

e-Cash

Lecture 22

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- Merchants/users (even colluding) should not be able to deposit money that was not withdrawn
- Users should not be able to cheat honest merchants. In particular, users should not be able to double-spend

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- At a merchant’s, the user gives the signed coin
- Merchant contacts the Bank (online) who ensures that the coin has not been used before (i.e., no double spending) and the signature is valid. If so adds the coin to the spent-coin list

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 - Unforgeability: After t sessions, User cannot output signatures on $t+1$ distinct messages

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- Blindness, because signer sees only $\text{Commit}(m)$. Unlinkability from encryption. Unforgeability from soundness of NIZK, efficient decryption of PKE, and unforgeability of the signature scheme
- Efficient variants (under suitable assumptions) using Groth-Sahai NIZK (or NIWI) scheme and compatible primitives

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 - Idea: verification in two sessions of the spending protocol with the same coin exposes the user's identity

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 - Merchant needs to transfer the User's proof to Bank (i.e., Bank should be convinced that the merchant didn't fake)

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- Fiat-Shamir heuristic (in the Random Oracle Model) to make proofs transferable (and non-interactive)
- Alternately, in the CRS setting, use a NIZK (e.g. Groth-Sahai)

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 - Interactive verification (transferable only using RO)

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 - e.g. (Weak) Boneh-Boyen signature: $\text{Sign}_{\text{SK}}(x) = g^{1/(\text{SK}+x)}$

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- Compact e-Cash: Remove linking multiple spending

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 - Issue: A PRF that supports efficient proofs

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- Security/Efficiency/Usability issues: need to cancel stolen electronic wallet; need to recharge electronic wallet (cellphone?) online, but protect it from malware; efficient multiple denomination coins; allow transferability; tracing may not deter double-spending

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 - Bob and Carol cannot link the persons who proved credentials to the persons who obtained credentials

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- Introduced by Chaum in 1985
- Similar to e-cash, but without the double-spending problem
- Alice should be able to prove to Bob that she has a credential from Carol (cf. Alice withdraws a coin from Carol and spends it with Bob)
 - Bob and Carol cannot link the persons who proved credentials to the persons who obtained credentials
 - And they cannot link together multiple proofs coming from the same user

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- Proving: Alice wants to prove to Carol that owner of A_C has a credential from Bob. She commits SK_A again to get A' and shows that she has a signature from Carol on the message in A' . She also proves that A' and A_C have the same message

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 - Anonymous, offline validation and compact

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- Next: More signatures