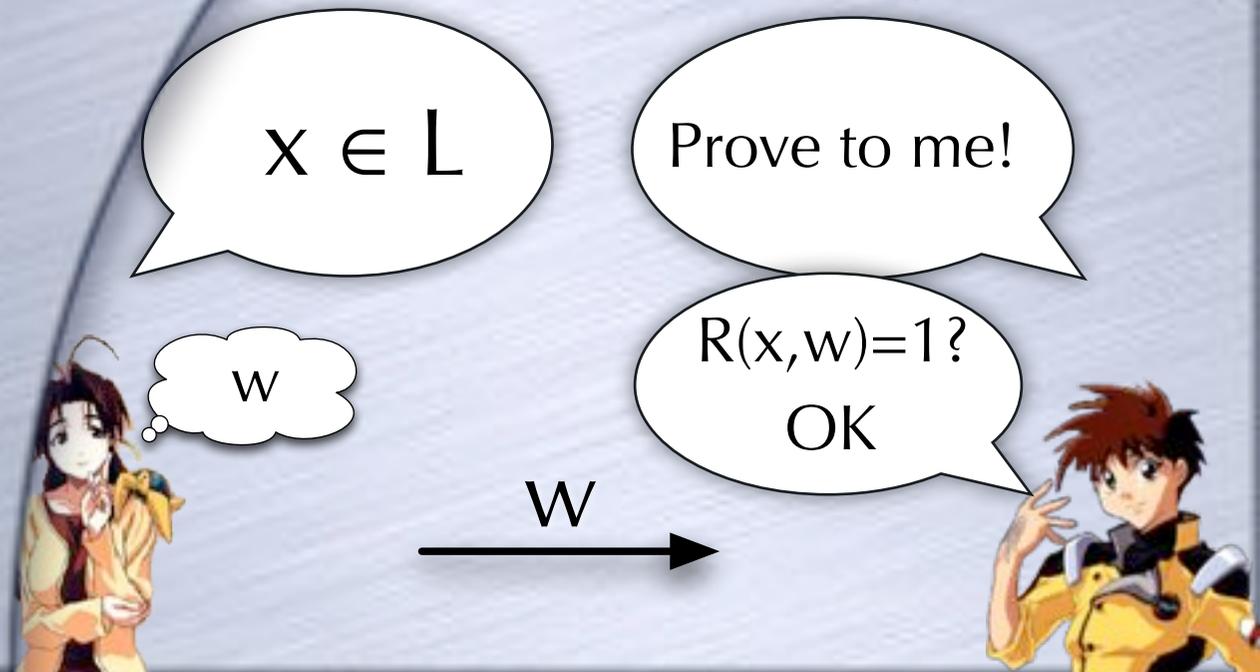
- Proving membership in an NP language L

- $x \in L$  iff  $\exists w R(x,w)=1$  (for R in P)

- e.g. Graph Isomorphism

- IP protocol:

- prover sends w (non-interactive)



# Proofs for NP languages

- Proving membership in an NP language L

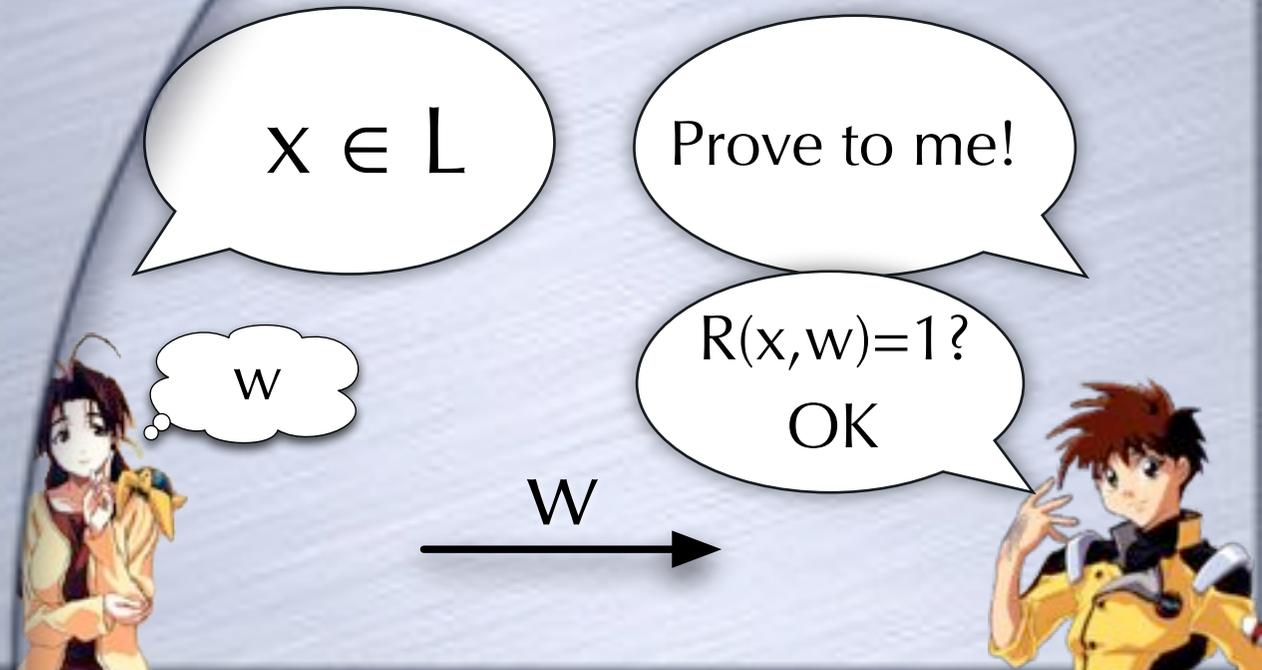
- $x \in L$  iff  $\exists w R(x,w)=1$  (for R in P)

- e.g. Graph Isomorphism

- IP protocol:

- prover sends w (non-interactive)

- *What if prover doesn't want to reveal w?*



# Zero-Knowledge Proofs



# Zero-Knowledge Proofs

- Verifier should not gain *any* knowledge from the honest prover



# Zero-Knowledge Proofs

- Verifier should not gain *any* knowledge from the honest prover
- except whether  $x$  is in  $L$



# Zero-Knowledge Proofs

- Verifier should not gain *any* knowledge from the honest prover
- except whether  $x$  is in  $L$



# Zero-Knowledge Proofs

- Verifier should not gain *any* knowledge from the honest prover
- except whether  $x$  is in  $L$

$x \in L$



# Zero-Knowledge Proofs

- Verifier should not gain *any* knowledge from the honest prover
- except whether  $x$  is in  $L$

$x \in L$

Prove to me!



# Zero-Knowledge Proofs

- Verifier should not gain *any* knowledge from the honest prover
- except whether  $x$  is in  $L$

$x \in L$

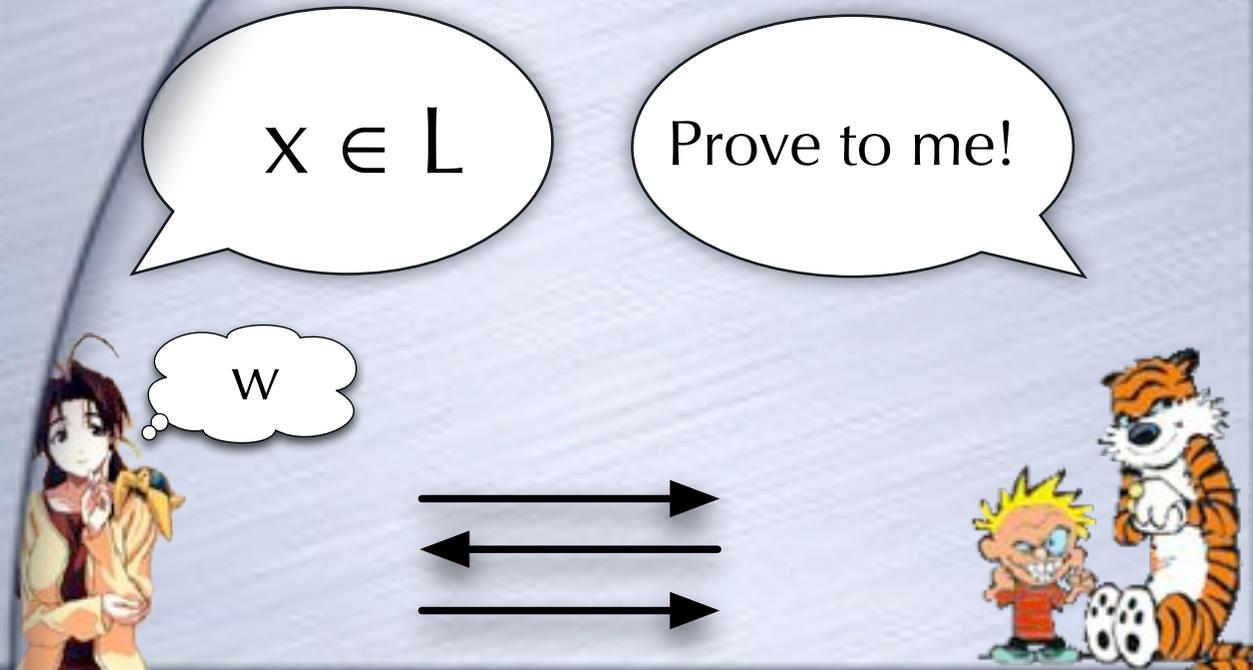
Prove to me!

$w$



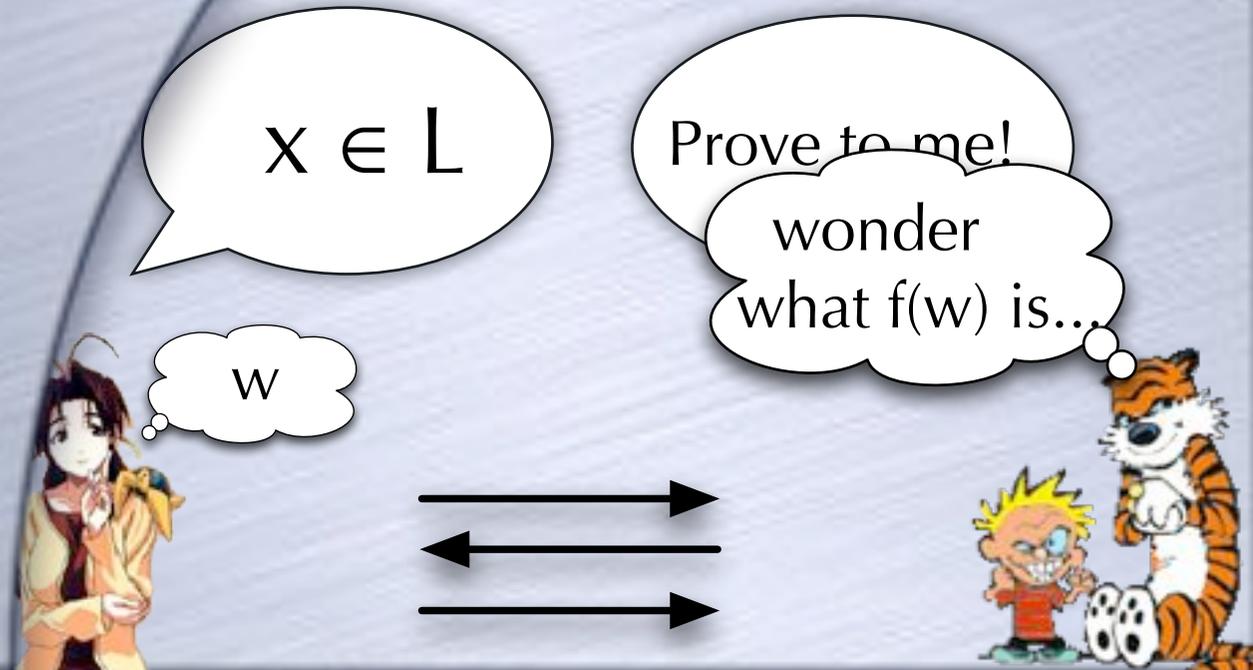
# Zero-Knowledge Proofs

- Verifier should not gain *any* knowledge from the honest prover
- except whether  $x$  is in  $L$



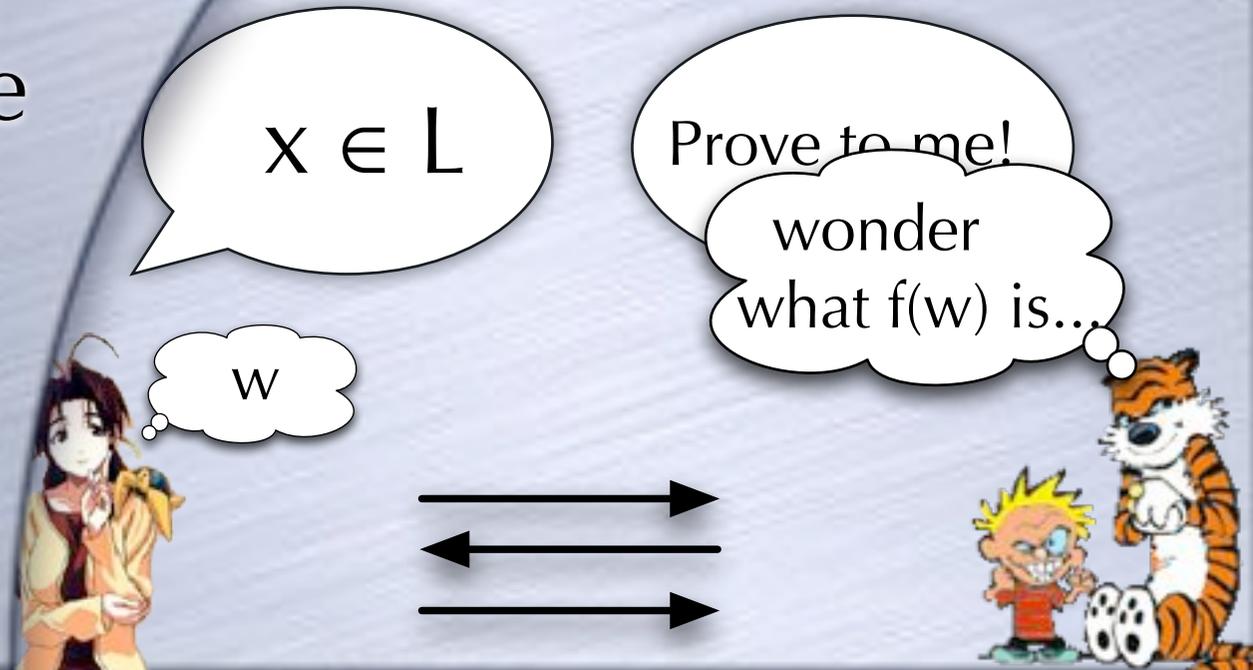
# Zero-Knowledge Proofs

- Verifier should not gain *any* knowledge from the honest prover
- except whether  $x$  is in  $L$



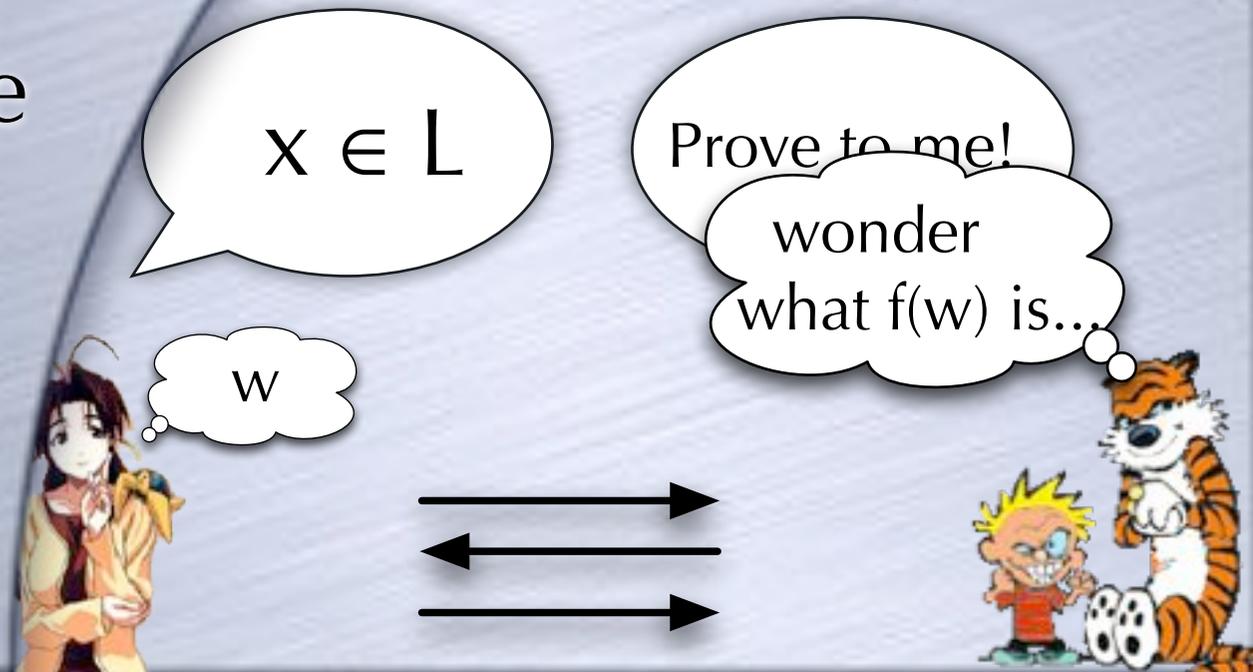
# Zero-Knowledge Proofs

- Verifier should not gain *any* knowledge from the honest prover
  - except whether  $x$  is in  $L$
- How to formalize this?



# Zero-Knowledge Proofs

- Verifier should not gain *any* knowledge from the honest prover
  - except whether  $x$  is in  $L$
- How to formalize this?
  - Simulation!



# An Example



# An Example

- **Graph Isomorphism**



# An Example

- **Graph Isomorphism**
- $(G_0, G_1)$  in L iff there exists an isomorphism  $\sigma$  such that  $\sigma(G_0) = G_1$



# An Example

- **Graph Isomorphism**
  - $(G_0, G_1)$  in L iff there exists an isomorphism  $\sigma$  such that  $\sigma(G_0) = G_1$
- IP protocol: send  $\sigma$



# An Example

- **Graph Isomorphism**
  - $(G_0, G_1)$  in L iff there exists an isomorphism  $\sigma$  such that  $\sigma(G_0) = G_1$
- IP protocol: send  $\sigma$
- ZK protocol?



# An Example

- **Graph Isomorphism**
  - $(G_0, G_1)$  in L iff there exists an isomorphism  $\sigma$  such that  $\sigma(G_0) = G_1$
  - IP protocol: send  $\sigma$
  - ZK protocol?

$G^* := \pi(G_1)$   
(random  $\pi$ )



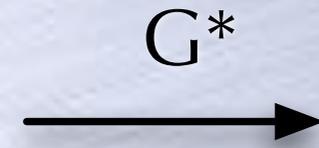
# An Example

- **Graph Isomorphism**

- $(G_0, G_1)$  in L iff there exists an isomorphism  $\sigma$  such that  $\sigma(G_0) = G_1$

- IP protocol: send  $\sigma$

- ZK protocol?



$G^* := \pi(G_1)$   
(random  $\pi$ )



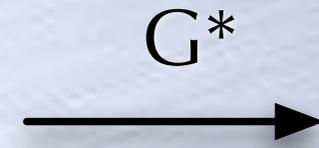
# An Example

- **Graph Isomorphism**

- $(G_0, G_1)$  in L iff there exists an isomorphism  $\sigma$  such that  $\sigma(G_0) = G_1$

- IP protocol: send  $\sigma$

- ZK protocol?



$G^* := \pi(G_1)$   
(random  $\pi$ )

random bit  
 $b$



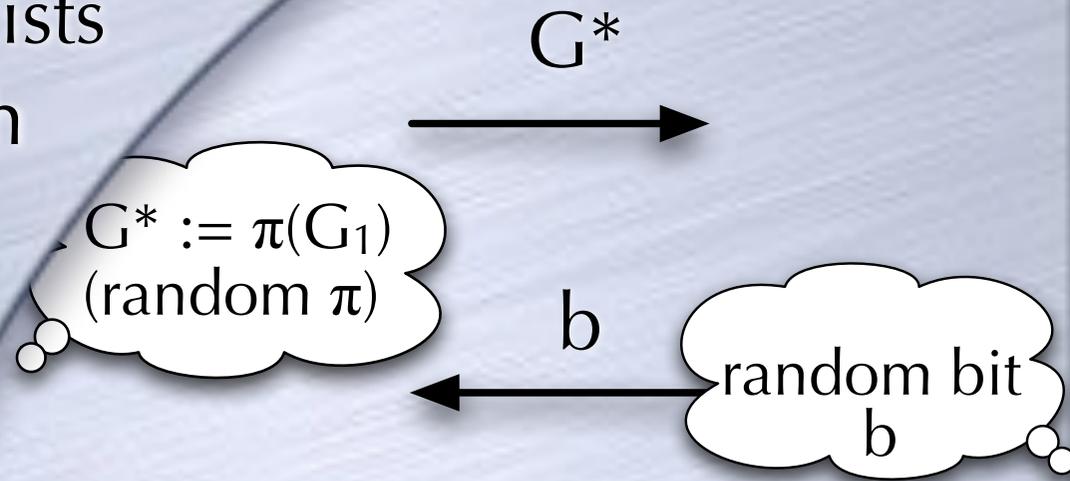
# An Example

- **Graph Isomorphism**

- $(G_0, G_1)$  in L iff there exists an isomorphism  $\sigma$  such that  $\sigma(G_0) = G_1$

- IP protocol: send  $\sigma$

- ZK protocol?



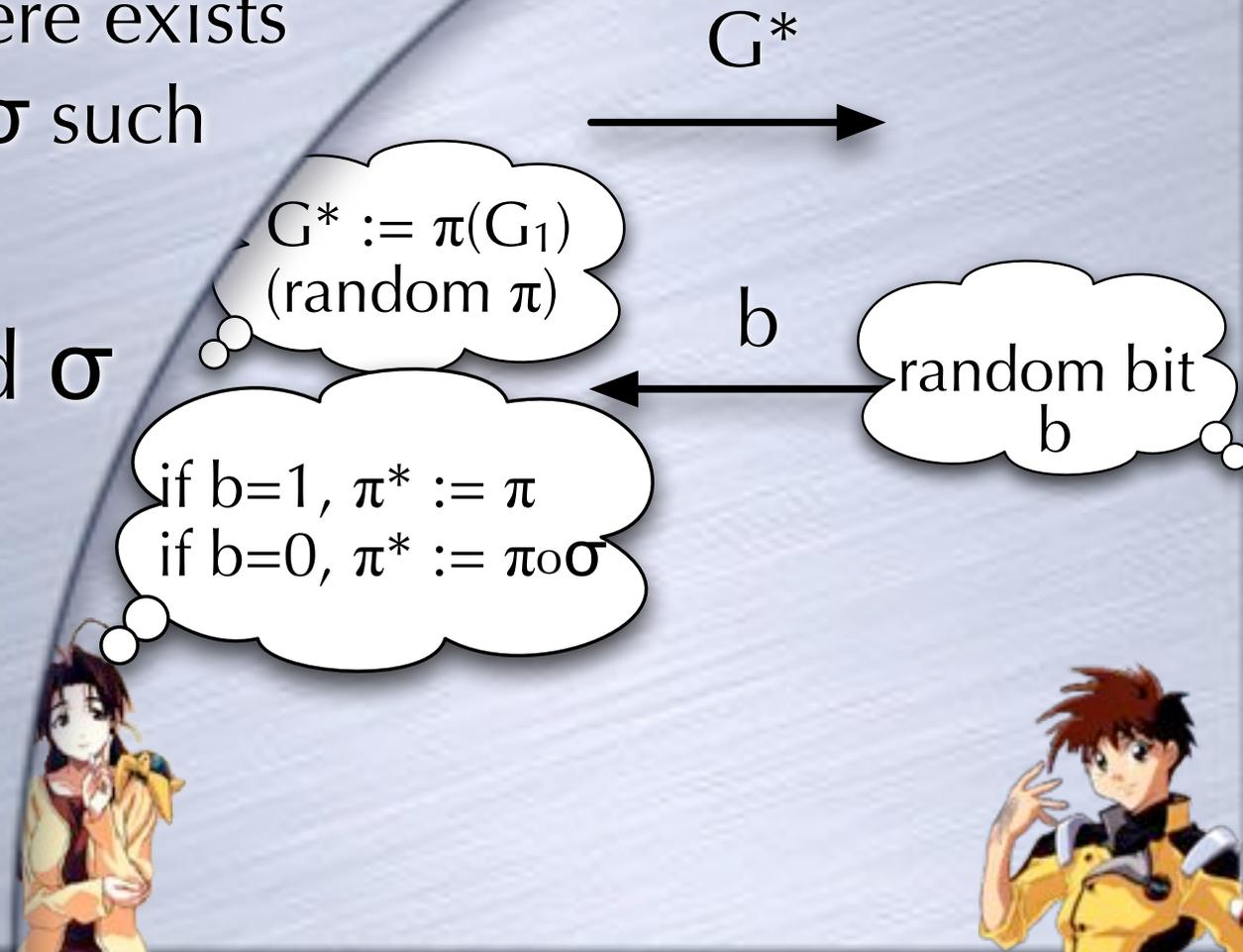
# An Example

- **Graph Isomorphism**

- $(G_0, G_1)$  in L iff there exists an isomorphism  $\sigma$  such that  $\sigma(G_0) = G_1$

- IP protocol: send  $\sigma$

- ZK protocol?



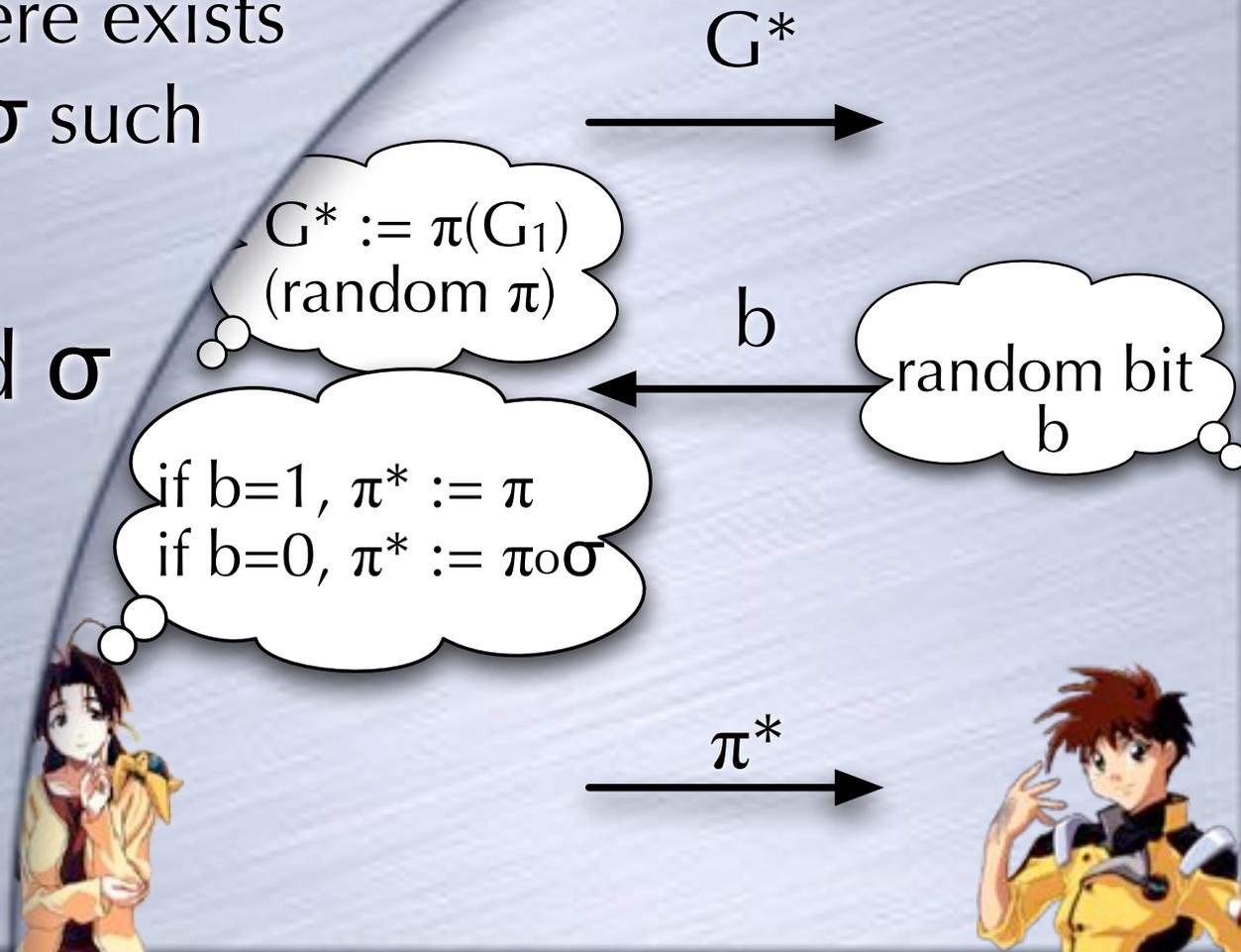
# An Example

- **Graph Isomorphism**

- $(G_0, G_1)$  in L iff there exists an isomorphism  $\sigma$  such that  $\sigma(G_0) = G_1$

- IP protocol: send  $\sigma$

- ZK protocol?



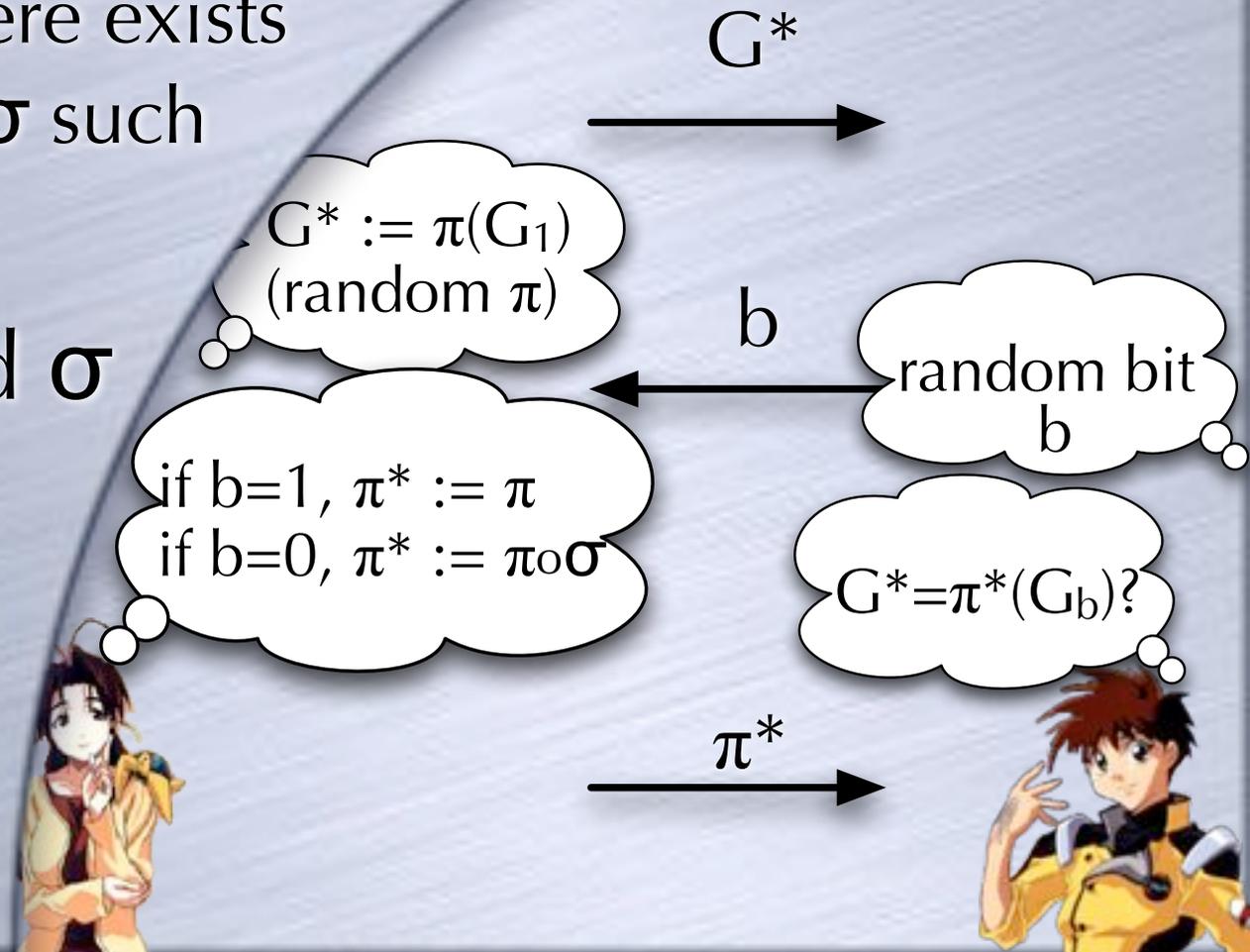
# An Example

- **Graph Isomorphism**

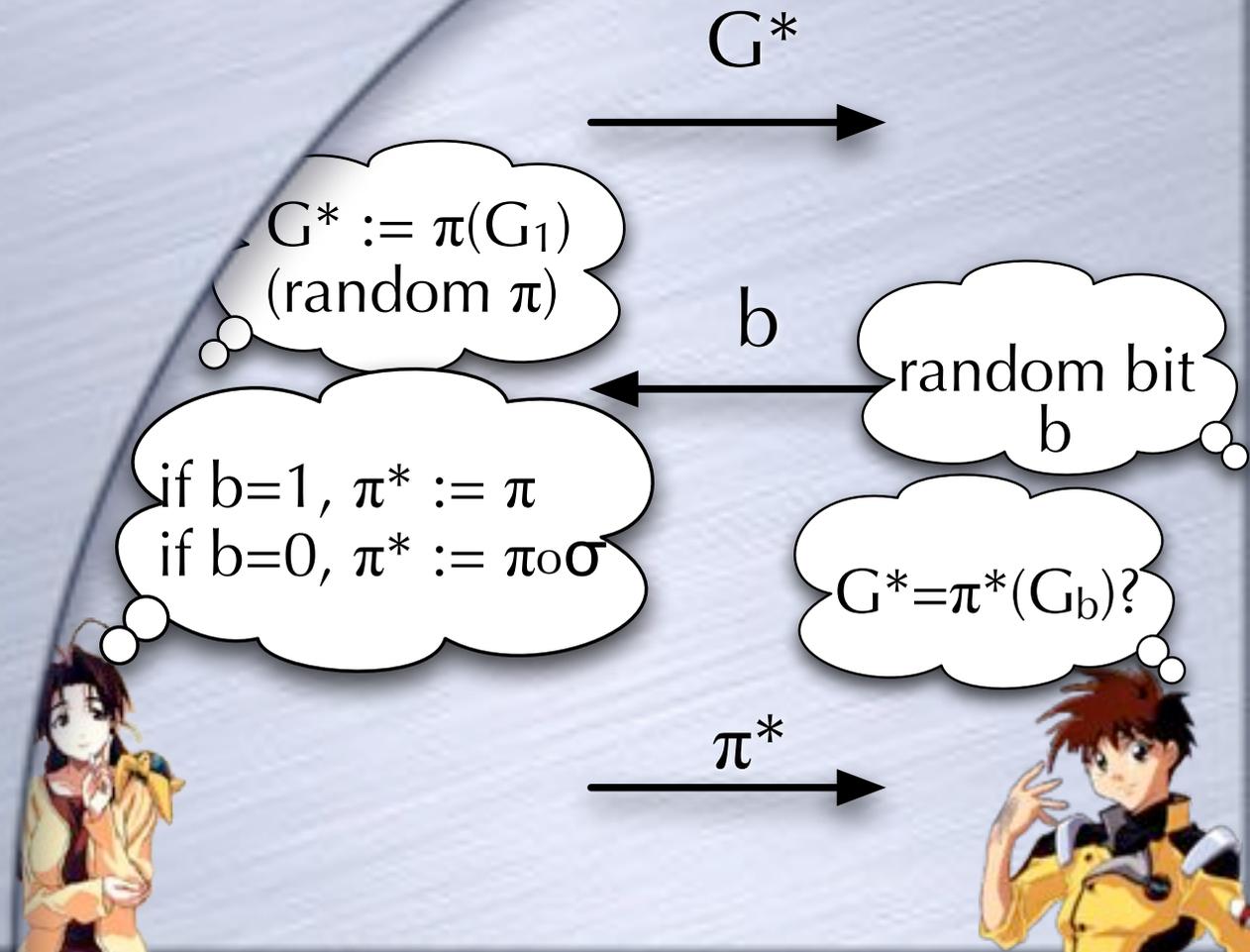
- $(G_0, G_1)$  in L iff there exists an isomorphism  $\sigma$  such that  $\sigma(G_0) = G_1$

- IP protocol: send  $\sigma$

- ZK protocol?

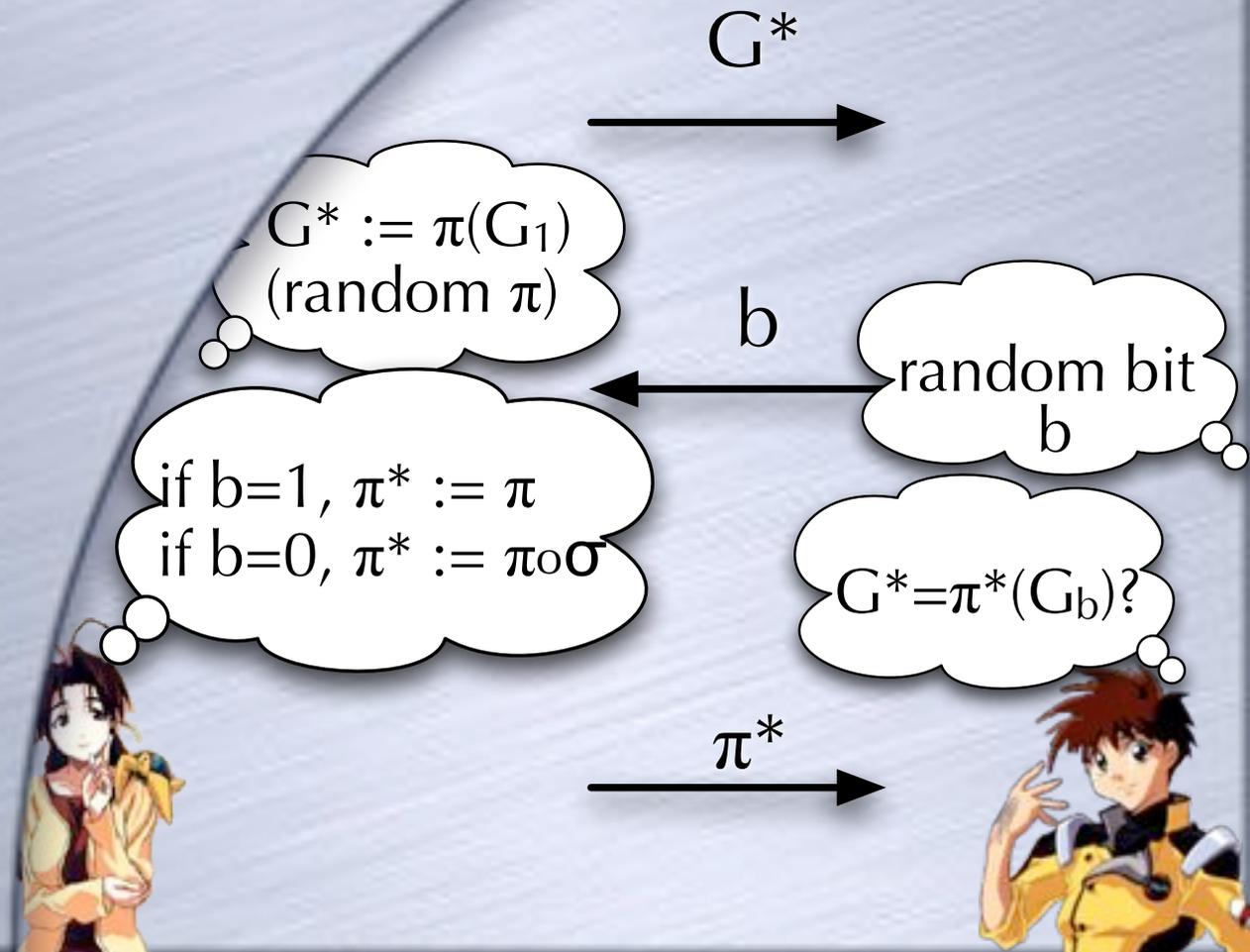


# An Example



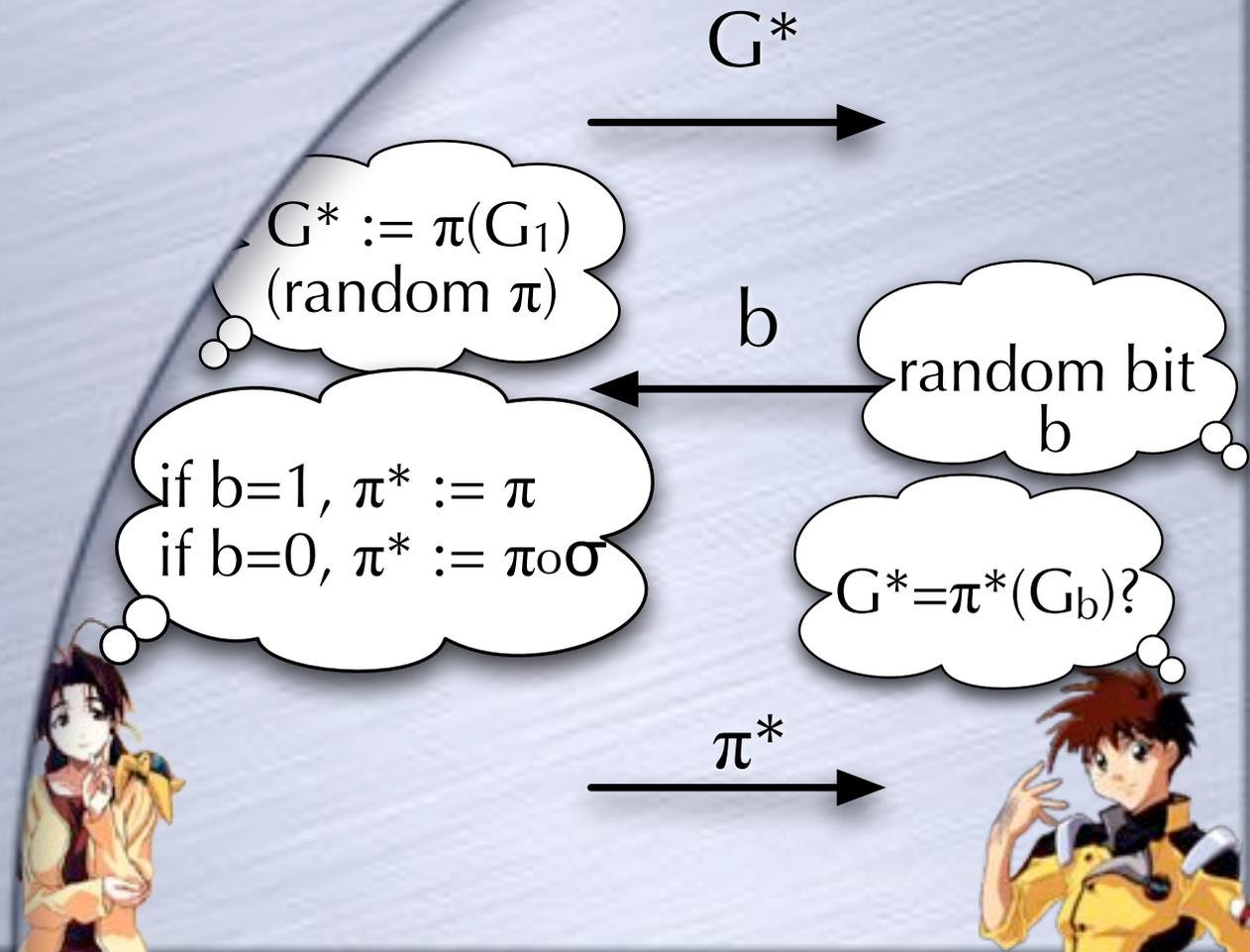
# An Example

- Why is this convincing?



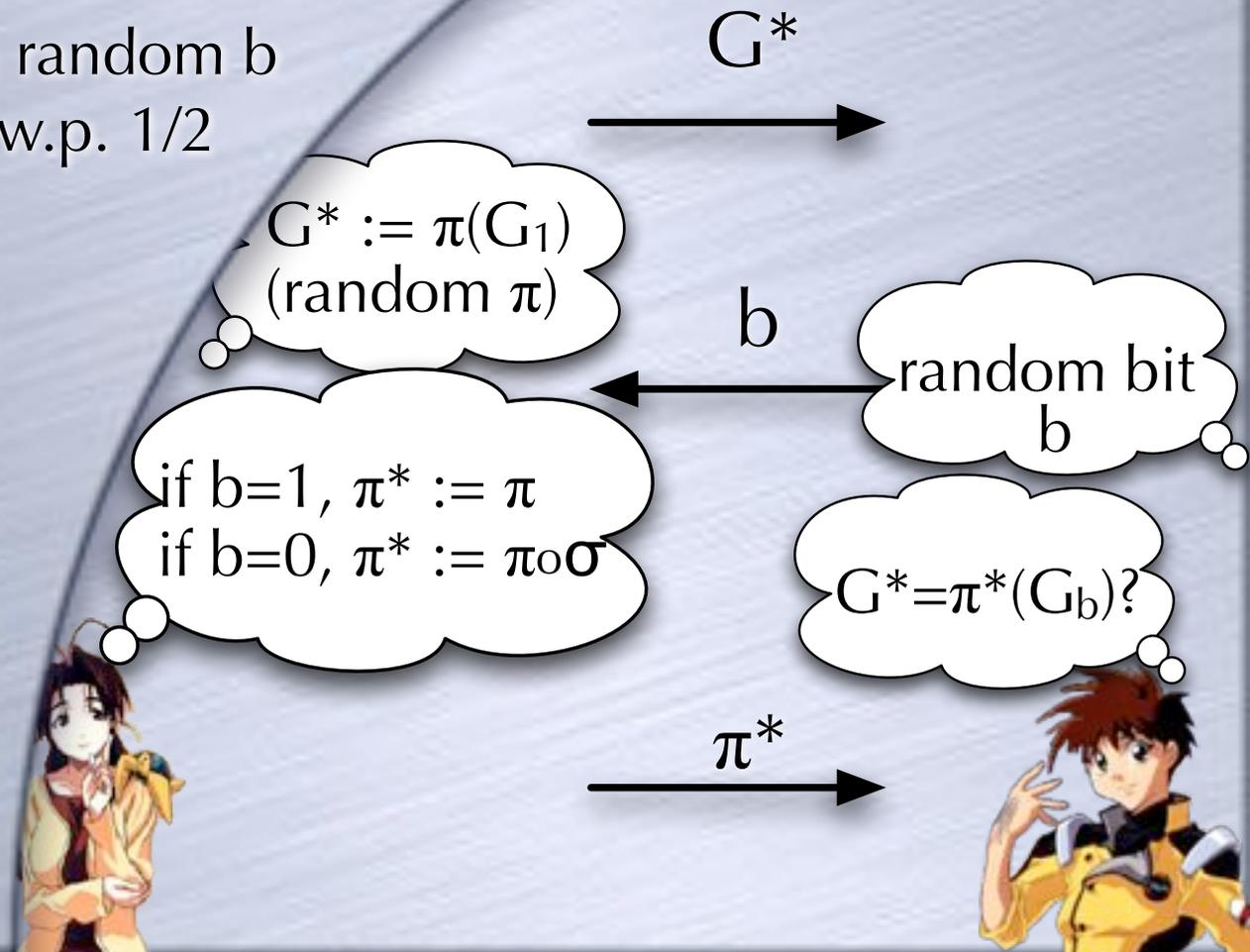
# An Example

- Why is this convincing?
- If prover can answer both  $b$ 's for the same  $G^*$  then  $G_0 \sim G_1$



# An Example

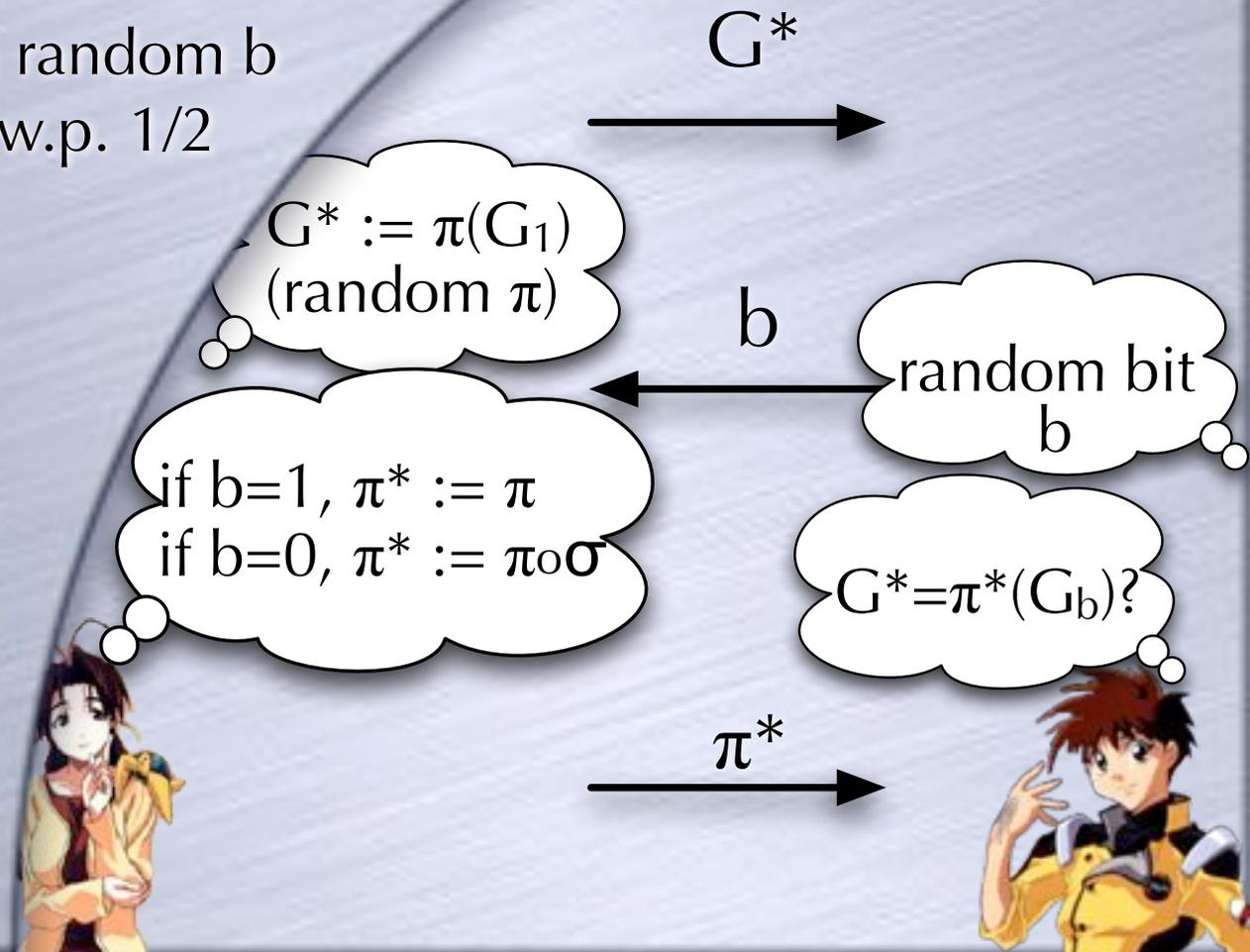
- Why is this convincing?
  - If prover can answer both  $b$ 's for the same  $G^*$  then  $G_0 \sim G_1$
  - Otherwise, testing on a random  $b$  will leave prover stuck w.p.  $1/2$



# An Example

- Why is this convincing?
  - If prover can answer both  $b$ 's for the same  $G^*$  then  $G_0 \sim G_1$
  - Otherwise, testing on a random  $b$  will leave prover stuck w.p.  $1/2$

- Why ZK?

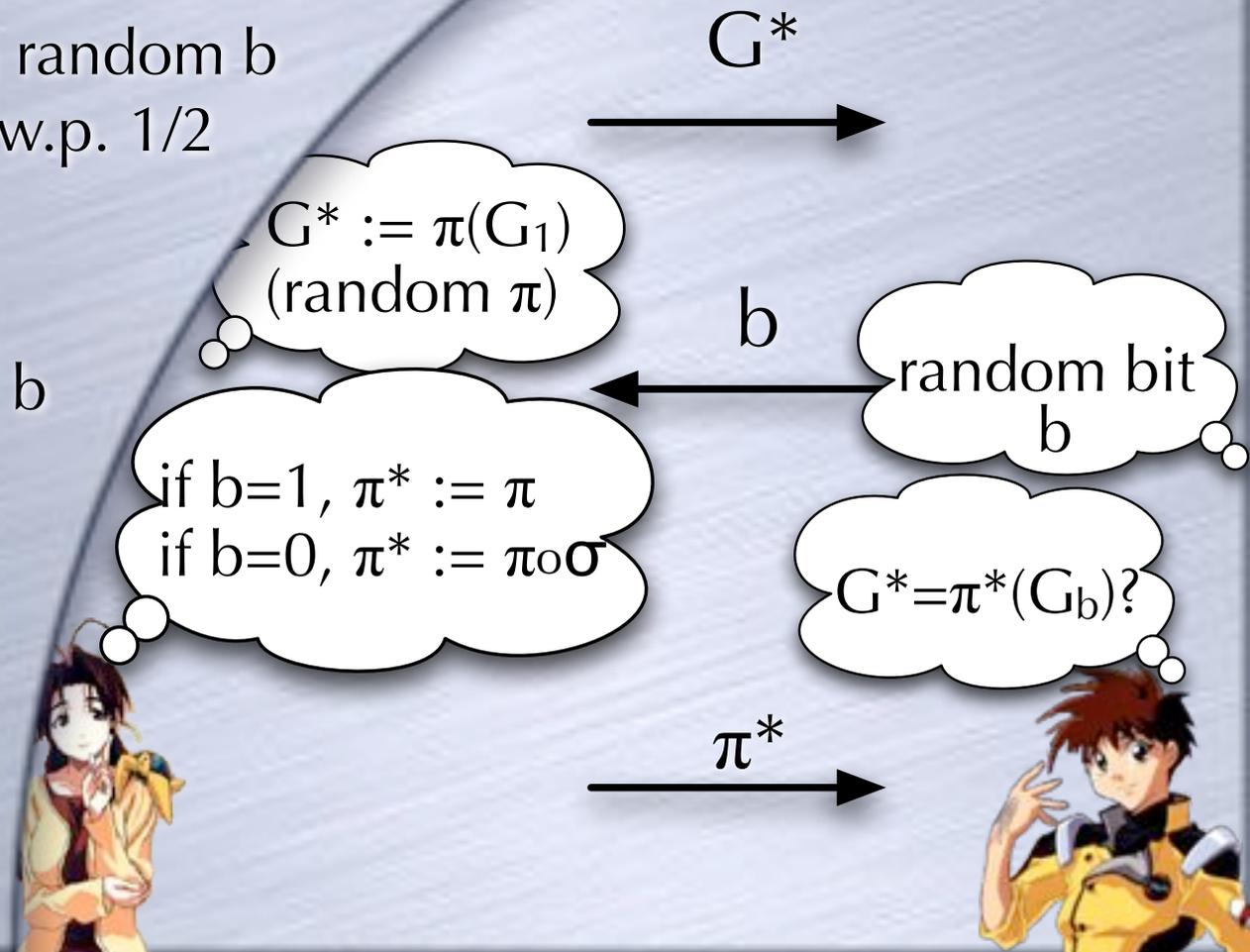


# An Example

- **Why is this convincing?**
  - If prover can answer both  $b$ 's for the same  $G^*$  then  $G_0 \sim G_1$
  - Otherwise, testing on a random  $b$  will leave prover stuck w.p.  $1/2$

- **Why ZK?**

- Verifier's view: random  $b$  and  $\pi^*$  s.t.  $G^* = \pi^*(G_b)$

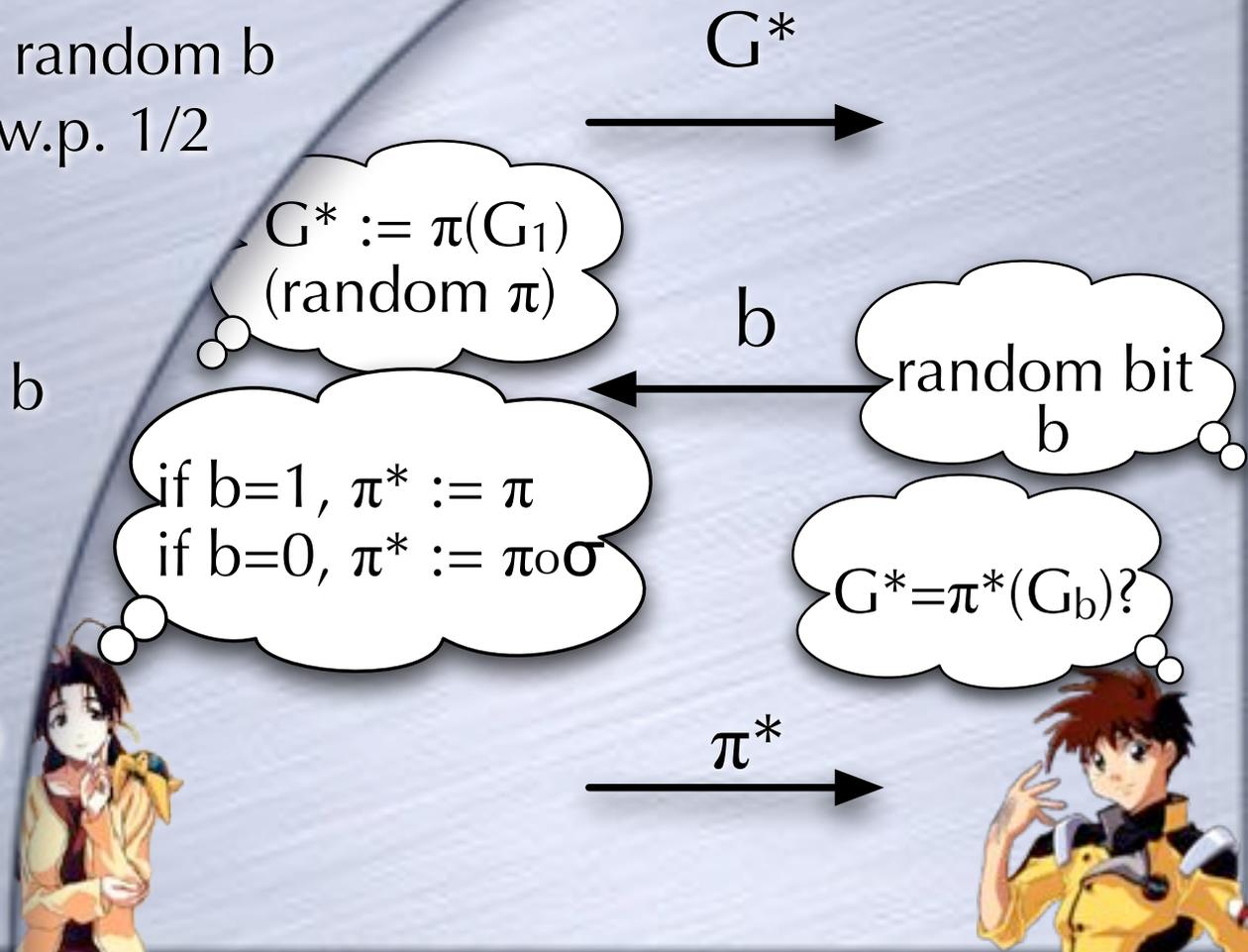


# An Example

- Why is this convincing?
  - If prover can answer both  $b$ 's for the same  $G^*$  then  $G_0 \sim G_1$
  - Otherwise, testing on a random  $b$  will leave prover stuck w.p.  $1/2$

- Why ZK?

- Verifier's view: random  $b$  and  $\pi^*$  s.t.  $G^* = \pi^*(G_b)$
- Which he could have generated by himself (whether  $G_0 \sim G_1$  or not)



# Zero-Knowledge Proofs



# Zero-Knowledge Proofs

- Interactive Proof



# Zero-Knowledge Proofs

- Interactive Proof
  - Complete and Sound



# Zero-Knowledge Proofs

- Interactive Proof
  - Complete and Sound
- ZK Property:



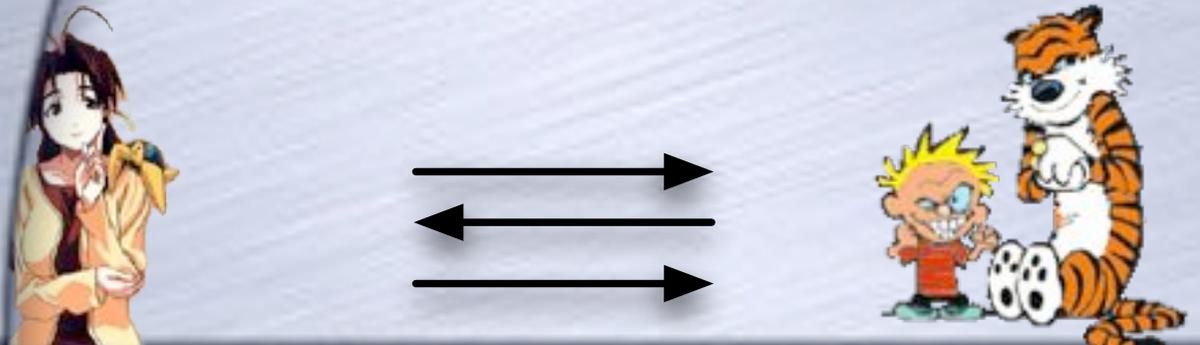
# Zero-Knowledge Proofs

- Interactive Proof
  - Complete and Sound
- ZK Property:



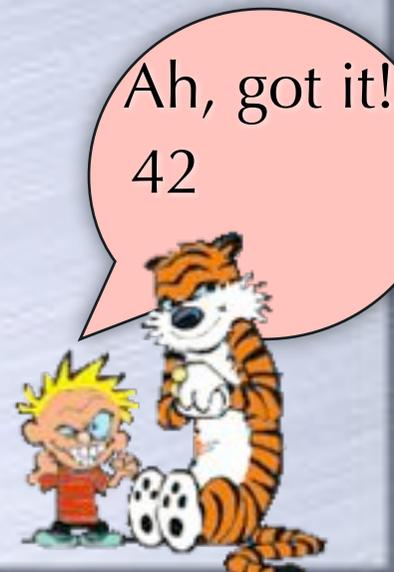
# Zero-Knowledge Proofs

- Interactive Proof
  - Complete and Sound
- ZK Property:



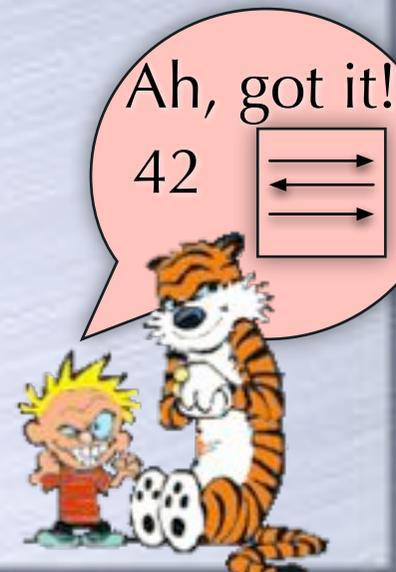
# Zero-Knowledge Proofs

- Interactive Proof
  - Complete and Sound
- ZK Property:



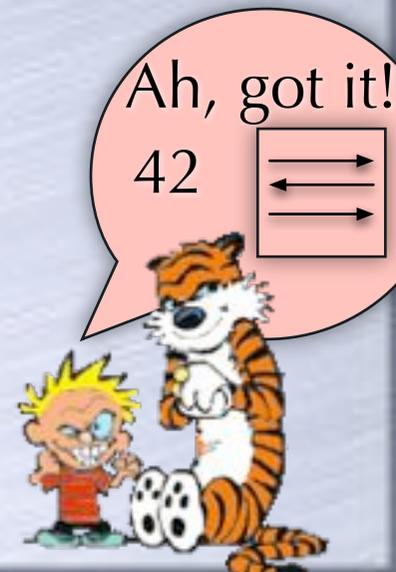
# Zero-Knowledge Proofs

- Interactive Proof
  - Complete and Sound
- ZK Property:



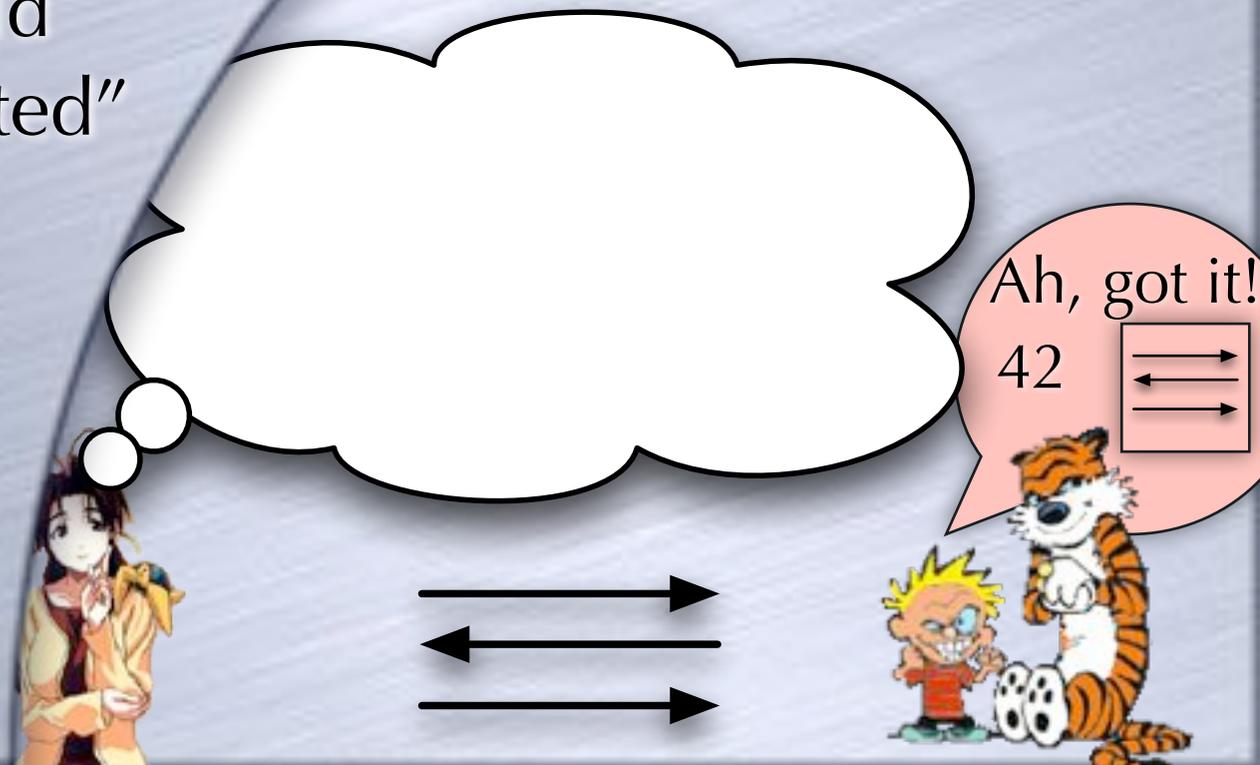
# Zero-Knowledge Proofs

- Interactive Proof
  - Complete and Sound
- ZK Property:
  - Verifier's view could have been "simulated"



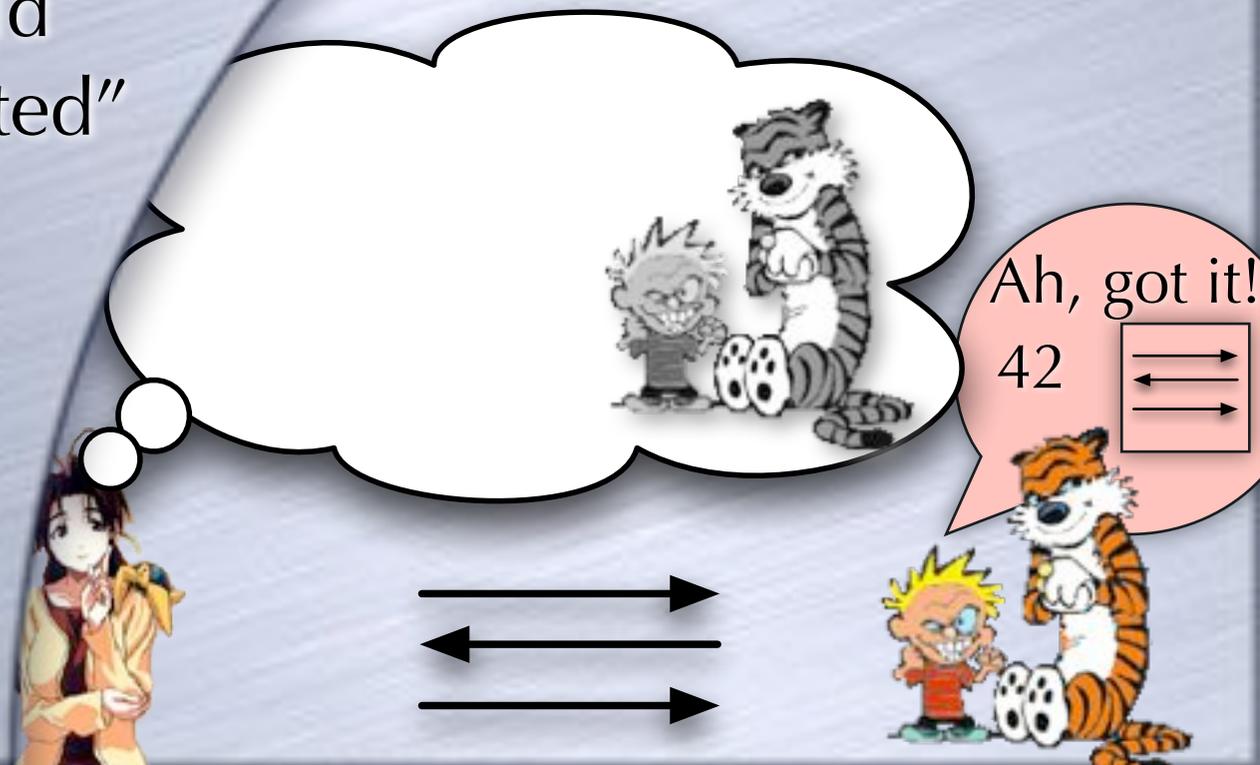
# Zero-Knowledge Proofs

- Interactive Proof
  - Complete and Sound
- ZK Property:
  - Verifier's view could have been "simulated"



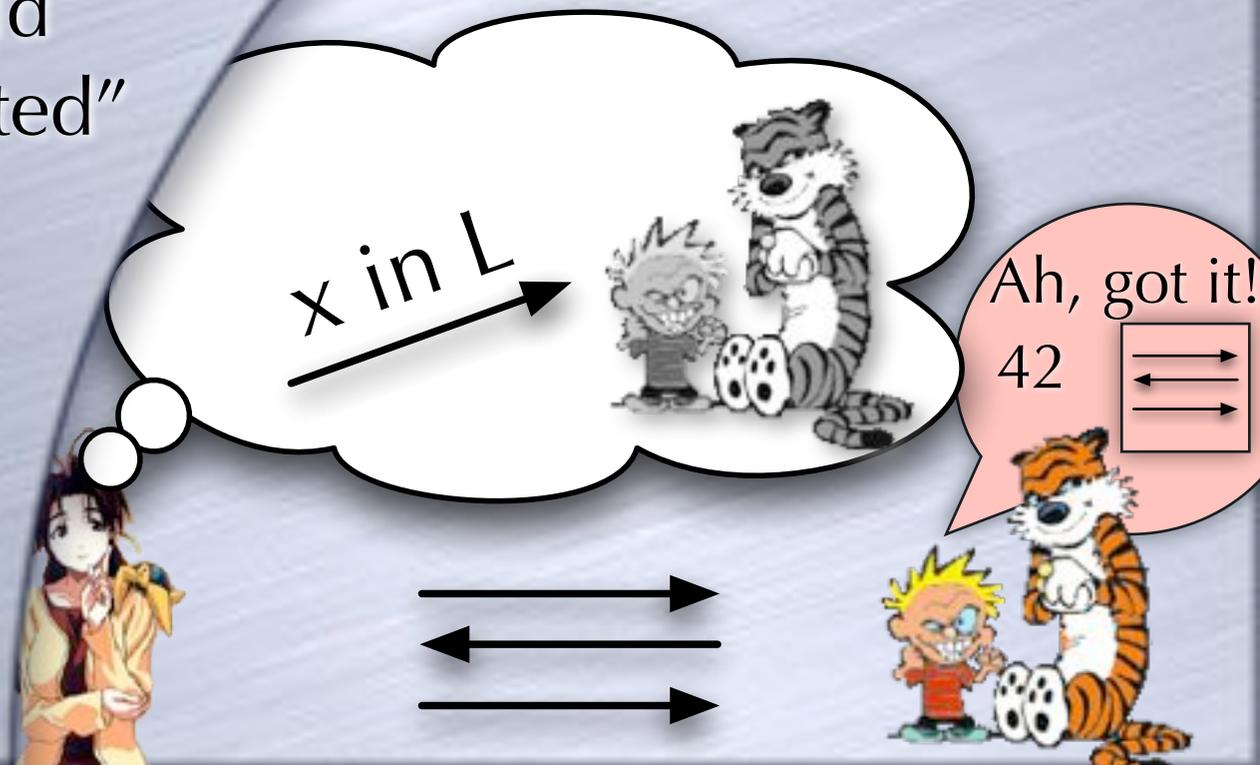
# Zero-Knowledge Proofs

- Interactive Proof
  - Complete and Sound
- ZK Property:
  - Verifier's view could have been "simulated"



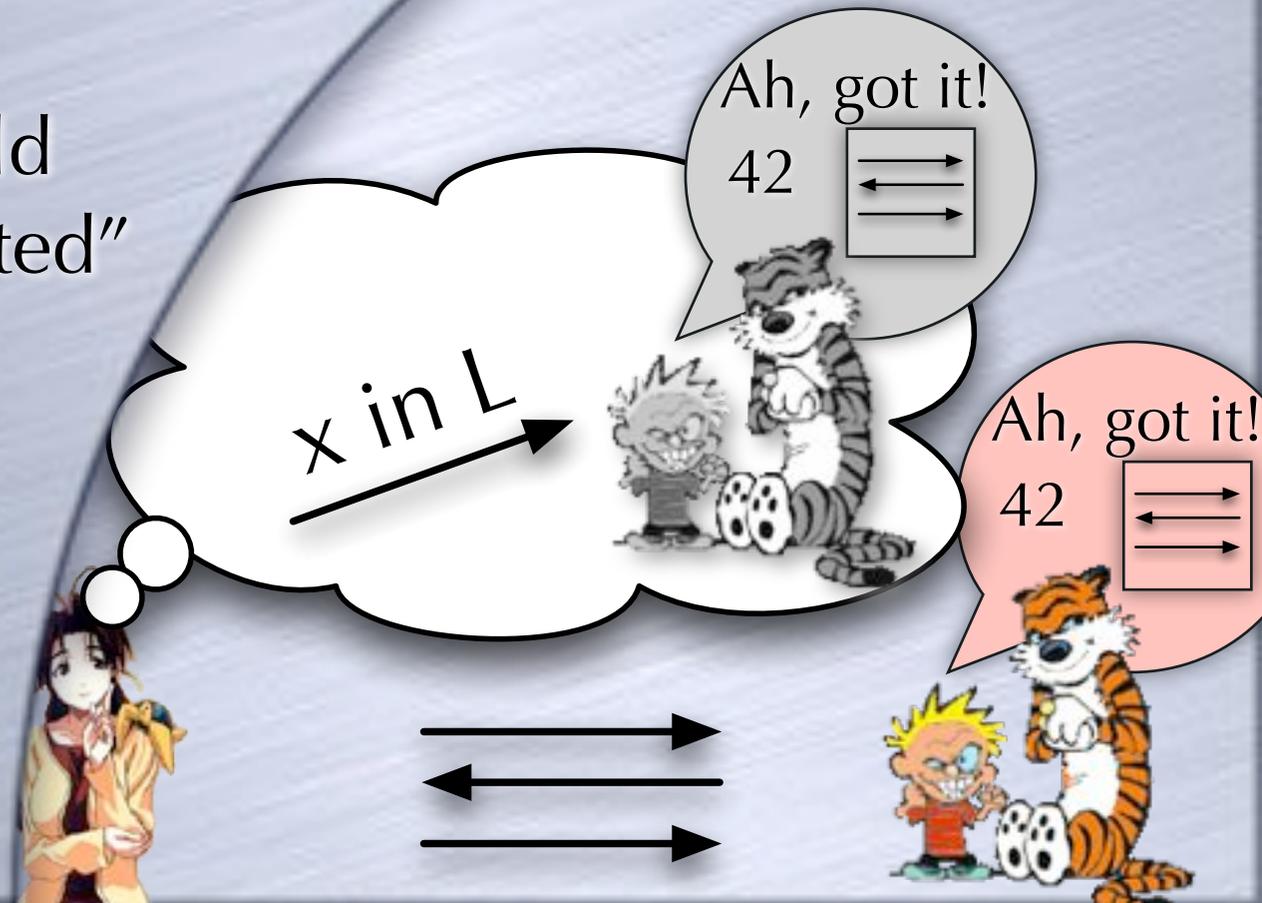
# Zero-Knowledge Proofs

- Interactive Proof
  - Complete and Sound
- ZK Property:
  - Verifier's view could have been "simulated"



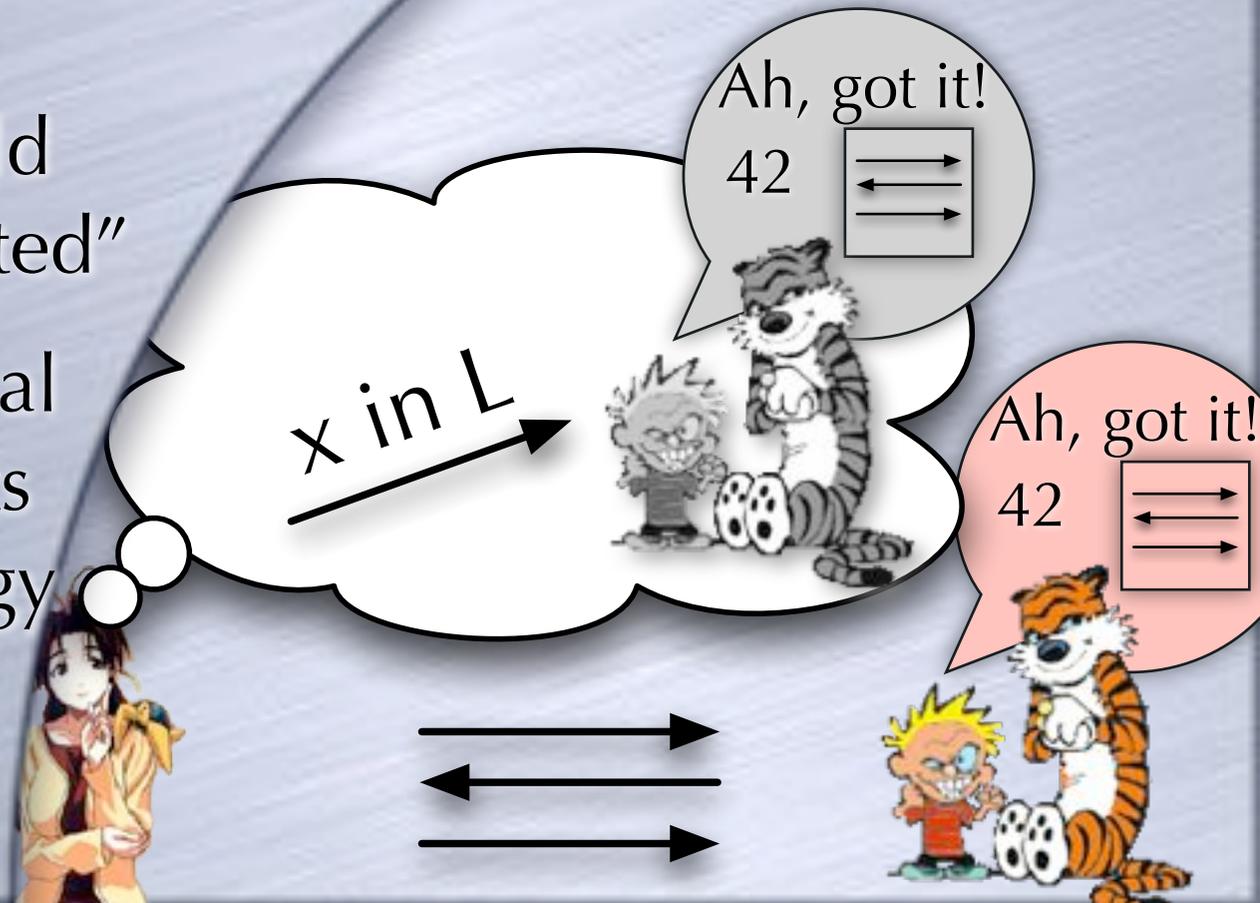
# Zero-Knowledge Proofs

- Interactive Proof
  - Complete and Sound
- ZK Property:
  - Verifier's view could have been "simulated"

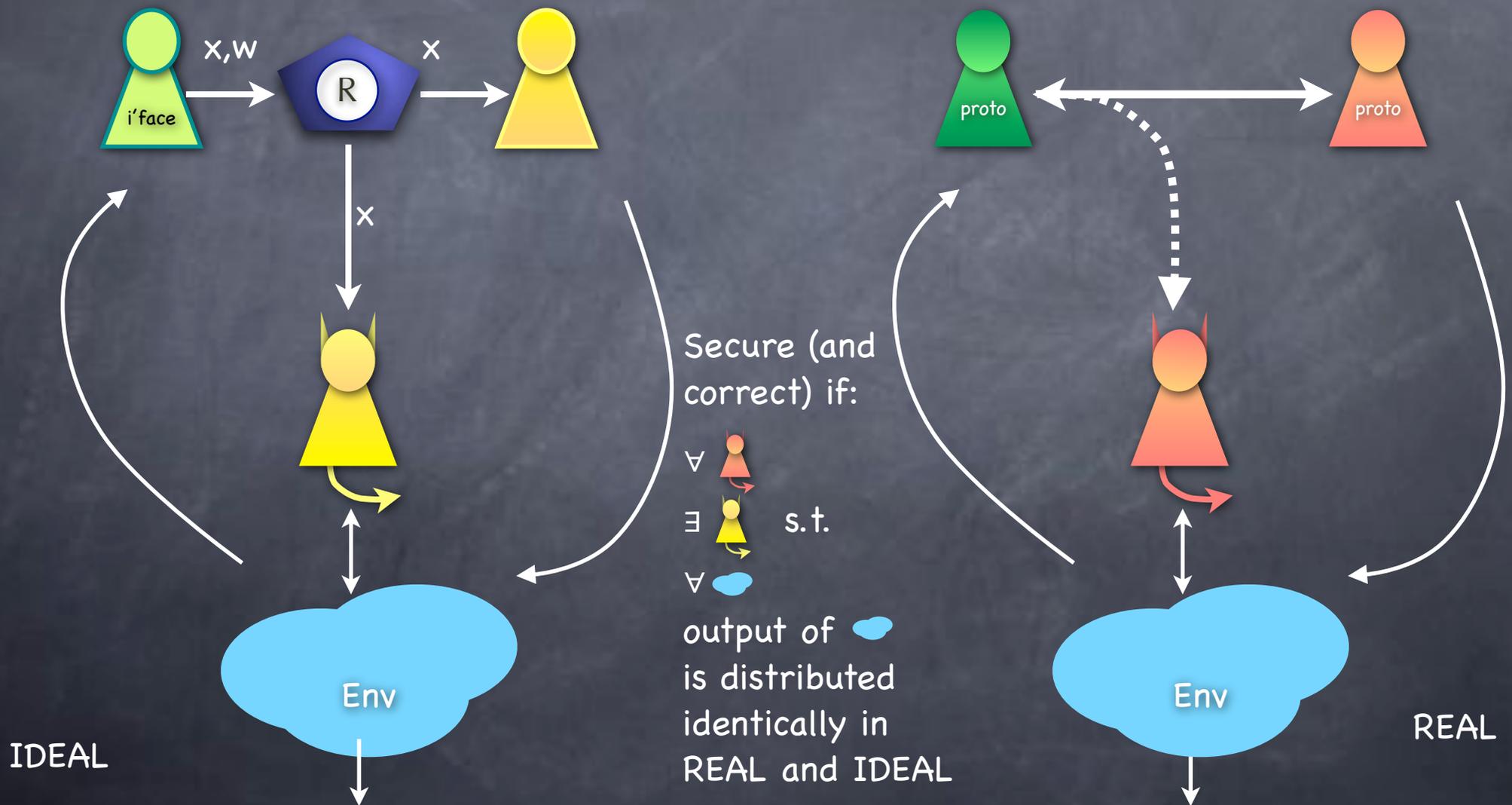


# Zero-Knowledge Proofs

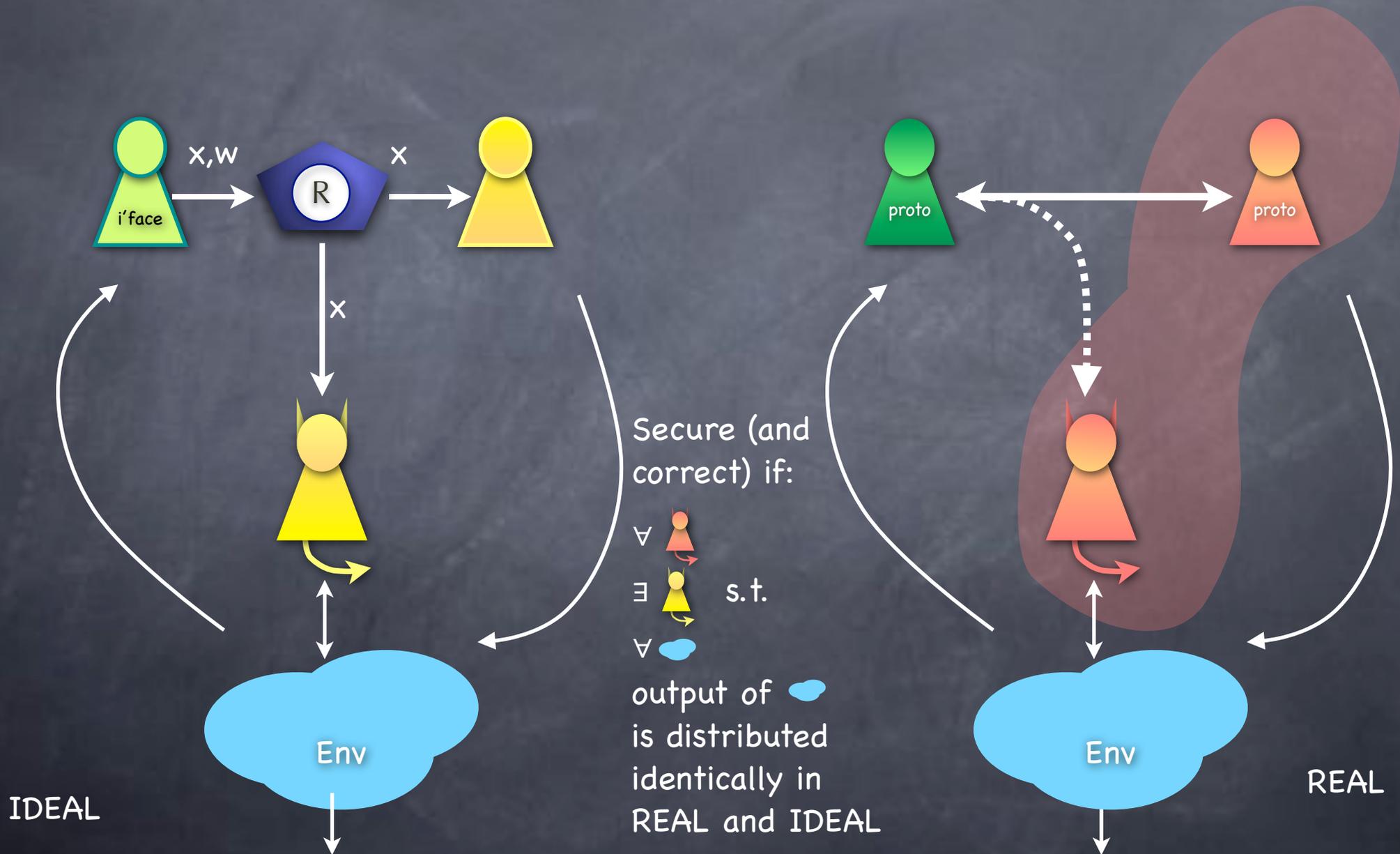
- Interactive Proof
  - Complete and Sound
- ZK Property:
  - Verifier's view could have been "simulated"
  - For every adversarial strategy, there exists a simulation strategy



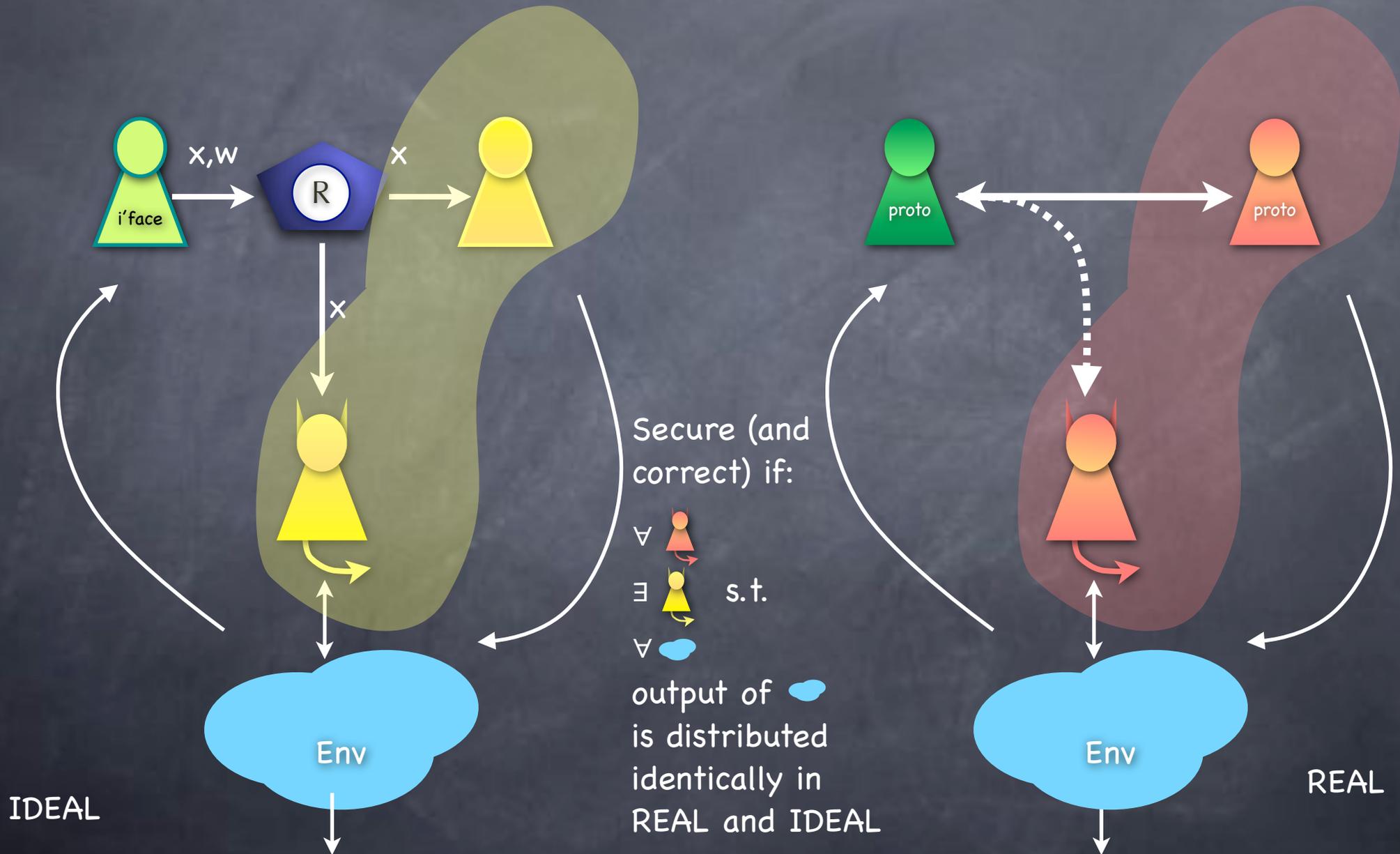
# ZK Property (in other pict's)



# ZK Property (in other pict's)

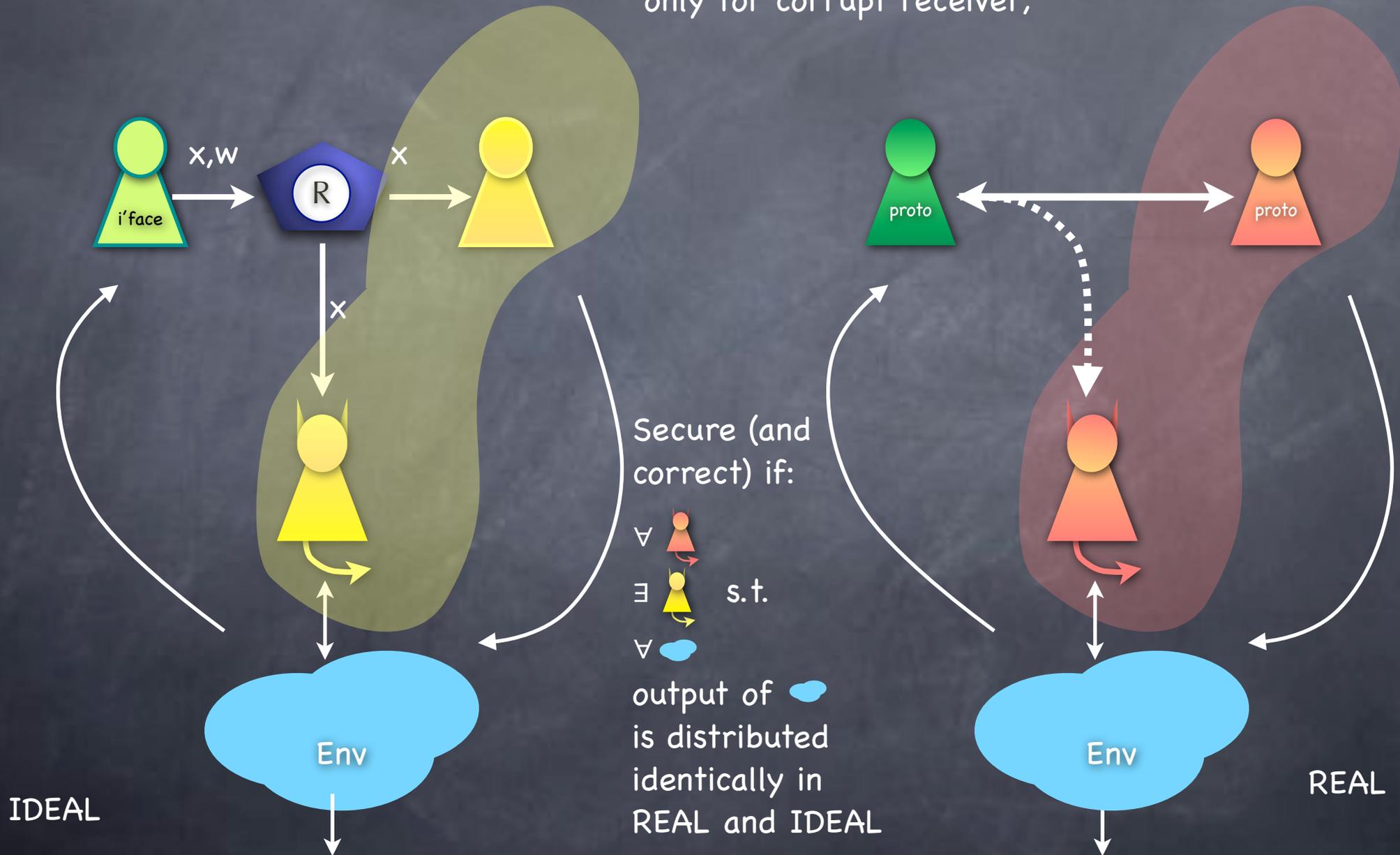


# ZK Property (in other pict's)



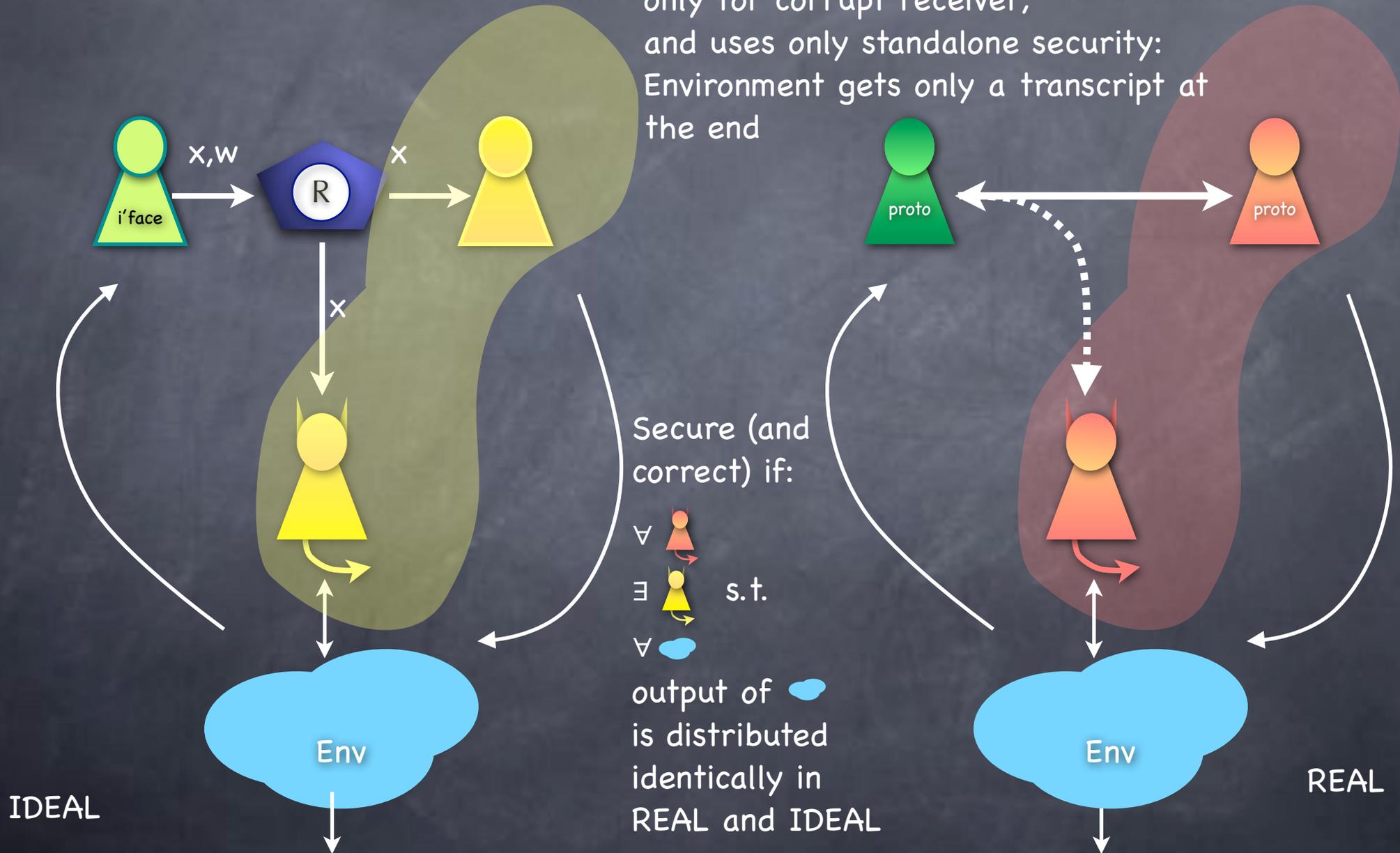
# ZK Property (in other pict's)

Classical definition uses simulation only for corrupt receiver;

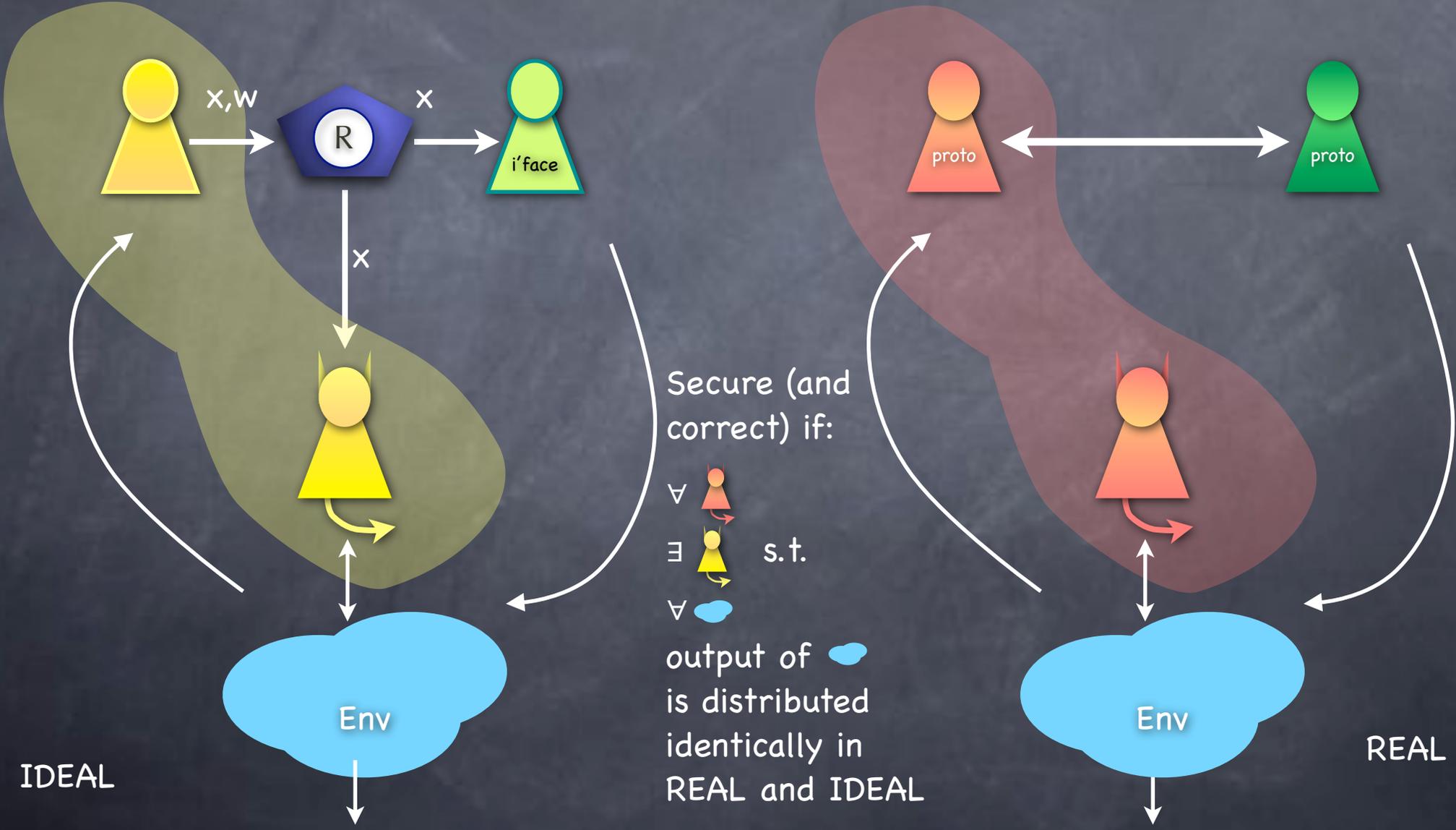


# ZK Property (in other pict's)

Classical definition uses simulation only for corrupt receiver;  
and uses only standalone security:  
Environment gets only a transcript at the end

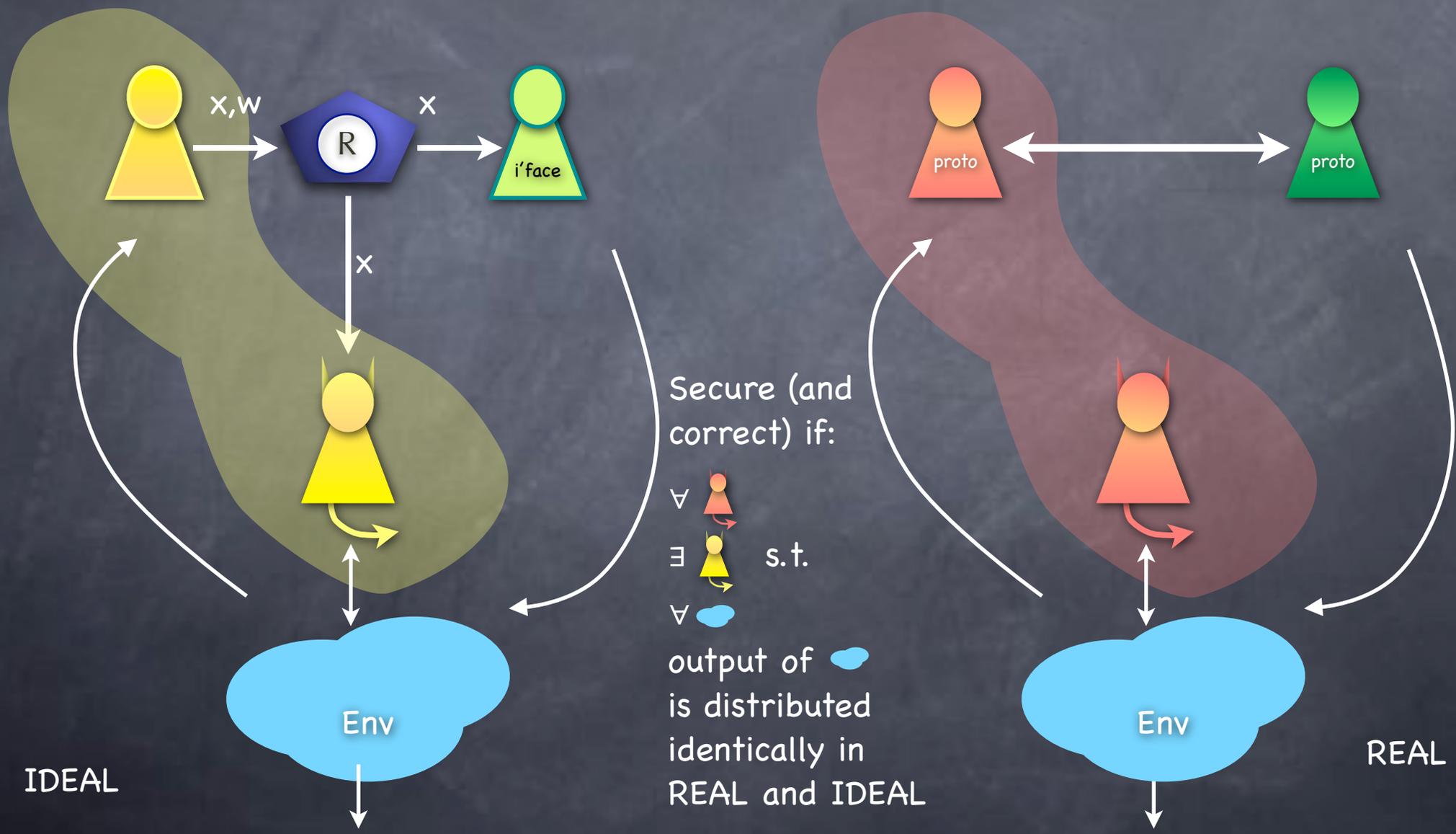


# SIM ZK



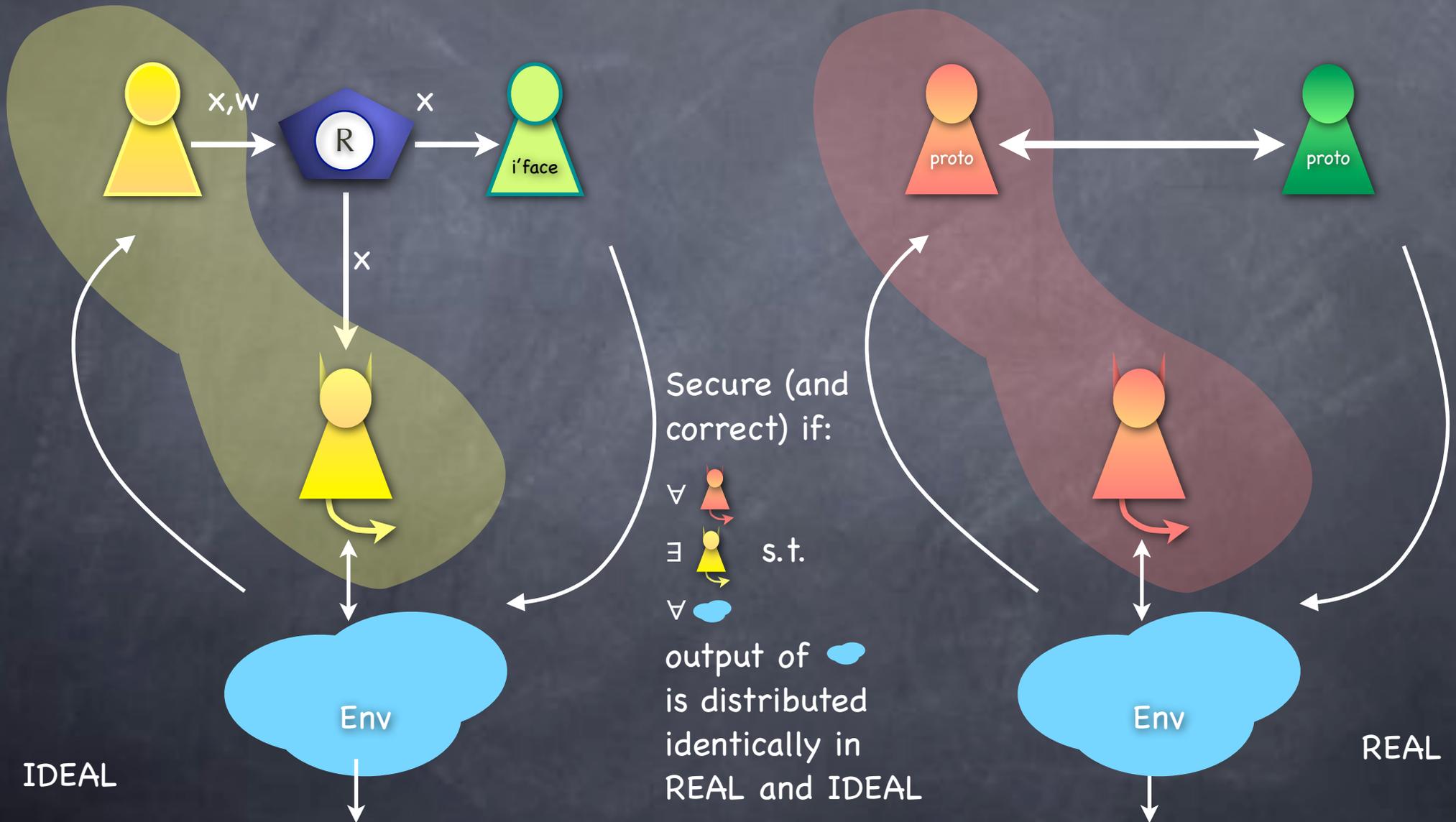
# SIM ZK

- SIM-ZK would require simulation also when prover is corrupt



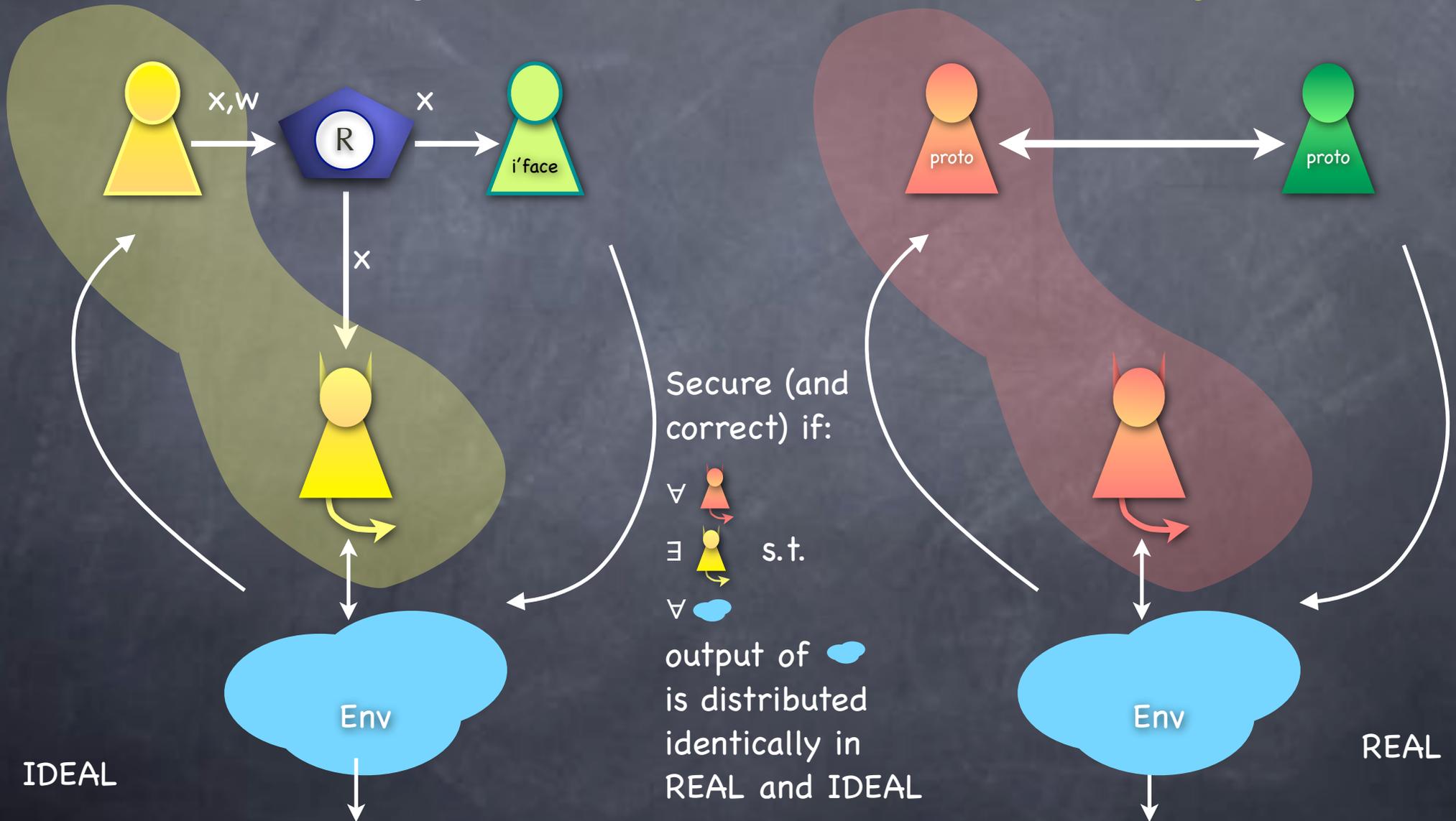
# SIM ZK

- SIM-ZK would require simulation also when prover is corrupt
- Then simulator is a witness extractor



# SIM ZK

- SIM-ZK would require simulation also when prover is corrupt
  - Then simulator is a witness extractor
- Adding this (in standalone) makes it a **Proof of Knowledge**



# Results

# Results

- IP and ZK defined [GMR'85]

# Results

- IP and ZK defined [GMR'85]
- ZK for all NP languages [GMW'86]

# Results

- IP and ZK defined [GMR'85]
- ZK for all NP languages [GMW'86]
  - Assuming one-way functions exist

# Results

- IP and ZK defined [GMR'85]
- ZK for all NP languages [GMW'86]
  - Assuming one-way functions exist
- ZK for all of IP [BGGHKMR'88]

# Results

- IP and ZK defined [GMR'85]
- ZK for all NP languages [GMW'86]
  - Assuming one-way functions exist
- ZK for all of IP [BGGHKMR'88]
  - Everything that can be proven can be proven in zero-knowledge! (Assuming OWF)

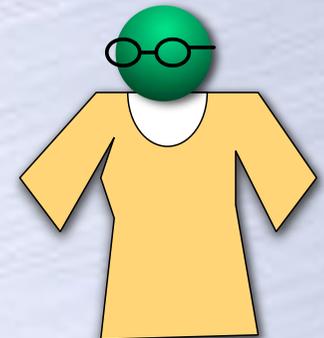
# Results

- IP and ZK defined [GMR'85]
- ZK for all NP languages [GMW'86]
  - Assuming one-way functions exist
- ZK for all of IP [BGGHKMR'88]
  - Everything that can be proven can be proven in zero-knowledge! (Assuming OWF)
- Variants (for NP)

# Results

- IP and ZK defined [GMR'85]
- ZK for all NP languages [GMW'86]
  - Assuming one-way functions exist
- ZK for all of IP [BGGHKMR'88]
  - Everything that can be proven can be proven in zero-knowledge! (Assuming OWF)
- Variants (for NP)
  - ZKPoK, Statistical ZK Arguments,  $O(1)$ -round ZK, ...

# A ZK Proof for Graph Colorability

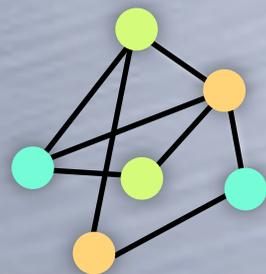


# A ZK Proof for Graph Colorability

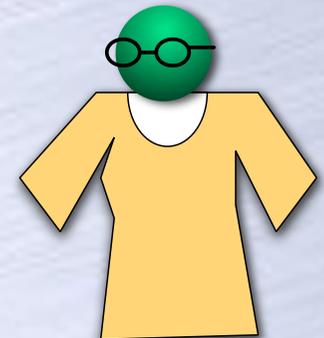


# A ZK Proof for Graph Colorability

- Uses a commitment protocol as a subroutine

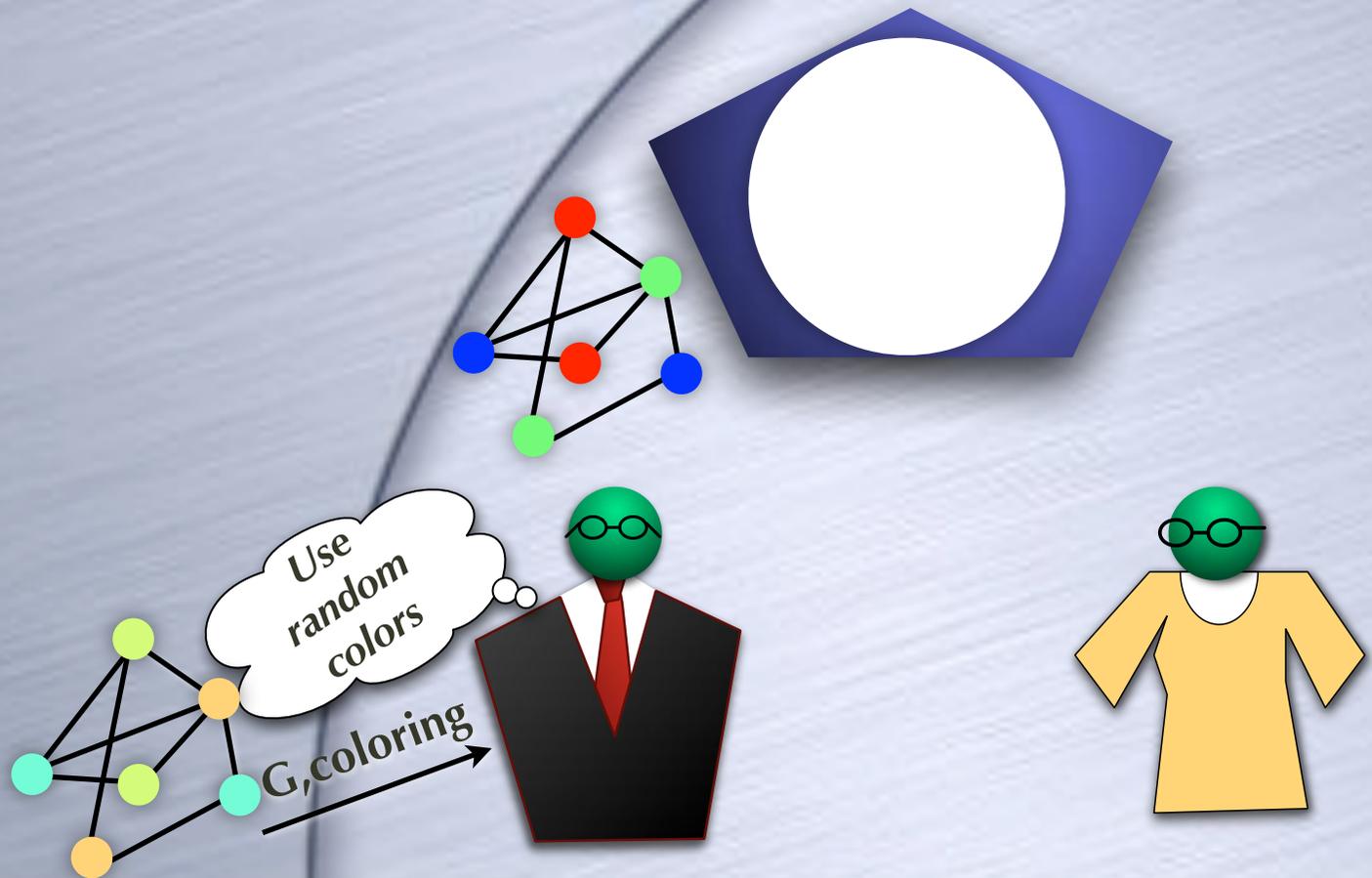


$G, \text{coloring}$



# A ZK Proof for Graph Colorability

- Uses a commitment protocol as a subroutine



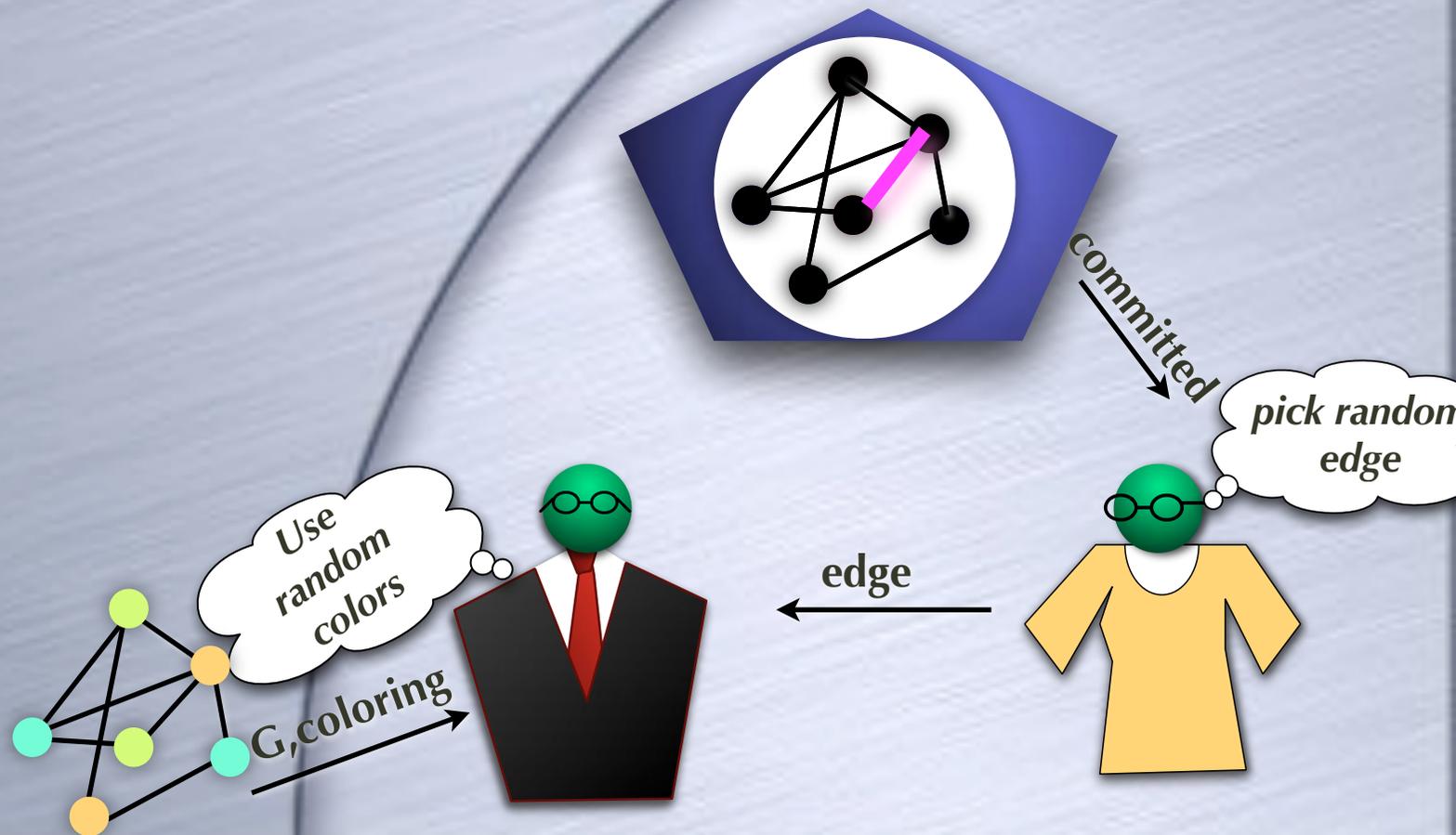
# A ZK Proof for Graph Colorability

- Uses a commitment protocol as a subroutine



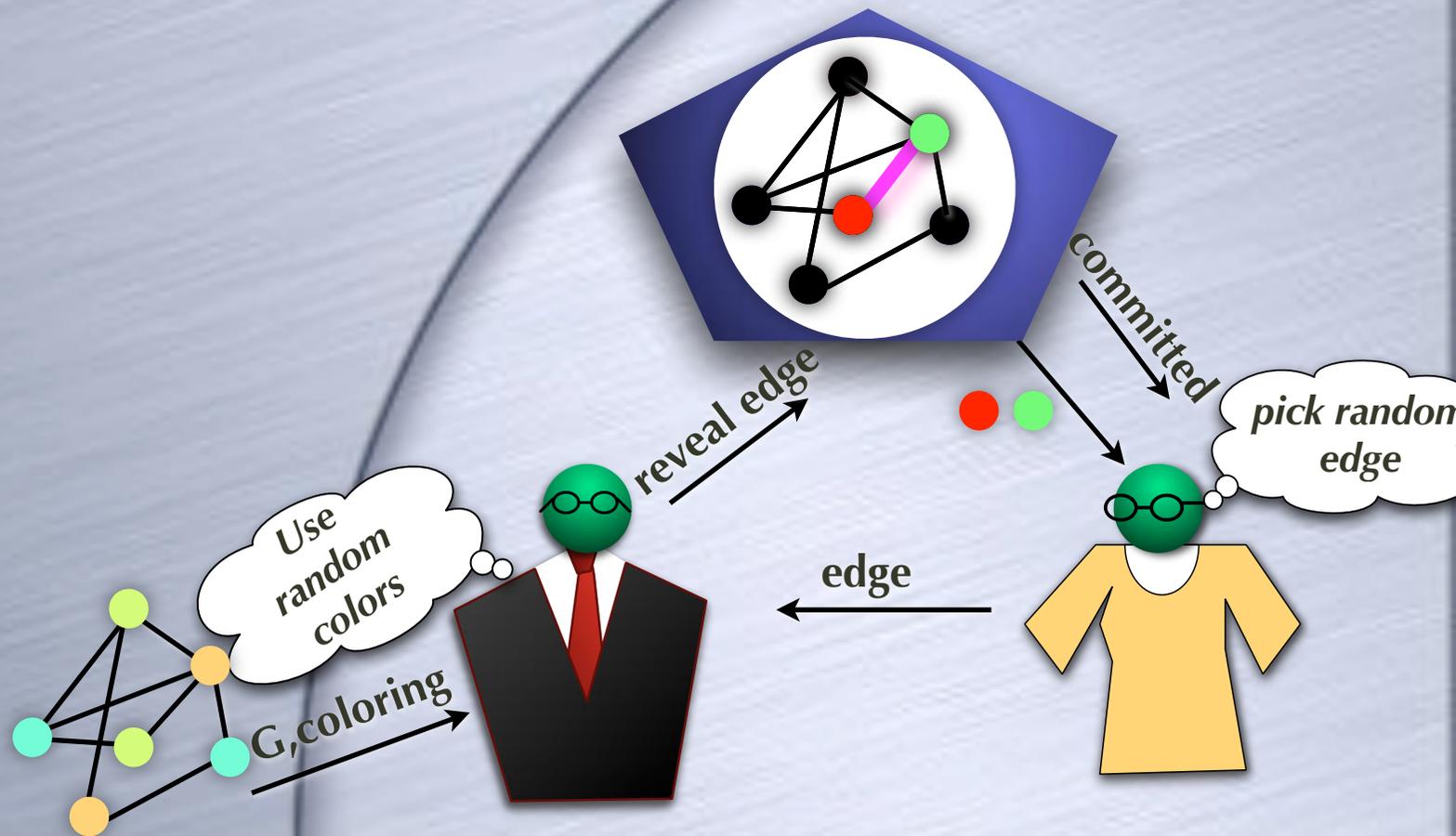
# A ZK Proof for Graph Colorability

- Uses a commitment protocol as a subroutine



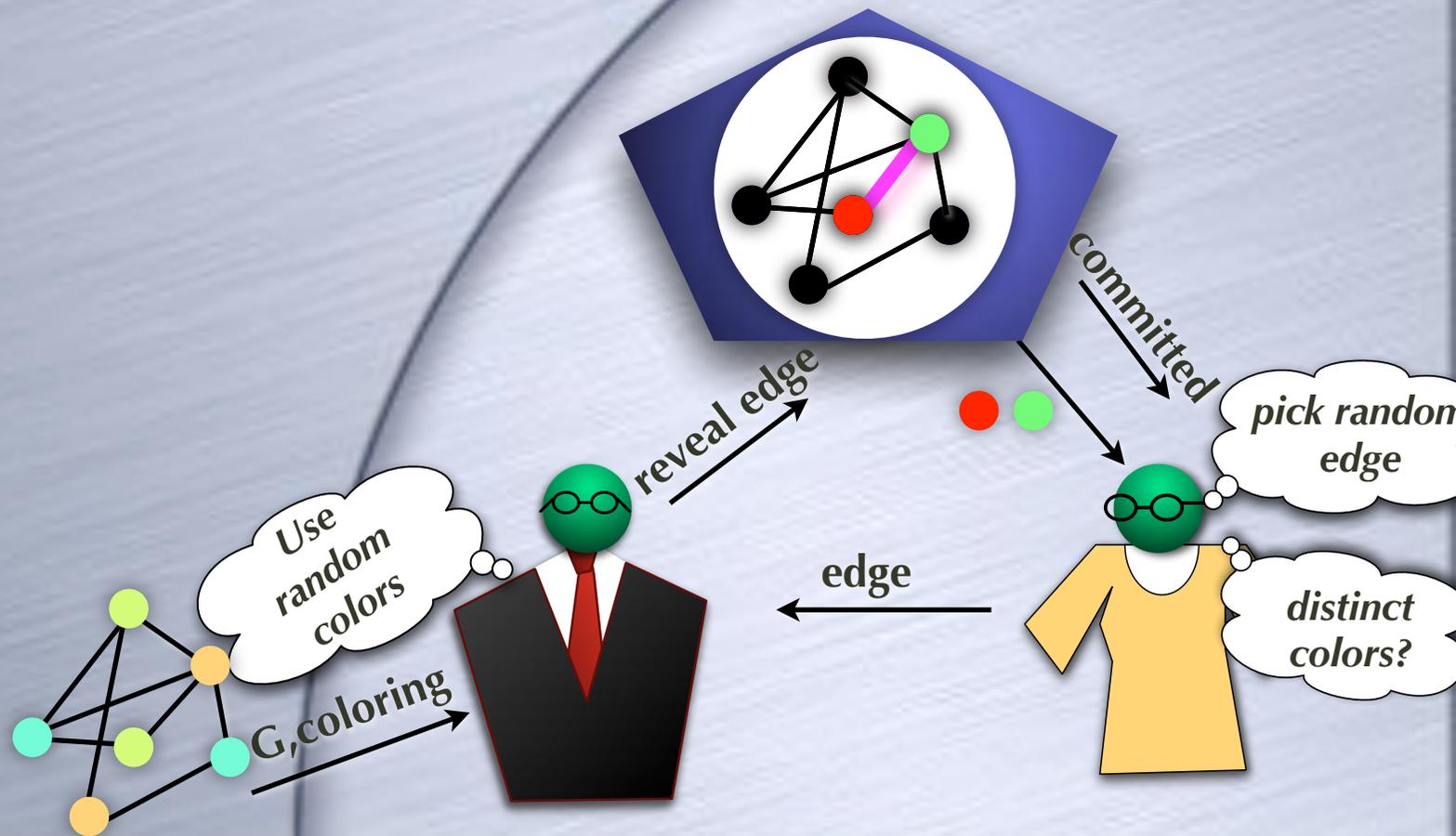
# A ZK Proof for Graph Colorability

- Uses a commitment protocol as a subroutine



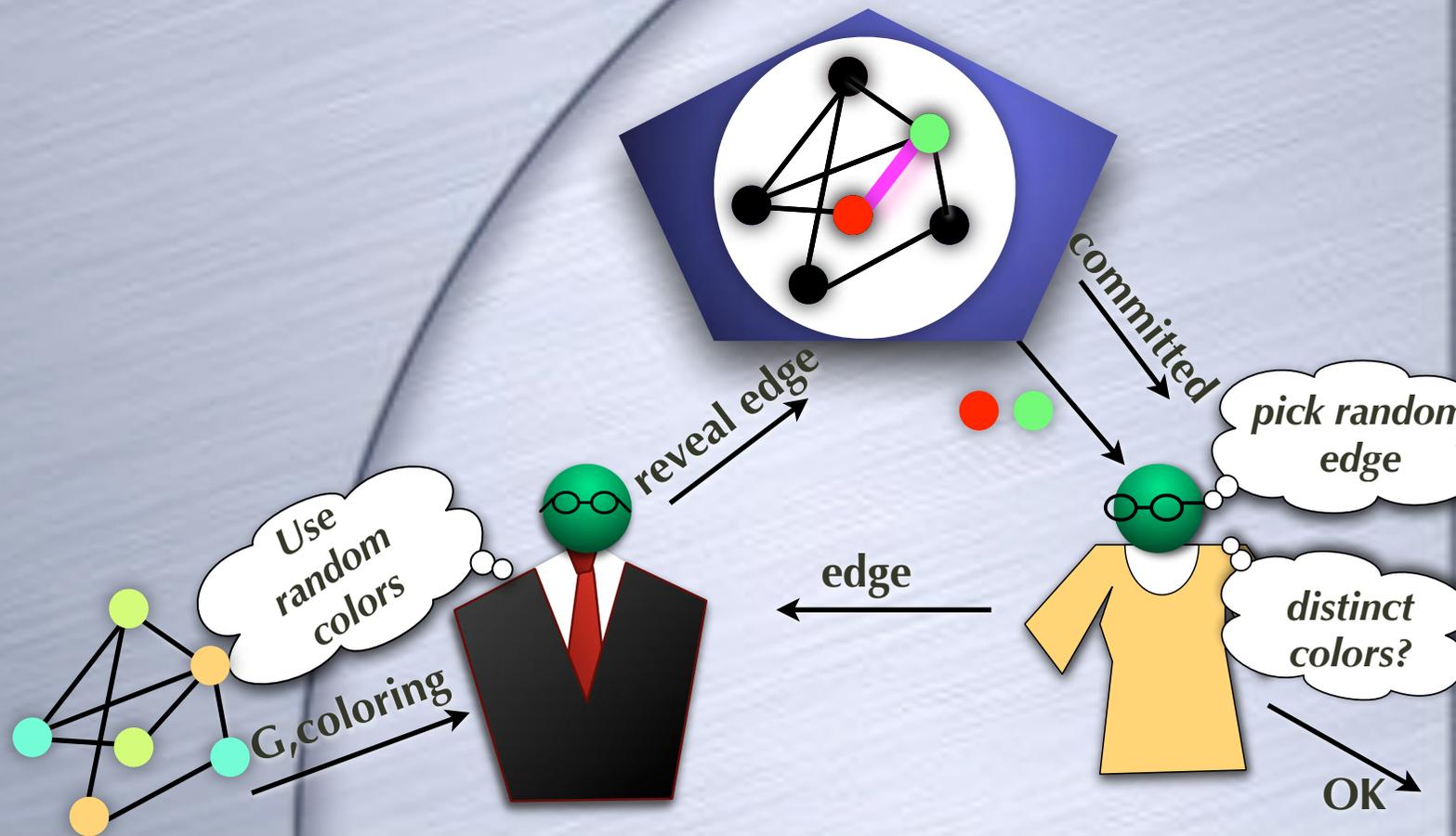
# A ZK Proof for Graph Colorability

- Uses a commitment protocol as a subroutine



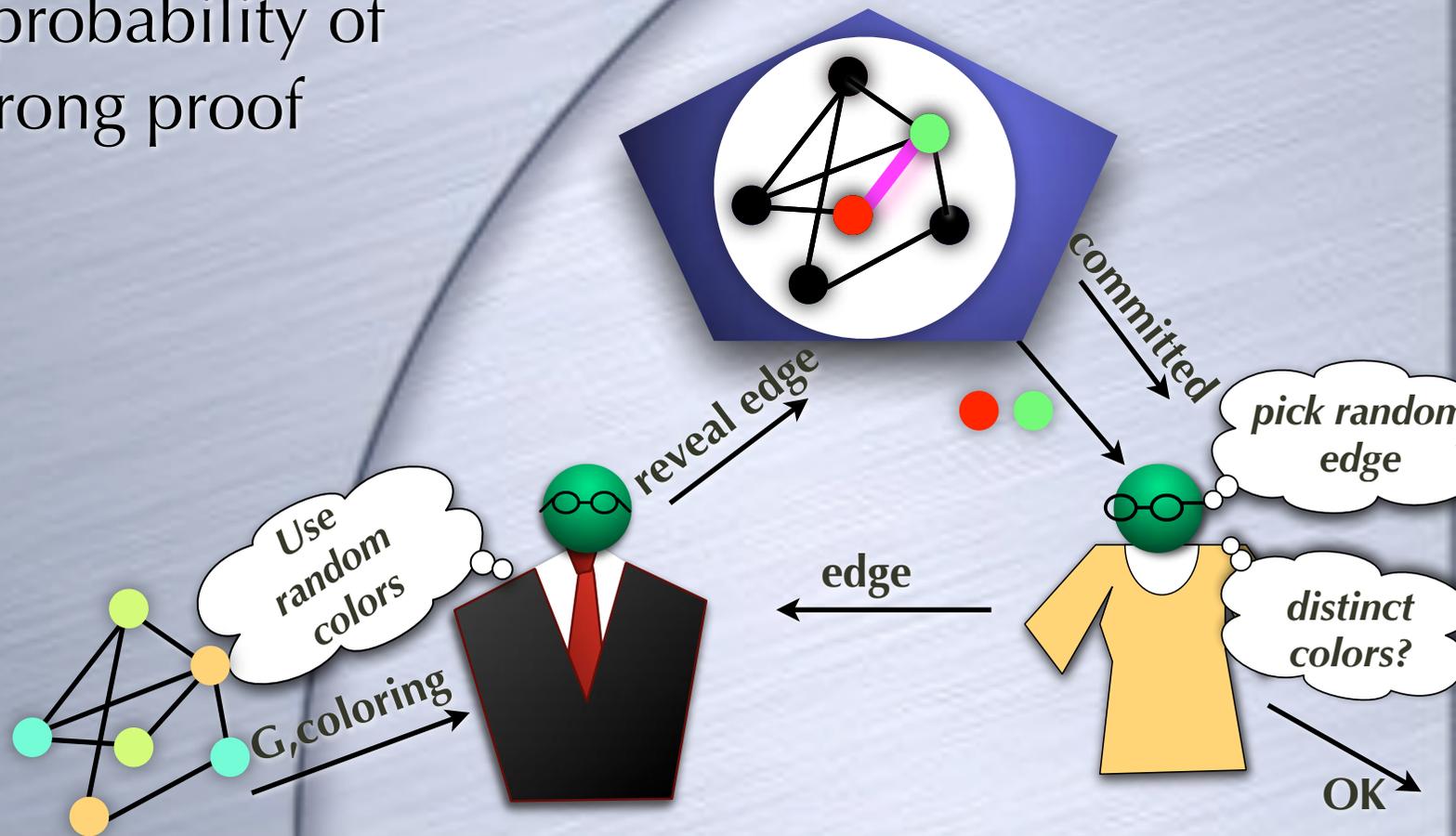
# A ZK Proof for Graph Colorability

- Uses a commitment protocol as a subroutine



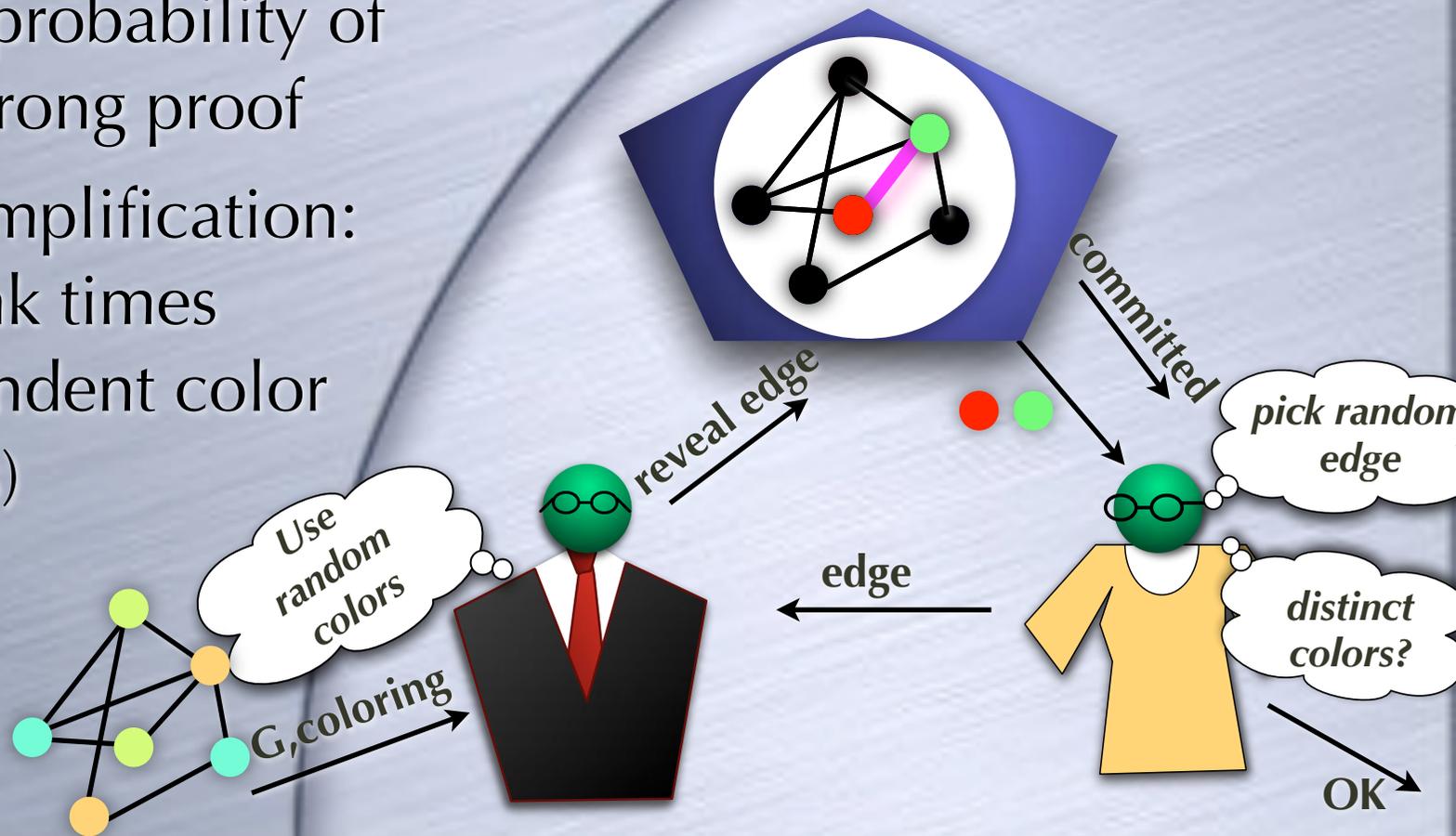
# A ZK Proof for Graph Colorability

- Uses a commitment protocol as a subroutine
- At least  $1/m$  probability of catching a wrong proof

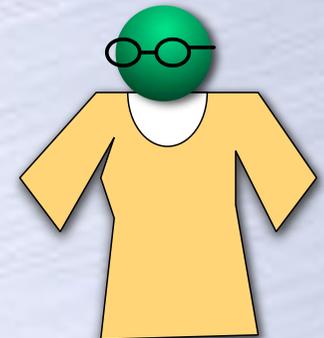


# A ZK Proof for Graph Colorability

- Uses a commitment protocol as a subroutine
- At least  $1/m$  probability of catching a wrong proof
- Soundness amplification: Repeat say  $mk$  times (with independent color permutations)

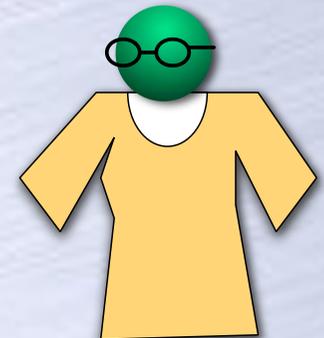


# A Commitment Protocol



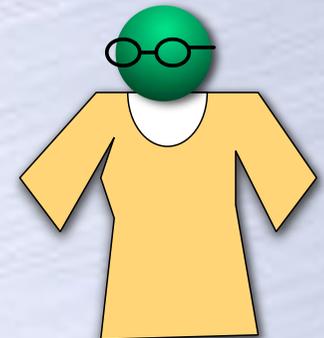
# A Commitment Protocol

- Uses a OWP  $f$  and a hardcore predicate for it  $B$



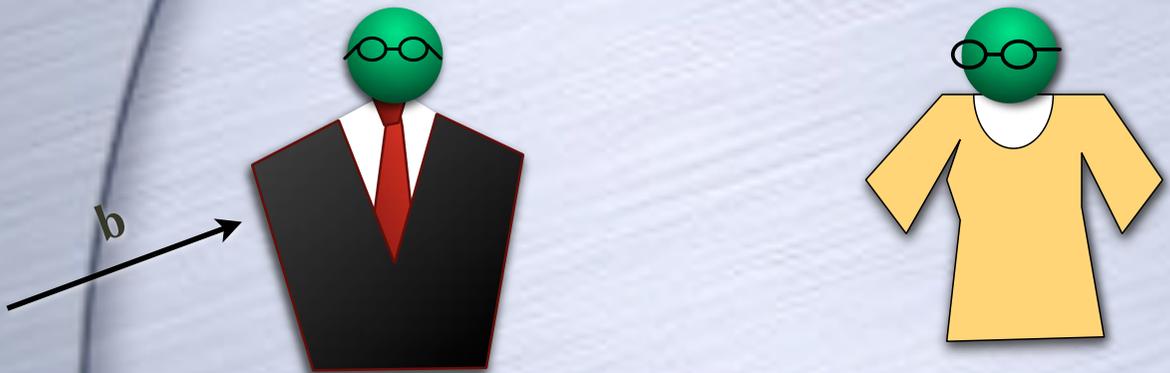
# A Commitment Protocol

- Uses a OWP  $f$  and a hardcore predicate for it  $B$
- Satisfies only classical (IND) security, in terms of hiding and binding



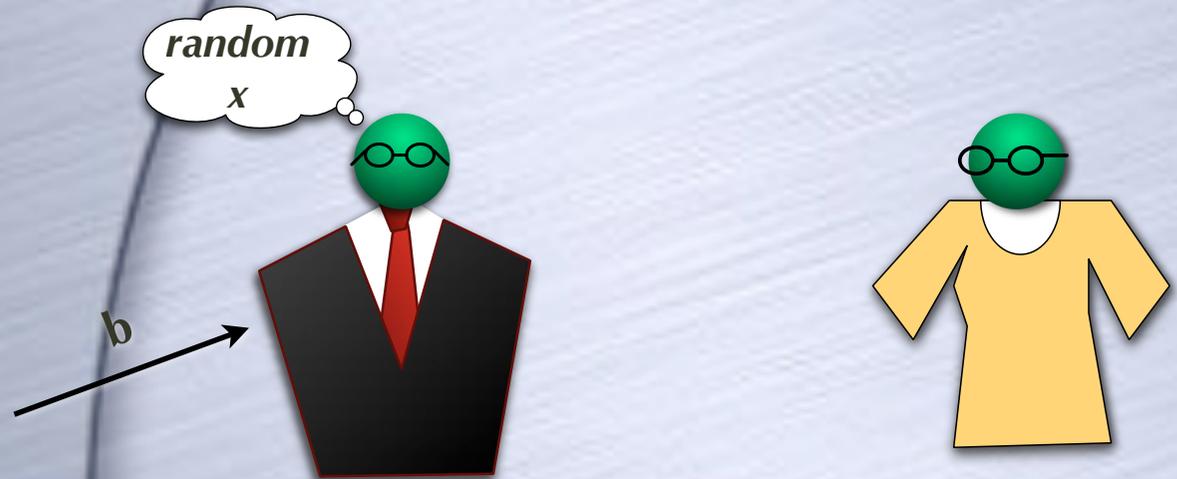
# A Commitment Protocol

- Uses a OWP  $f$  and a hardcore predicate for it  $B$
- Satisfies only classical (IND) security, in terms of hiding and binding



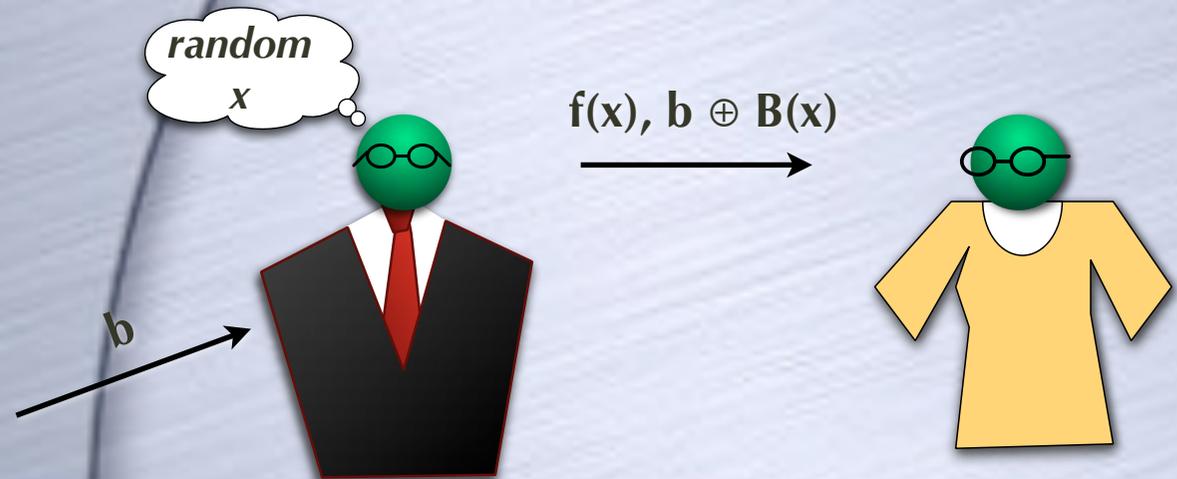
# A Commitment Protocol

- Uses a OWP  $f$  and a hardcore predicate for it  $B$
- Satisfies only classical (IND) security, in terms of hiding and binding



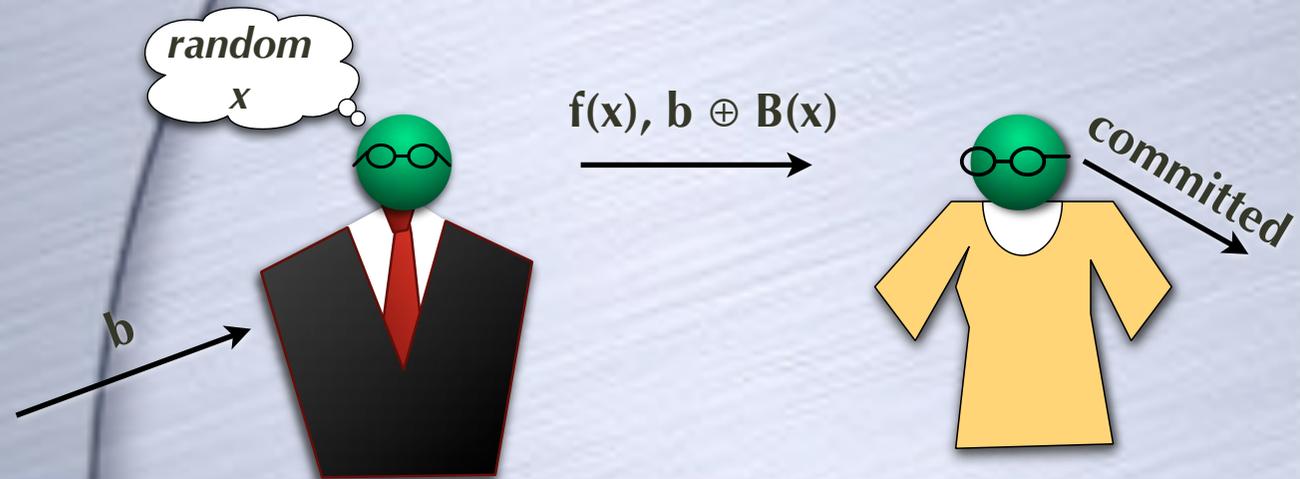
# A Commitment Protocol

- Uses a OWP  $f$  and a hardcore predicate for it  $B$
- Satisfies only classical (IND) security, in terms of hiding and binding



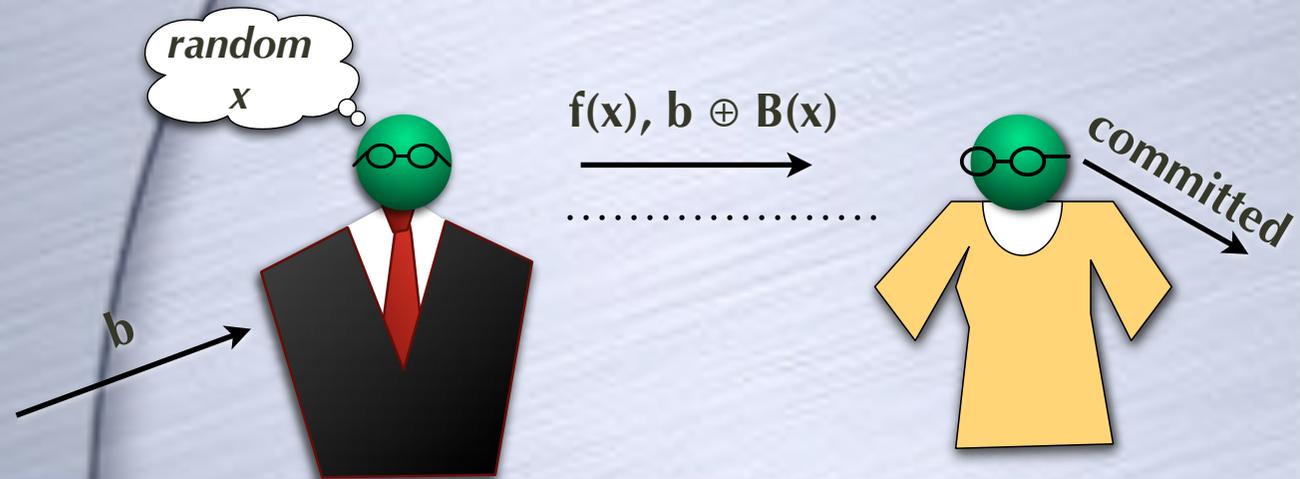
# A Commitment Protocol

- Uses a OWP  $f$  and a hardcore predicate for it  $B$
- Satisfies only classical (IND) security, in terms of hiding and binding



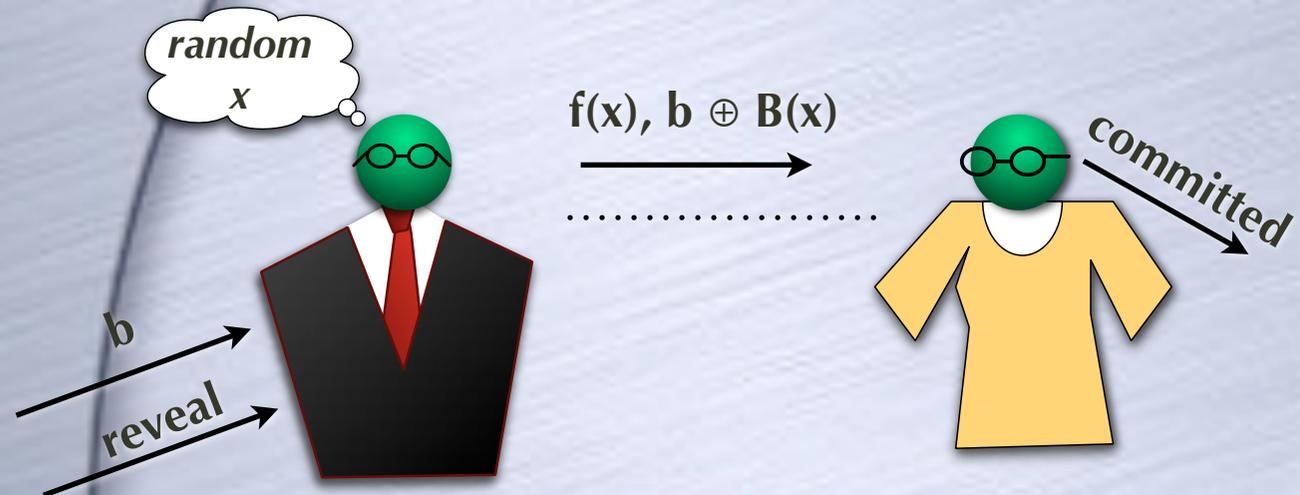
# A Commitment Protocol

- Uses a OWP  $f$  and a hardcore predicate for it  $B$
- Satisfies only classical (IND) security, in terms of hiding and binding



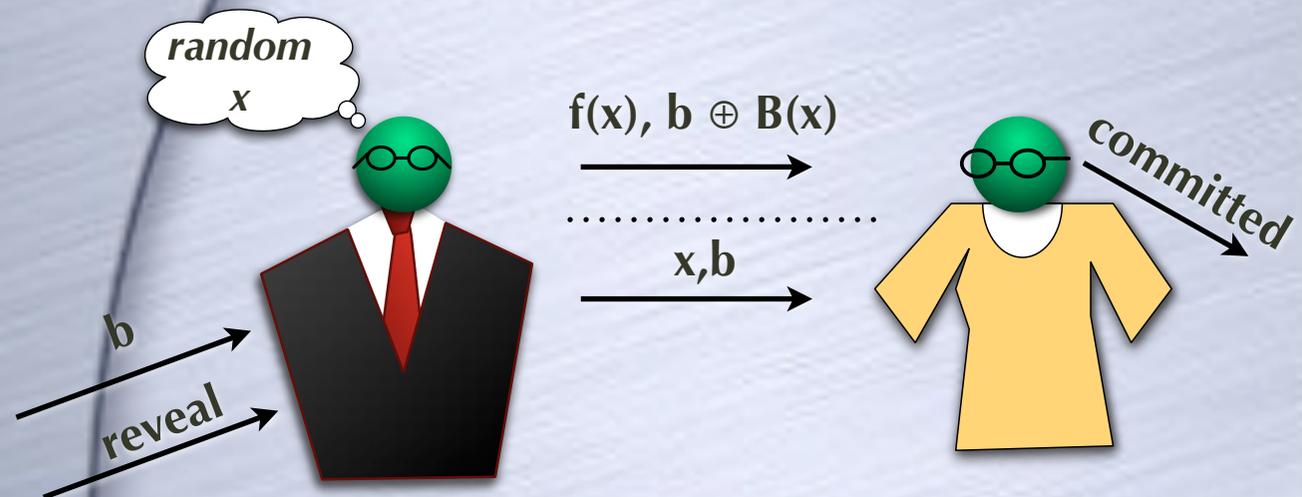
# A Commitment Protocol

- Uses a OWP  $f$  and a hardcore predicate for it  $B$
- Satisfies only classical (IND) security, in terms of hiding and binding



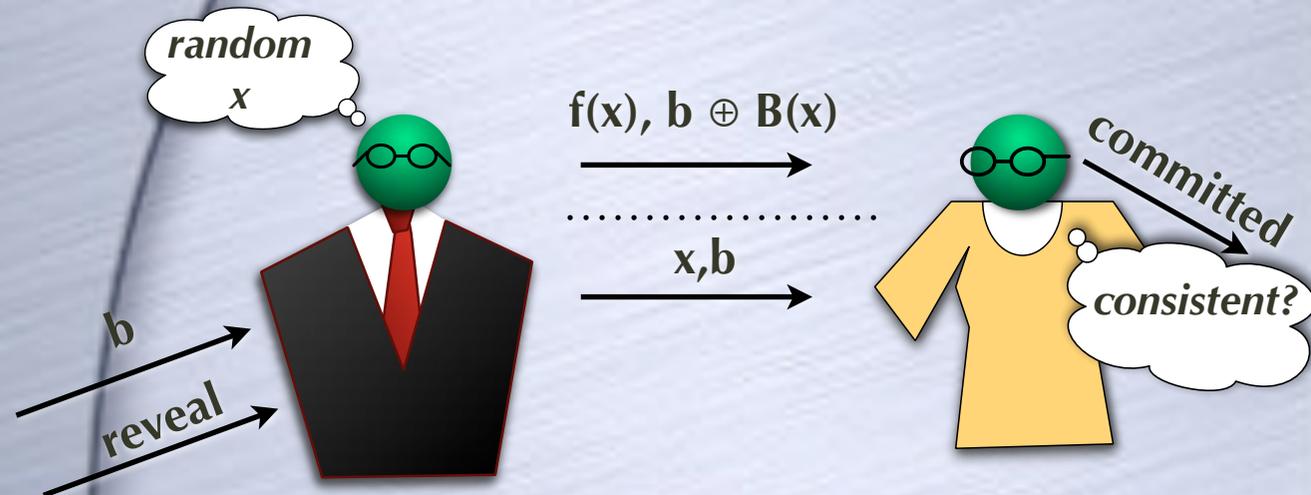
# A Commitment Protocol

- Uses a OWP  $f$  and a hardcore predicate for it  $B$
- Satisfies only classical (IND) security, in terms of hiding and binding



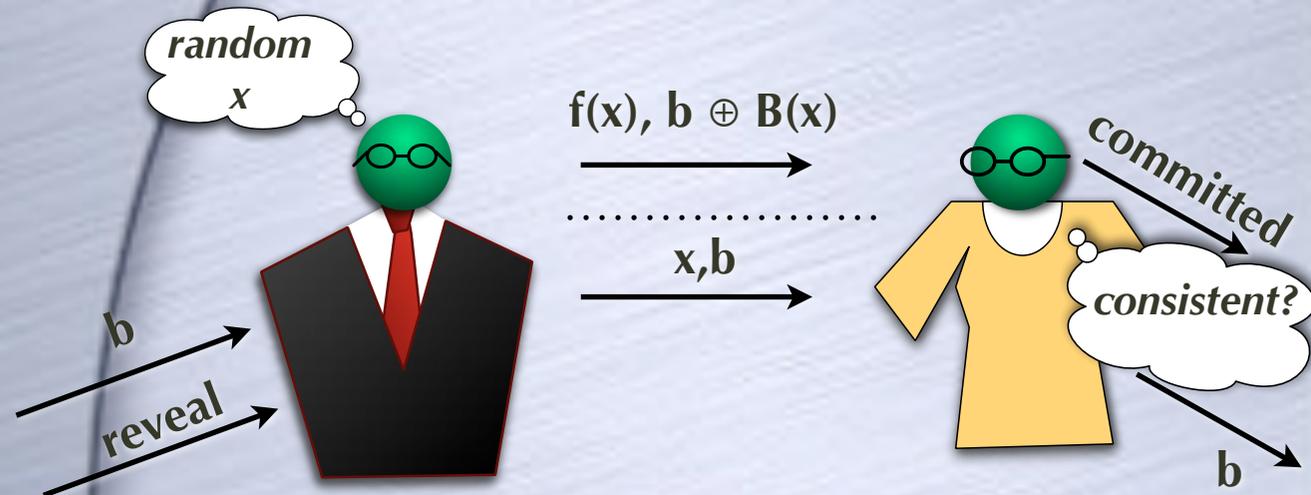
# A Commitment Protocol

- Uses a OWP  $f$  and a hardcore predicate for it  $B$
- Satisfies only classical (IND) security, in terms of hiding and binding



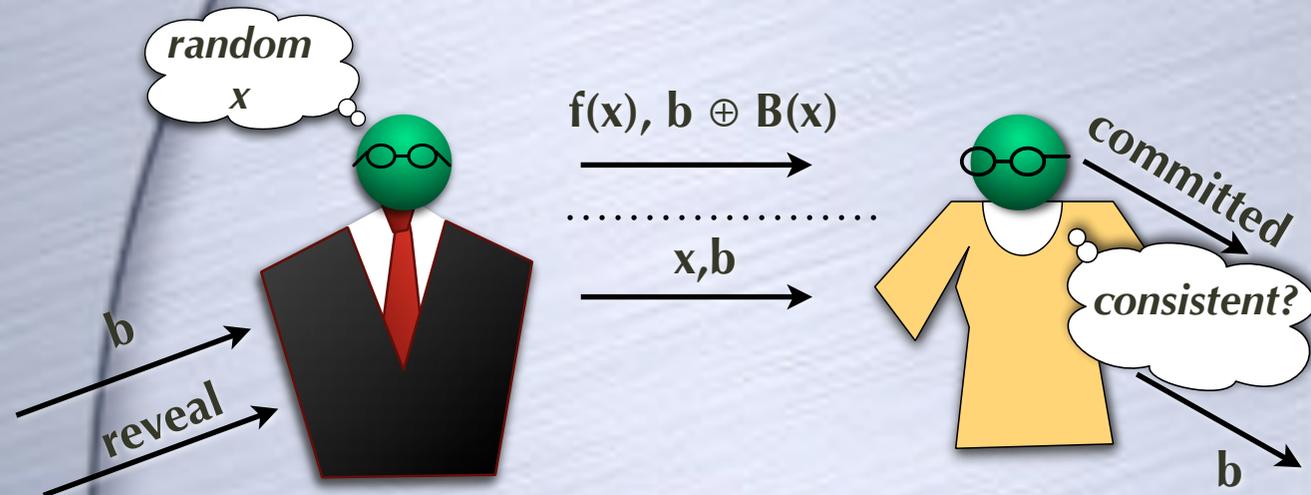
# A Commitment Protocol

- Uses a OWP  $f$  and a hardcore predicate for it  $B$
- Satisfies only classical (IND) security, in terms of hiding and binding



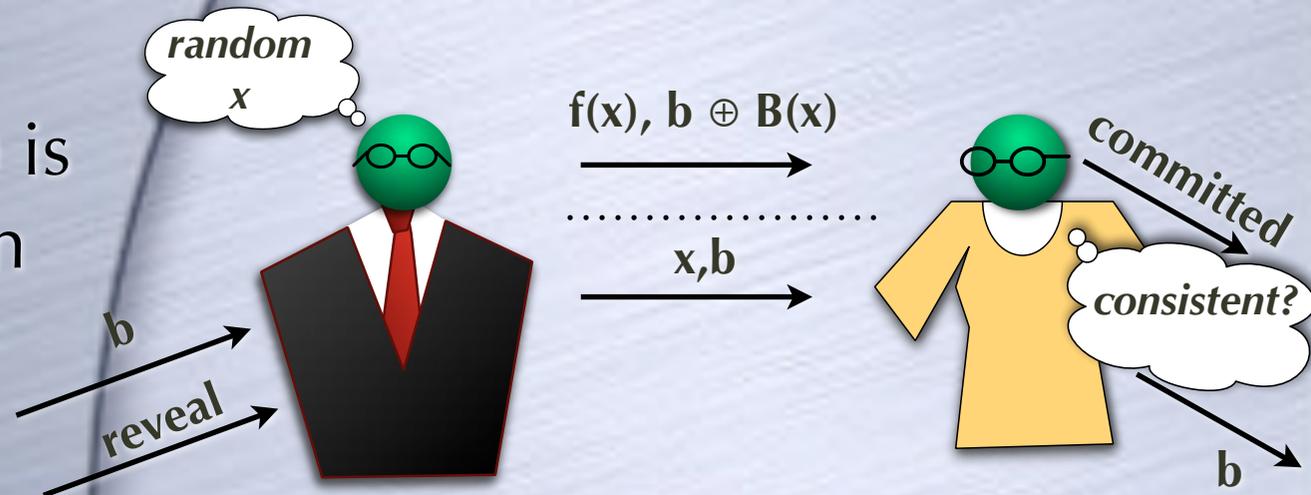
# A Commitment Protocol

- Uses a OWP  $f$  and a hardcore predicate for it  $B$
- Satisfies only classical (IND) security, in terms of hiding and binding
- Perfectly binding because  $f$  is a permutation



# A Commitment Protocol

- Uses a OWP  $f$  and a hardcore predicate for it  $B$
- Satisfies only classical (IND) security, in terms of hiding and binding
- Perfectly binding because  $f$  is a permutation
- Hiding because  $B(x)$  is pseudorandom given  $f(x)$



# ZK Proofs: What for?



# ZK Proofs: What for?

- Authentication



# ZK Proofs: What for?

- Authentication
  - Using ZK Proof of Knowledge



# ZK Proofs: What for?

- Authentication
  - Using ZK Proof of Knowledge
- Canonical use: As a tool in larger protocols



# ZK Proofs: What for?

- Authentication
  - Using ZK Proof of Knowledge
- Canonical use: As a tool in larger protocols
  - To enforce “honest behavior” in protocols



# ZK Proofs: What for?

- Authentication
  - Using ZK Proof of Knowledge
- Canonical use: As a tool in larger protocols
  - To enforce “honest behavior” in protocols
  - At each step prove in ZK it was done as prescribed



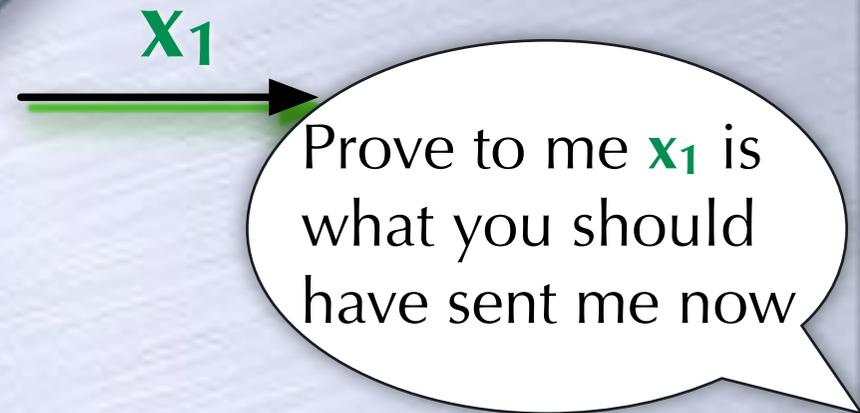
# ZK Proofs: What for?

- Authentication
  - Using ZK Proof of Knowledge
- Canonical use: As a tool in larger protocols
  - To enforce “honest behavior” in protocols
  - At each step prove in ZK it was done as prescribed



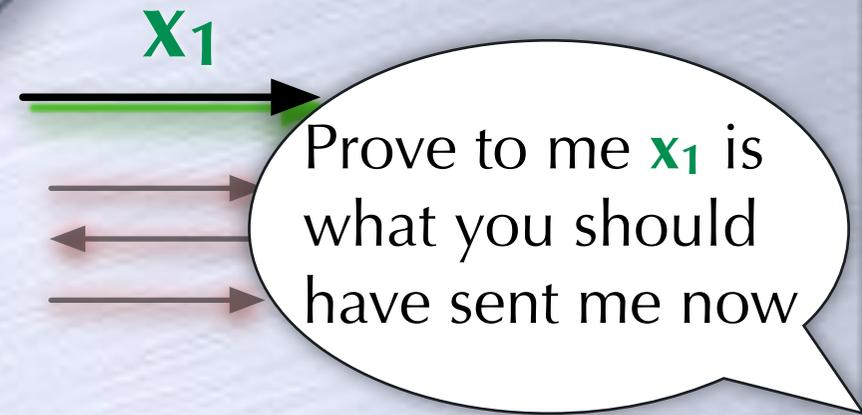
# ZK Proofs: What for?

- Authentication
  - Using ZK Proof of Knowledge
- Canonical use: As a tool in larger protocols
  - To enforce “honest behavior” in protocols
  - At each step prove in ZK it was done as prescribed



# ZK Proofs: What for?

- Authentication
  - Using ZK Proof of Knowledge
- Canonical use: As a tool in larger protocols
  - To enforce “honest behavior” in protocols
  - At each step prove in ZK it was done as prescribed



# ZK Proofs: What for?

- Authentication
  - Using ZK Proof of Knowledge
- Canonical use: As a tool in larger protocols
  - To enforce “honest behavior” in protocols
  - At each step prove in ZK it was done as prescribed



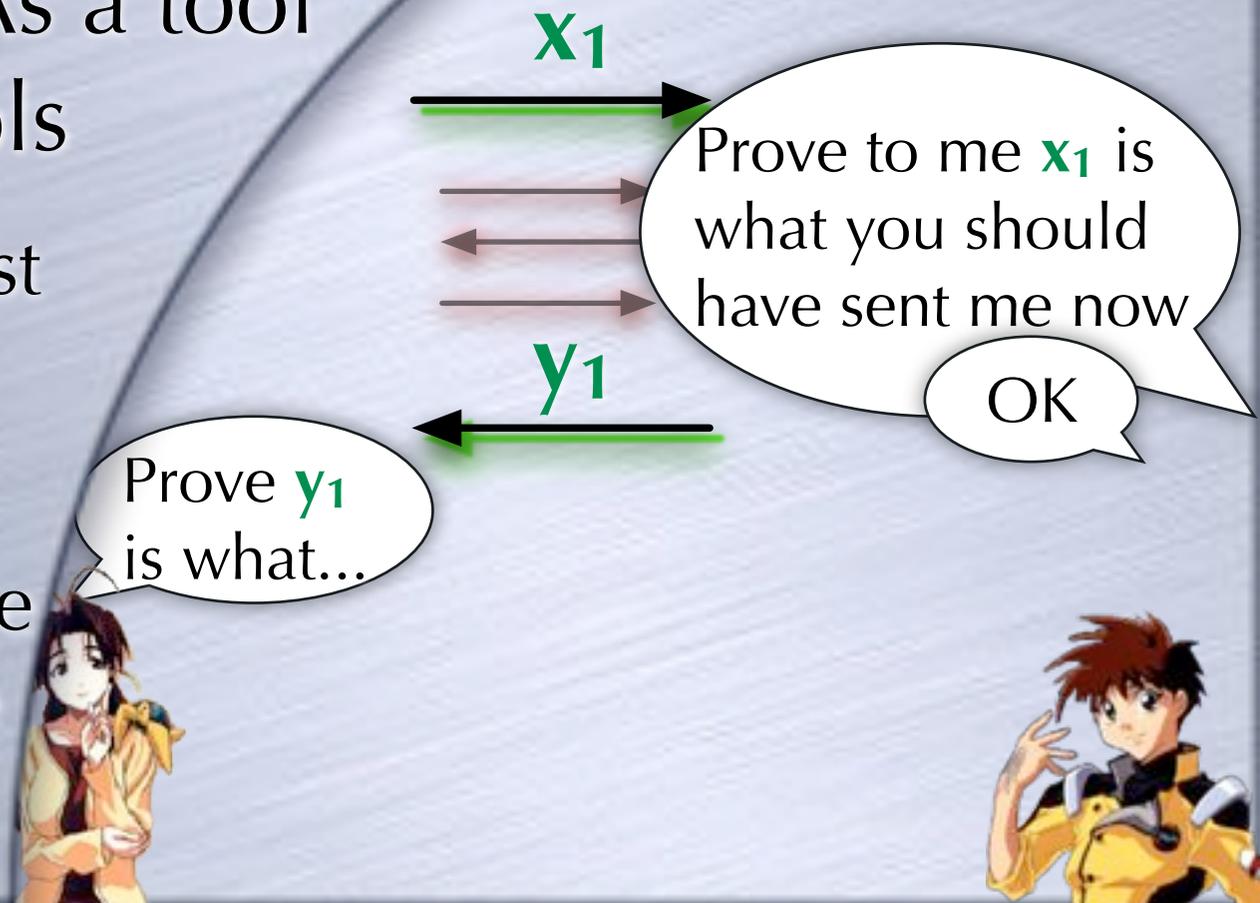
# ZK Proofs: What for?

- Authentication
  - Using ZK Proof of Knowledge
- Canonical use: As a tool in larger protocols
  - To enforce “honest behavior” in protocols
  - At each step prove in ZK it was done as prescribed



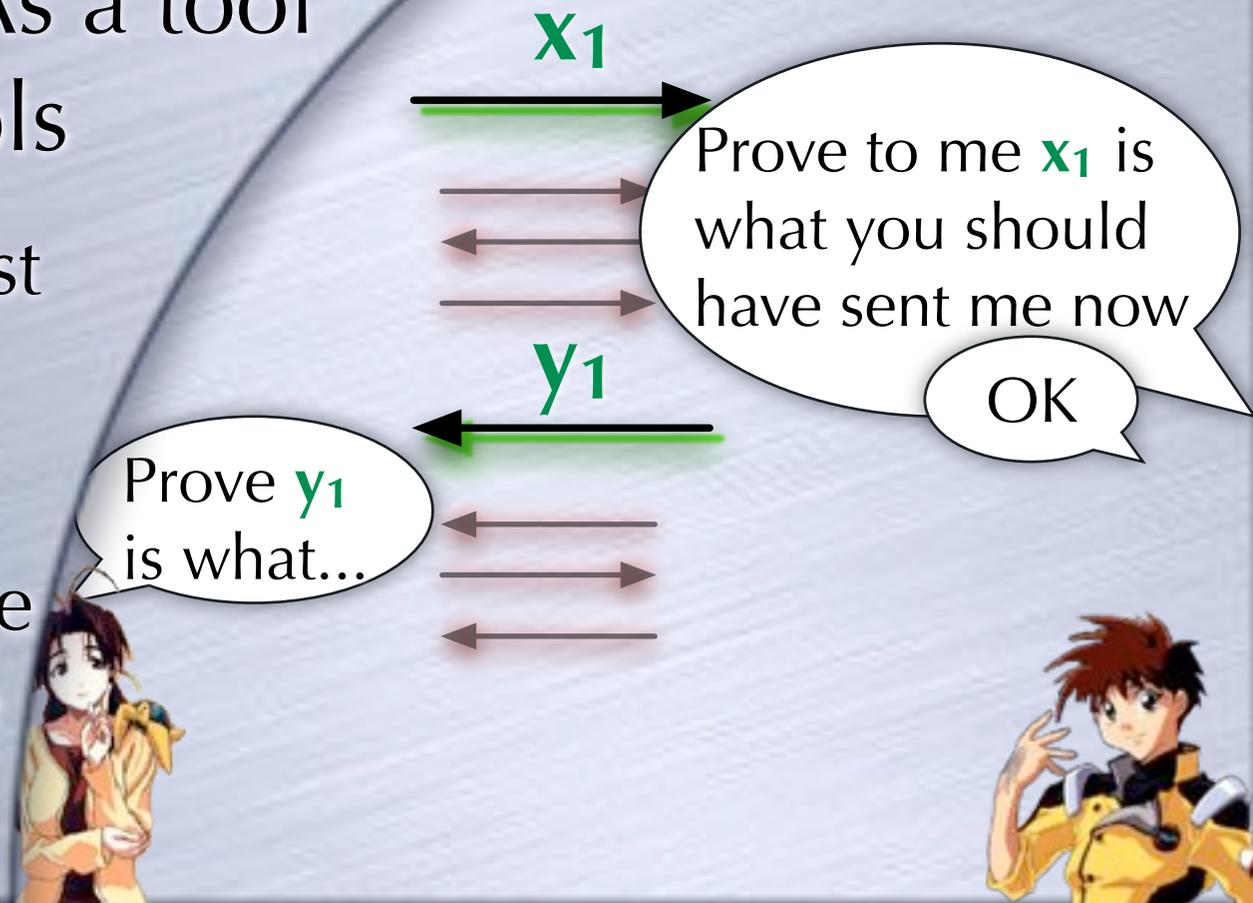
# ZK Proofs: What for?

- Authentication
  - Using ZK Proof of Knowledge
- Canonical use: As a tool in larger protocols
  - To enforce “honest behavior” in protocols
  - At each step prove in ZK it was done as prescribed



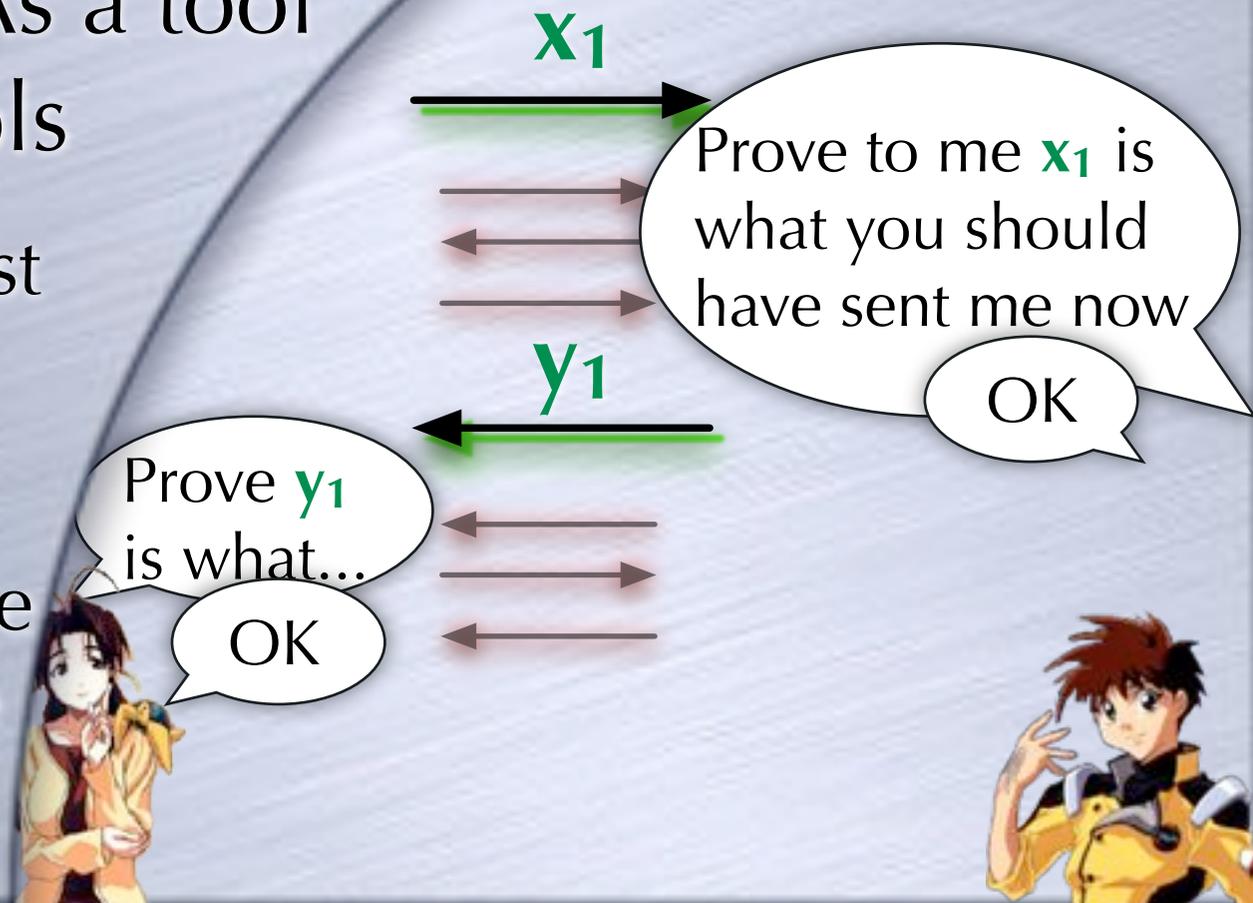
# ZK Proofs: What for?

- Authentication
  - Using ZK Proof of Knowledge
- Canonical use: As a tool in larger protocols
  - To enforce “honest behavior” in protocols
  - At each step prove in ZK it was done as prescribed



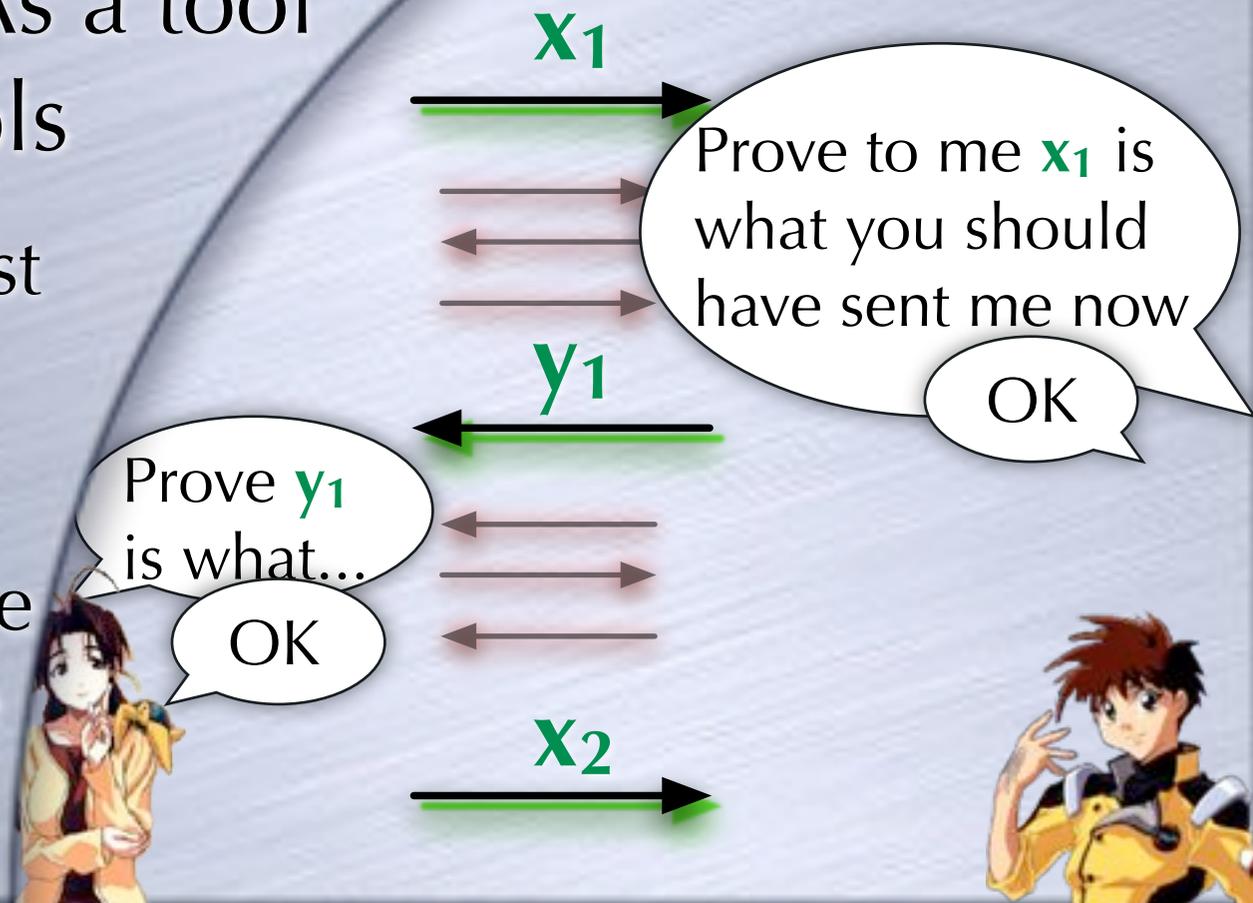
# ZK Proofs: What for?

- Authentication
  - Using ZK Proof of Knowledge
- Canonical use: As a tool in larger protocols
  - To enforce “honest behavior” in protocols
  - At each step prove in ZK it was done as prescribed



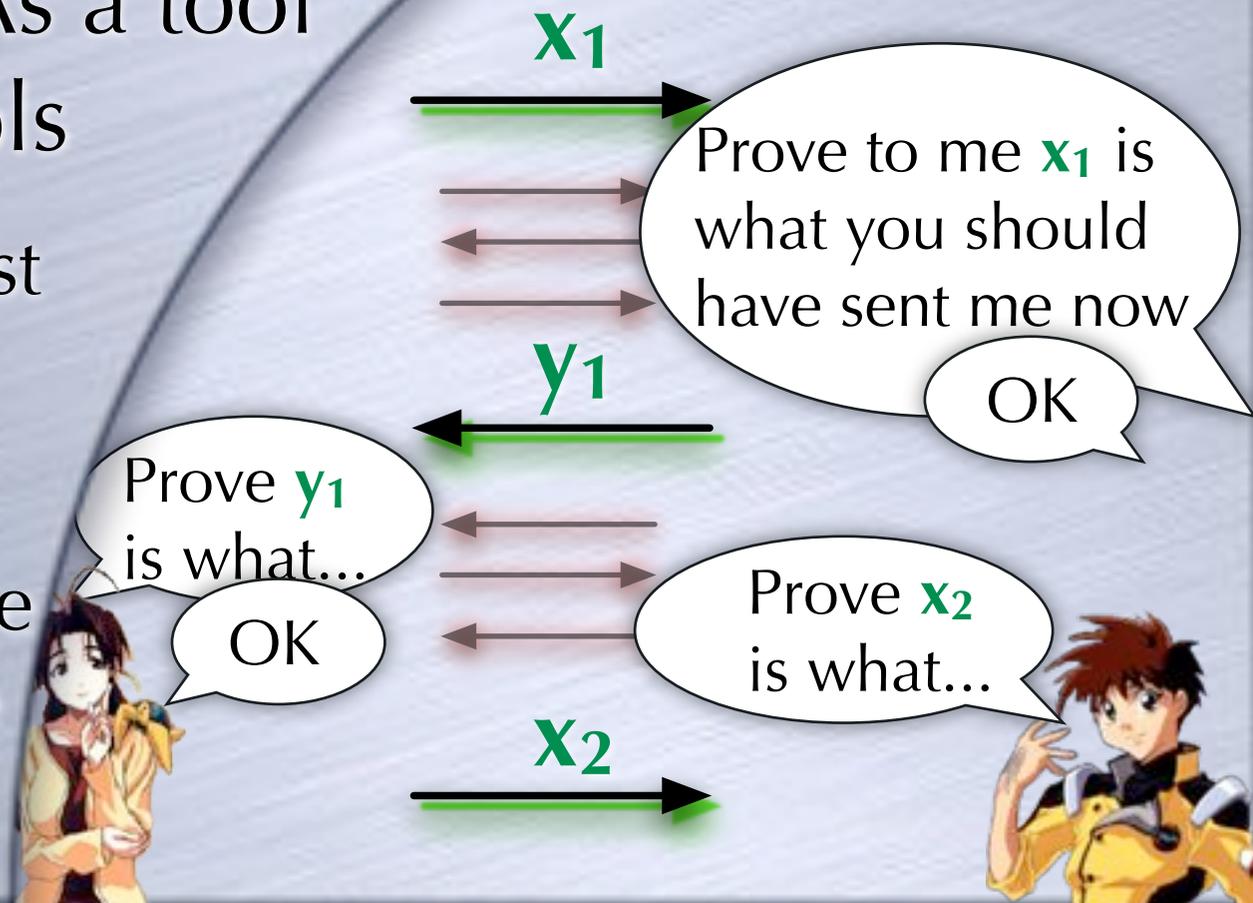
# ZK Proofs: What for?

- Authentication
  - Using ZK Proof of Knowledge
- Canonical use: As a tool in larger protocols
  - To enforce “honest behavior” in protocols
  - At each step prove in ZK it was done as prescribed

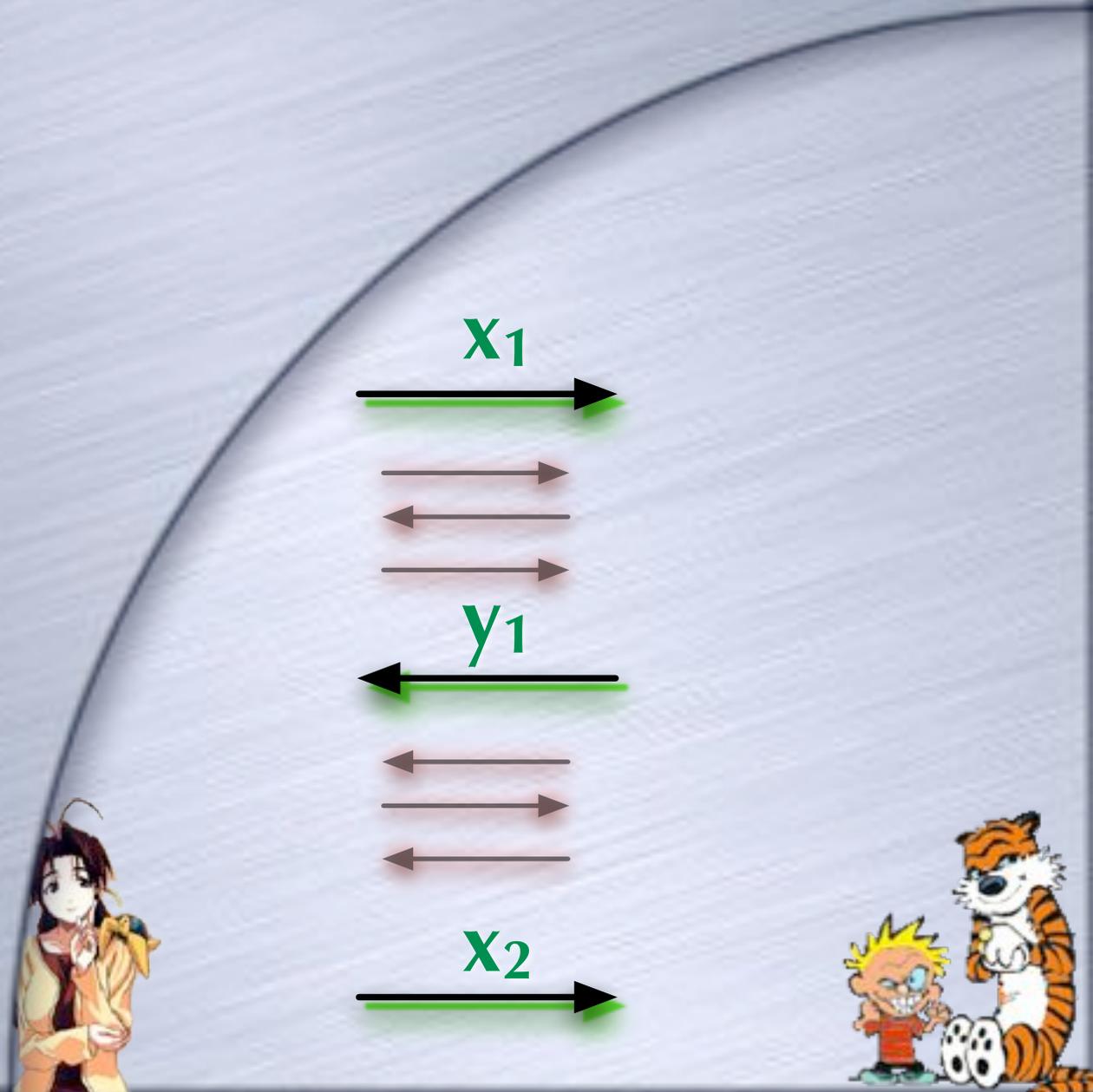


# ZK Proofs: What for?

- Authentication
  - Using ZK Proof of Knowledge
- Canonical use: As a tool in larger protocols
  - To enforce “honest behavior” in protocols
  - At each step prove in ZK it was done as prescribed

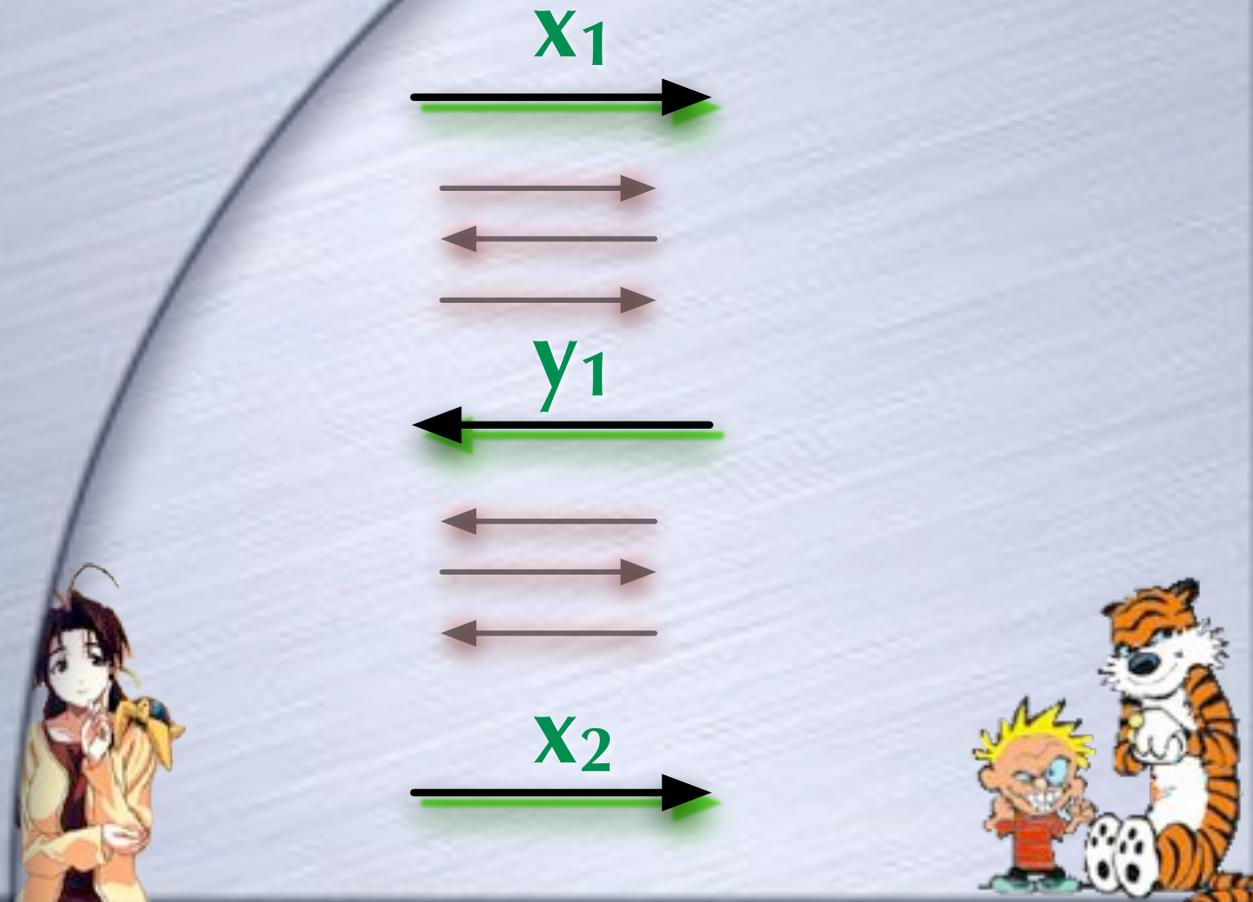


# Does it fit in?



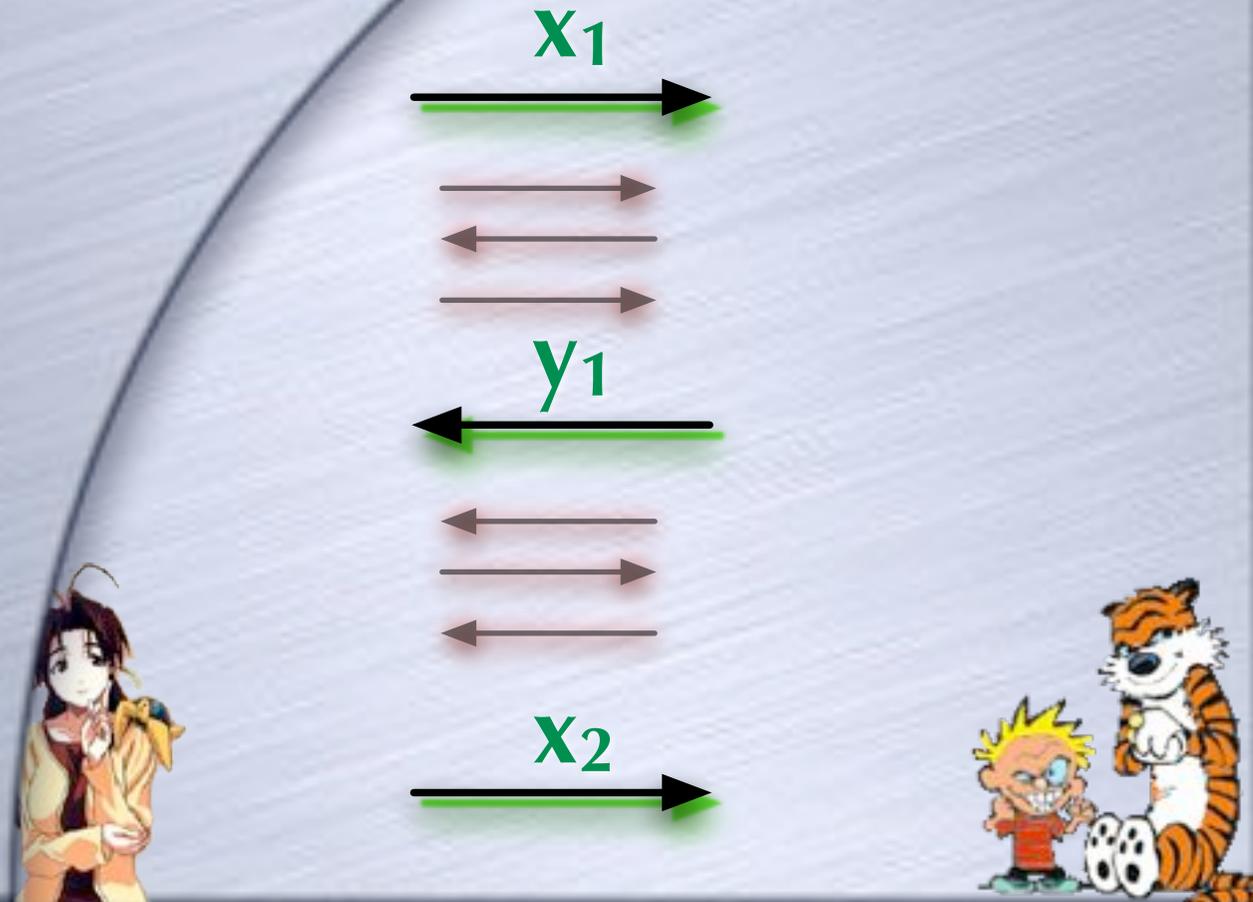
# Does it fit in?

- Does the proof stay ZK in the big picture?



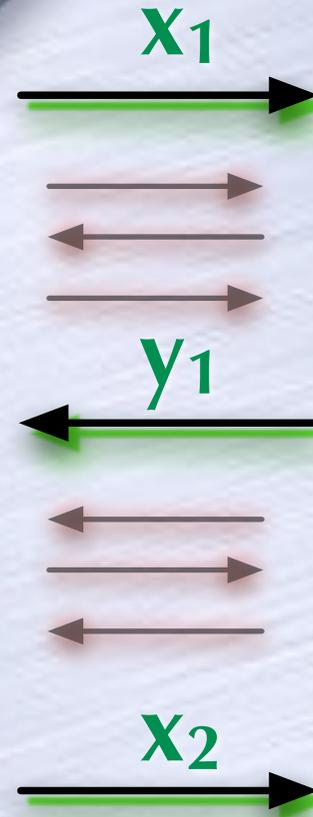
# Does it fit in?

- Does the proof stay ZK in the big picture?
- Composition



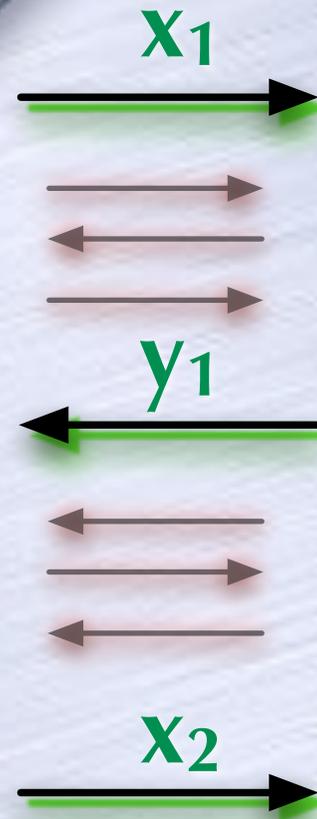
# Does it fit in?

- Does the proof stay ZK in the big picture?
- Composition
  - Several issues: auxiliary information from previous runs, concurrency issues, malleability/man-in-the-middle



# Does it fit in?

- Does the proof stay ZK in the big picture?
- Composition
  - Several issues: auxiliary information from previous runs, concurrency issues, malleability/man-in-the-middle
- In general, to allow composition more complicated protocols



# An IND-security Notion

# An IND-security Notion

- ZK (as opposed to SIM-ZK/ZK-PoK) weakens soundness guarantee

# An IND-security Notion

- ZK (as opposed to SIM-ZK/ZK-PoK) weakens soundness guarantee
- A weakening of ZK property: **Witness Indistinguishability (WI)**

# An IND-security Notion

- ZK (as opposed to SIM-ZK/ZK-PoK) weakens soundness guarantee
- A weakening of ZK property: **Witness Indistinguishability (WI)**
  - Adversarial verifier gives  $(x, w_0, w_1)$  and prover uses  $(x, w_b)$  for a random  $b$ . Adversary has negligible advantage in guessing  $b$ .

# An IND-security Notion

- ZK (as opposed to SIM-ZK/ZK-PoK) weakens soundness guarantee
- A weakening of ZK property: **Witness Indistinguishability** (WI)
  - Adversarial verifier gives  $(x, w_0, w_1)$  and prover uses  $(x, w_b)$  for a random  $b$ . Adversary has negligible advantage in guessing  $b$ .
    - A ZK proof is always WI, but not vice-versa

# An IND-security Notion

- ZK (as opposed to SIM-ZK/ZK-PoK) weakens soundness guarantee
- A weakening of ZK property: **Witness Indistinguishability** (WI)
  - Adversarial verifier gives  $(x, w_0, w_1)$  and prover uses  $(x, w_b)$  for a random  $b$ . Adversary has negligible advantage in guessing  $b$ .
    - A ZK proof is always WI, but not vice-versa
- WI Proofs used as components inside larger protocols

# An IND-security Notion

- ZK (as opposed to SIM-ZK/ZK-PoK) weakens soundness guarantee
- A weakening of ZK property: **Witness Indistinguishability** (WI)
  - Adversarial verifier gives  $(x, w_0, w_1)$  and prover uses  $(x, w_b)$  for a random  $b$ . Adversary has negligible advantage in guessing  $b$ .
    - A ZK proof is always WI, but not vice-versa
- WI Proofs used as components inside larger protocols
  - Sometimes with certain other useful properties

# An IND-security Notion

- ZK (as opposed to SIM-ZK/ZK-PoK) weakens soundness guarantee
- A weakening of ZK property: **Witness Indistinguishability** (WI)
  - Adversarial verifier gives  $(x, w_0, w_1)$  and prover uses  $(x, w_b)$  for a random  $b$ . Adversary has negligible advantage in guessing  $b$ .
    - A ZK proof is always WI, but not vice-versa
- WI Proofs used as components inside larger protocols
  - Sometimes with certain other useful properties
    - e.g. WI-PoK, "Sigma protocols"

# An IND-security Notion

- ZK (as opposed to SIM-ZK/ZK-PoK) weakens soundness guarantee
- A weakening of ZK property: **Witness Indistinguishability** (WI)
  - Adversarial verifier gives  $(x, w_0, w_1)$  and prover uses  $(x, w_b)$  for a random  $b$ . Adversary has negligible advantage in guessing  $b$ .
    - A ZK proof is always WI, but not vice-versa
- WI Proofs used as components inside larger protocols
  - Sometimes with certain other useful properties
    - e.g. WI-PoK, "Sigma protocols"
- Defined in standalone setting, but WI property is preserved under "parallel composition"

# Non-Interactive ZK

# Non-Interactive ZK

- Can the prover just give a written proof (no interaction) which any one can verify and can simulate too?

# Non-Interactive ZK

- Can the prover just give a written proof (no interaction) which any one can verify and can simulate too?
  - No soundness: prover can give the simulated proof!

# Non-Interactive ZK

- Can the prover just give a written proof (no interaction) which any one can verify and can simulate too?
  - No soundness: prover can give the simulated proof!
- NIZK: a trusted “common random string” (CRS) is published, and the proof/verification is w.r.t CRS

# Non-Interactive ZK

- Can the prover just give a written proof (no interaction) which any one can verify and can simulate too?
  - No soundness: prover can give the simulated proof!
- NIZK: a trusted “common random string” (CRS) is published, and the proof/verification is w.r.t CRS
  - NIZK property: a simulator can simulate the CRS and the proofs

# Non-Interactive ZK

- Can the prover just give a written proof (no interaction) which any one can verify and can simulate too?
  - No soundness: prover can give the simulated proof!
- NIZK: a trusted “common random string” (CRS) is published, and the proof/verification is w.r.t CRS
  - NIZK property: a simulator can simulate the CRS and the proofs
  - Note: CRS is a part of the proof, but prover is not allowed to choose it (otherwise no soundness)

# Non-Interactive ZK

- Can the prover just give a written proof (no interaction) which any one can verify and can simulate too?
  - No soundness: prover can give the simulated proof!
- NIZK: a trusted “common random string” (CRS) is published, and the proof/verification is w.r.t CRS
  - NIZK property: a simulator can simulate the CRS and the proofs
  - Note: CRS is a part of the proof, but prover is not allowed to choose it (otherwise no soundness)
- NIZK schemes exist for all NP languages (using “enhanced” T-OWP)

# Non-Interactive ZK

- Can the prover just give a written proof (no interaction) which any one can verify and can simulate too?
  - No soundness: prover can give the simulated proof!
- NIZK: a trusted “common random string” (CRS) is published, and the proof/verification is w.r.t CRS
  - NIZK property: a simulator can simulate the CRS and the proofs
  - Note: CRS is a part of the proof, but prover is not allowed to choose it (otherwise no soundness)
- NIZK schemes exist for all NP languages (using “enhanced” T-OWP)
  - Also can NIZK-ify some ZK protocols in the RO Model (no CRS)

# Today

# Today

- Zero-Knowledge Proofs

# Today

- Zero-Knowledge Proofs
  - Interactive Proofs (complete and sound), in which the verifier's view is simulatable given just the statement being proven

# Today

- Zero-Knowledge Proofs
  - Interactive Proofs (complete and sound), in which the verifier's view is simulatable given just the statement being proven
- Classical security definition

# Today

- Zero-Knowledge Proofs
  - Interactive Proofs (complete and sound), in which the verifier's view is simulatable given just the statement being proven
- Classical security definition
  - Standalone SIM-security for corrupt verifier (ZK property). Soundness (for corrupt prover) separately

# Today

- Zero-Knowledge Proofs
  - Interactive Proofs (complete and sound), in which the verifier's view is simulatable given just the statement being proven
- Classical security definition
  - Standalone SIM-security for corrupt verifier (ZK property). Soundness (for corrupt prover) separately
- Protocols for Graph 3-colorability (and hence all NP properties) using commitment schemes (in turn using OWP)

# Today

- Zero-Knowledge Proofs
  - Interactive Proofs (complete and sound), in which the verifier's view is simulatable given just the statement being proven
- Classical security definition
  - Standalone SIM-security for corrupt verifier (ZK property). Soundness (for corrupt prover) separately
- Protocols for Graph 3-colorability (and hence all NP properties) using commitment schemes (in turn using OWP)
  - Omitted: ZK for several specific statements

# Today

- Zero-Knowledge Proofs
  - Interactive Proofs (complete and sound), in which the verifier's view is simulatable given just the statement being proven
- Classical security definition
  - Standalone SIM-security for corrupt verifier (ZK property). Soundness (for corrupt prover) separately
- Protocols for Graph 3-colorability (and hence all NP properties) using commitment schemes (in turn using OWP)
  - Omitted: ZK for several specific statements
- Useful in "enforcing" honest (but curious) behavior

# Today

- Zero-Knowledge Proofs
  - Interactive Proofs (complete and sound), in which the verifier's view is simulatable given just the statement being proven
- Classical security definition
  - Standalone SIM-security for corrupt verifier (ZK property). Soundness (for corrupt prover) separately
- Protocols for Graph 3-colorability (and hence all NP properties) using commitment schemes (in turn using OWP)
  - Omitted: ZK for several specific statements
- Useful in "enforcing" honest (but curious) behavior
- Some variants (NIZK, WI)