CS598JHM: **Advanced NLP** (Spring '10)

# Forward/Backward

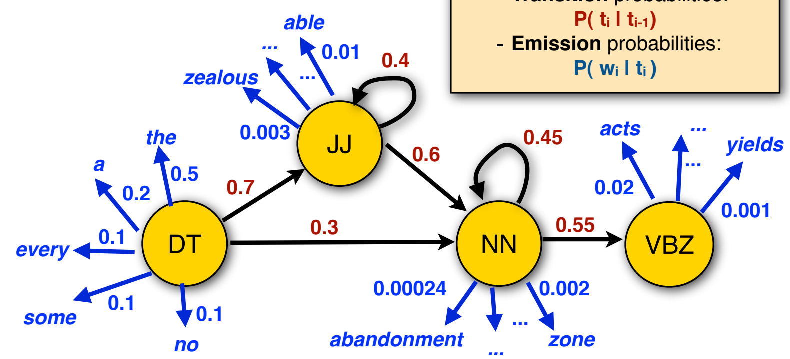

Julia Hockenmaier
juliahmr@illinois.edu
3324 Siebel Center


http://www.cs.uiuc.edu/class/sp10/cs598jhm

---

# HMMs as probabilistic automata

An HMM defines
- **Transition** probabilities: $P(t_i \mid t_{i-1})$
- **Emission** probabilities: $P(w_i \mid t_i)$



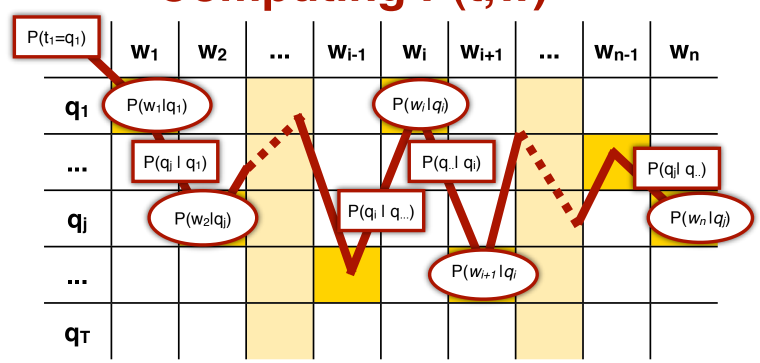



---

# The Forward algorithm

---

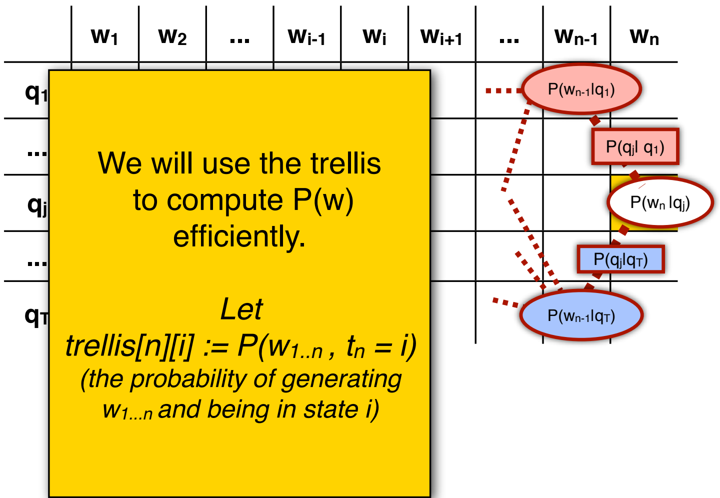
# Computing P(t,w)




- One path through the trellis = one tag sequence
- We just multiply the probabilities

$$P(\mathbf{t}, \mathbf{w}) = P(t_1)P(w_1|t_1)\prod_{i=2}^{T} P(t_i|t_{i-1})P(w_i|t_i)$$

# 2. Finding P(w) = ∑t P(t,w)




We will use the trellis to compute P(w) efficiently.

*Let*

*trellis[n][i] := P($w_{1..n}$ , $t_n$ = i)*

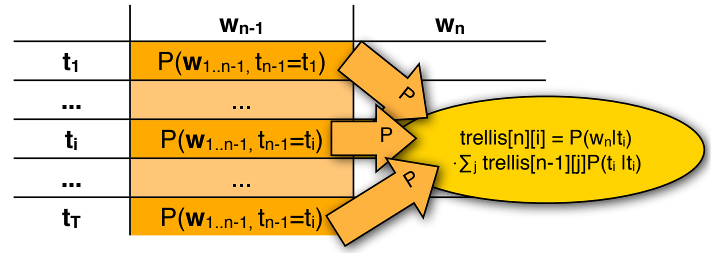
*(the probability of generating $w_{1...n}$ and being in state i)*



# 3. Finding P(w) = ∑t P(t,w)

$$\underbrace{P(\mathbf{w_{1..n}}, \mathbf{t_n} = \mathbf{t_i})}_{\texttt{trellis[n][i]}} = \sum_{\mathbf{t_{1..n}}|\mathbf{t_n}=\mathbf{t_i}} P(\mathbf{w_{1...n}}, \mathbf{t_{1..n}})$$

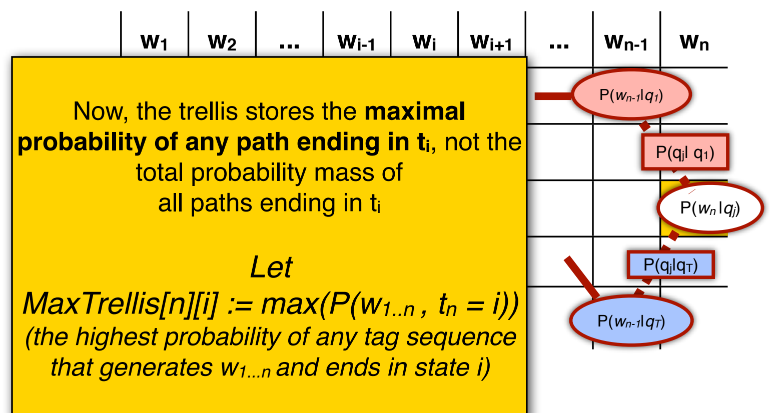
$$= P(w_n|t_i) \sum_{j=1}^{T} P(t_i|t_j) \underbrace{P(\mathbf{w_{1..n-1}}, t_{n-1} = t_j)}_{\texttt{trellis[n-1][j]}}$$






# The Viterbi algorithm

# Finding p* = max$_t$ P(t,w)




Now, the trellis stores the **maximal probability of any path ending in t$_i$**, not the total probability mass of all paths ending in t$_i$

*Let*

*MaxTrellis[n][i] := max(P($w_{1..n}$ , $t_n$ = i))*

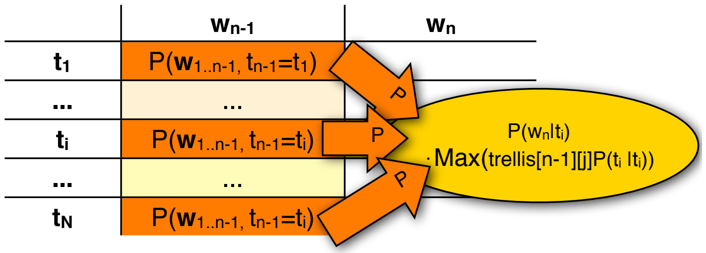
*(the highest probability of any tag sequence that generates $w_{1...n}$ and ends in state i)*

# Finding p* = max$_t$ P(t,w)

$$\underbrace{\max(P(\mathbf{w_{1..n}}, \mathbf{t_n} = \mathbf{t_i}))}_{\texttt{maxTrellis[n][i]}} = \sum_{\mathbf{t_{1..n}}|\mathbf{t_n}=\mathbf{t_i}} P(\mathbf{w_{1...n}}, \mathbf{t_{1..n}})$$

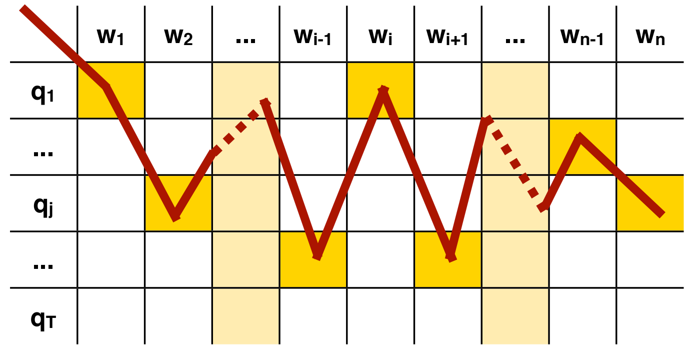
$$= P(w_n|t_i)\max_{j=1}\Big(P(t_i|t_j)\underbrace{P(\mathbf{w_{1..n-1}}, t_{n-1}=t_j)}_{\texttt{maxTrellis[n-1][j]}}\Big)$$

$$= b_{in}\max_{j=1}\Big(a_{ji}\underbrace{P(\mathbf{w_{1..n-1}}, t_{n-1}=t_j)}_{\texttt{maxTrellis[n-1][j]}}\Big)$$






# Retrieving t* = argmax$_t$ P(t,w)




- By keeping only **one backpointer** from each cell to the tag in the previous column that yields the highest probability, we can retrieve the most likely tag sequence when we're done.



# The Forward-Backward algorithm

# Learning an HMM from unlabeled data

```
Pierre Vinken , 61 years old , will
join the board as a nonexecutive
director Nov. 29 .
```

Tagset:
NNP: proper noun
CD: numeral,
JJ: adjective,...

**We can't count anymore.**
We have to *guess* how often we'd *expect* to see $t_i t_j$ *etc.* in our data set. Call this **expected count** ‹*C(...)*›

- Our estimate for the transition probabilities:

$$\hat{P}(t_j|t_i) = \frac{\langle C(t_i t_j)\rangle}{\langle C(t_i)\rangle}$$

- Our estimate for the emission probabilities:

$$\hat{P}(w_j|t_i) = \frac{\langle C(w_j\_t_i)\rangle}{\langle C(t_i)\rangle}$$

# Learning an HMM: the EM algorithm

**Initialization:**

- Take a data set **S**
- Guess some initial $A_0$ and $B_0$
  Let $\lambda_i = \lambda_0 = (A_0, B_0)$

**The Expectation (E) step:**

- Use $\lambda_i$ to compute $\langle C(t) \mid \lambda_i, \mathbf{S} \rangle$

**The Maximization (M) step:**

- Calculate a new HMM $\lambda_{i+1}$ using $\langle C(t) \mid \lambda_i, \mathbf{S} \rangle$

*Repeat the E and M steps until λ converges*



# How do we compute $\langle C(t_i) \rangle$ ?

- Our corpus ***S*** consists of *K* sentences:

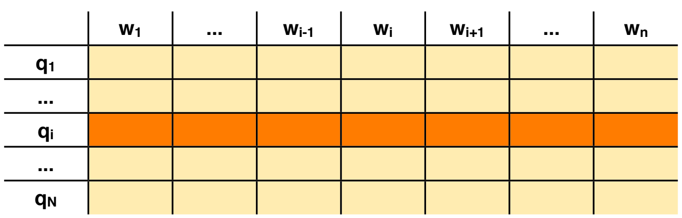
**S** = { $S_1$: "Pierre Vinken…"
$S_2$: "Vinken joined the board…"
…. …..
$S_K$: "Yesterday, the Dow Jones..."}

- We have to sum how often we expect $t_i$ in each sentence

$$\langle C(t_i)|\mathbf{S}\rangle_P = \sum_k^K \langle C(t_i|S_k)\rangle_P$$



# How do we compute $\langle C(t_i) \mid S_k \rangle$

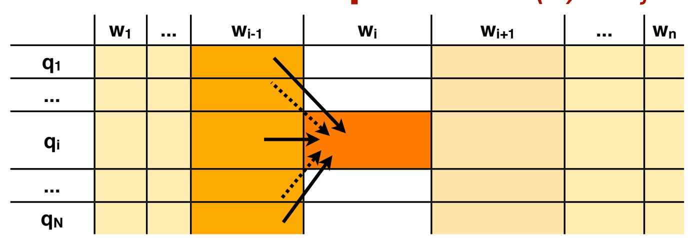



- $t_i$ can be assigned to any word in the sentence (it corresponds to one row in the trellis)
- We have to sum how often we expect $t_i$ in each cell of this row

$$\langle C(t_i)|\mathbf{w}_{1..n}\rangle_P = \sum_j^n \langle C(t_i|w_j)\rangle_P$$



# How do we compute $\langle C(t_i) \mid w_j \rangle$

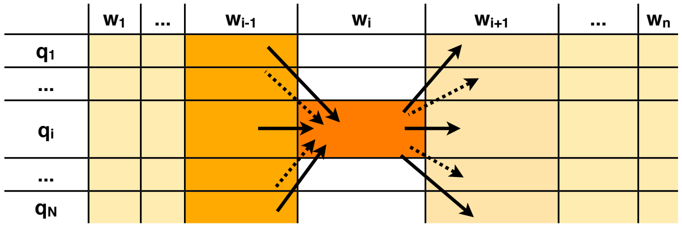



- We need to know $P(\mathbf{t}_j = t_i|\mathbf{w}_{1..n})$
- We can use Bayes Rule:

$$P(\mathbf{t}_j = t_i|\mathbf{w}_{1...n}) = \frac{P(\mathbf{t_j} = t_i, \mathbf{w}_{1...n})}{P(\mathbf{w}_{1...n})}$$

- The forward trellis tells us $\underbrace{P(\mathbf{w_{1..j}}, \mathbf{t_j} = t_i)}_{\texttt{trellis[j][i]}}$ and $P(\mathbf{w}_{1..n})$

# How do we compute $\langle C(t_i) \mid w_j \rangle$




- We need to know $P(\mathbf{w_{1..n}}, \mathbf{t_j} = t_i)$
- The trellis tells us $\underbrace{P(\mathbf{w_{1..j}}, \mathbf{t_j} = t_i)}_{\texttt{trellis[j][i]}}$
- We can use the Chain rule:

$$P(\mathbf{w_{1..j\ j+1..n}}, \mathbf{t_j} = t_i) = P(\mathbf{w_{1..j}}, \mathbf{t_j} = t_i) P(\mathbf{w_{j+1...n}} | \mathbf{w_{1..j}}, \mathbf{t_j} = \mathbf{t_i})$$



# Computing $P(w_{j+1...n} \mid w_{1..j}, t_j = t_i)$

In our HMM model, words depend only on their tags, thus:

$$P(\mathbf{w_{j+1...n}} | \mathbf{w_{1..j}}, \mathbf{t_j} = \mathbf{t_i}) \quad = \quad P(\mathbf{w_{j+1...n}} | \mathbf{t_j} = t_i)$$

We can calculate this recursively:

$$P(\mathbf{w_{j+1...n}} | \mathbf{t_j} = t_i) \quad = \quad \sum_k P(t_k | t_i) P(\mathbf{w_{j+1}} | t_k) P(\mathbf{w_{j+2...n}} | \mathbf{t_{j+1}} = t_k)$$



# Putting it all together

1. In our model, P(**w** | **t**$_\mathbf{j}$ = t$_i$) decomposes into two terms: a **forward** and a **backward** probability

$$P(\mathbf{w_{1...j\ j+1...n}} | \mathbf{t_j} = t_i)$$

$$= \underbrace{P(\mathbf{w_{1...j}} | \mathbf{t_j} = t_i)}_{\text{Forward probability of } \mathbf{w}_{1..j}, t_i} \times \underbrace{P(\mathbf{w_{j+1...n}} | \mathbf{t_j} = t_i)}_{\text{Backward probability of } \mathbf{w}_{j..n}, t_i}$$



# Forward and backward probabilities

2. Both can be calculated recursively:

$$\underbrace{P(\mathbf{w_{1...j}} | \mathbf{t_j} = t_i)}_{\text{Forward probability of } \mathbf{w}_{1..j}, t_i}$$

$$= \sum_k P(t_i | t_k) P(\mathbf{w_j} | t_k) \underbrace{P(\mathbf{w_{1...j-1}} | \mathbf{t_{j-1}} = t_k)}_{\text{Forward probability of } \mathbf{w}_{1..j-1}, t_k}$$

$$\underbrace{P(\mathbf{w_{j+1...n}} | \mathbf{t_j} = t_i)}_{\text{Backward probability of } \mathbf{w}_{j..n}, t_i}$$

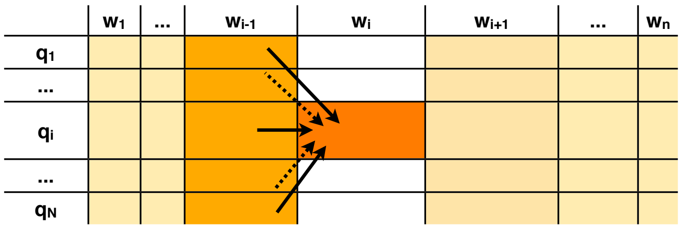
$$= \sum_k P(t_k | t_i) P(\mathbf{w_{j+1}} | t_k) \underbrace{P(\mathbf{w_{j+2...n}} | \mathbf{t_{j+1}} = t_k)}_{\text{Backward probability of } \mathbf{w}_{j+1..n}, t_k}$$

# Using the trellis

3. The trellis tells us already the forward probabilities:

$$\underbrace{P(\mathbf{w_{1\ldots j}}|\mathbf{t_j}=t_i)}_{\texttt{FWtrellis[j][i]}}$$

$$= \sum_k a_{ki} b_{kj} \underbrace{P(\mathbf{w_{1\ldots j-1}}|\mathbf{t_{j-1}}=t_k)}_{\texttt{FWtrellis[j-1][k]}}$$



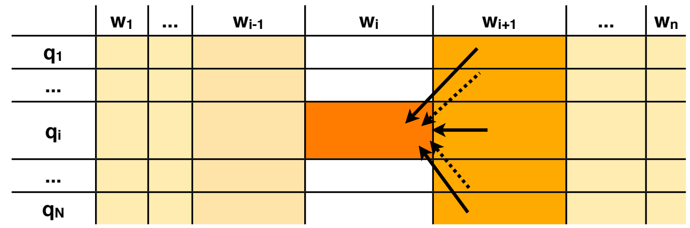



# Using the trellis

4. We can also use it to keep track of the backward probabilities:

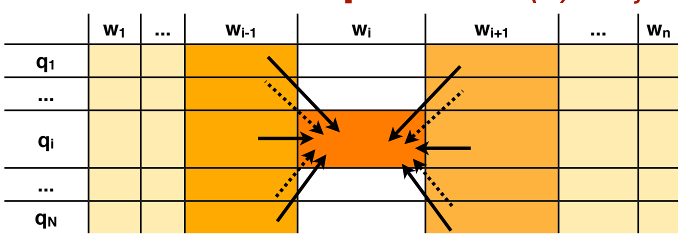
$$\underbrace{P(\mathbf{w_{j+1\ldots n}}|\mathbf{t_j}=t_i)}_{\texttt{BWtrellis[j][i]}}$$

$$= \sum_k a_{ik} b_{kj+1} \underbrace{P(\mathbf{w_{j+2\ldots n}}|\mathbf{t_{j+1}}=t_k)}_{\texttt{BWtrellis[j+1][k]}}$$






# How do we compute $\langle C(t_i) | w_j \rangle$




- The trellis tellls us everything we need to know to compute

$$P(\mathbf{t}_j = t_i|\mathbf{w}_{1\ldots n}) = \frac{P(\mathbf{t_j}=t_i, \mathbf{w}_{1\ldots n})}{P(\mathbf{w}_{1\ldots n})}$$