Knapsack Voting

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Overview

<u>Knapsack Voting</u>

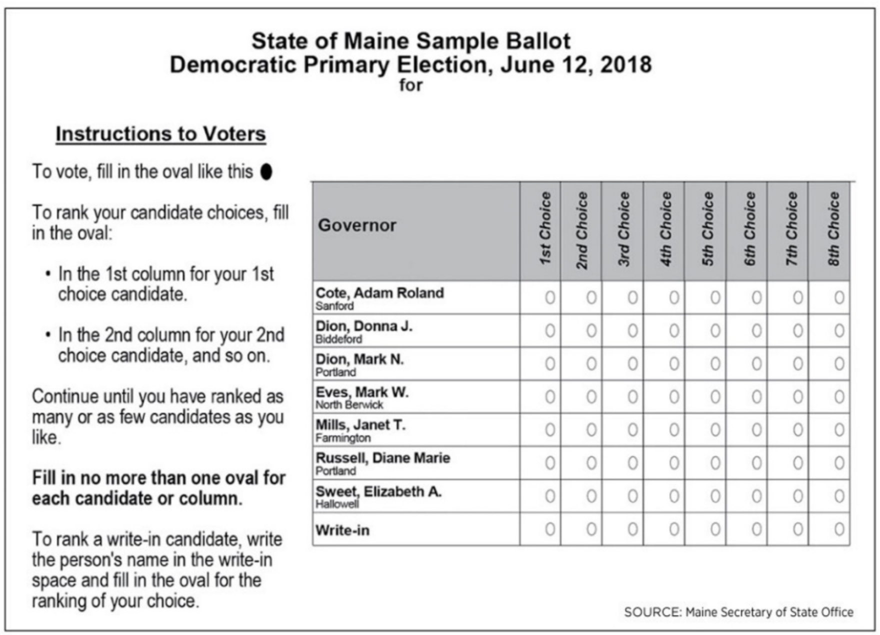
- Introduces definitions, theorems, and related concepts to knapsack voting
- Cursory look at preliminary trial
- Focus on definitions and propositions

<u>Budget Aggregation via Knapsack Voting: Welfare-maximization and Strategy-proofness</u>

- Summarizes definitions, theorems, and ideas from first paper
- Analyzes data from digital voting platform
- Split focus on definitions and empirical results

Background – Participatory Budgeting

- Residents vote on how to divide government's total budget between different proposals



State of Maine Sample Ballot
Democratic Primary Election, June 12, 2018
for

Instructions to Voters

To vote, fill in the oval like this ●

To rank your candidate choices, fill in the oval:

- In the 1st column for your 1st choice candidate.
- In the 2nd column for your 2nd choice candidate, and so on.

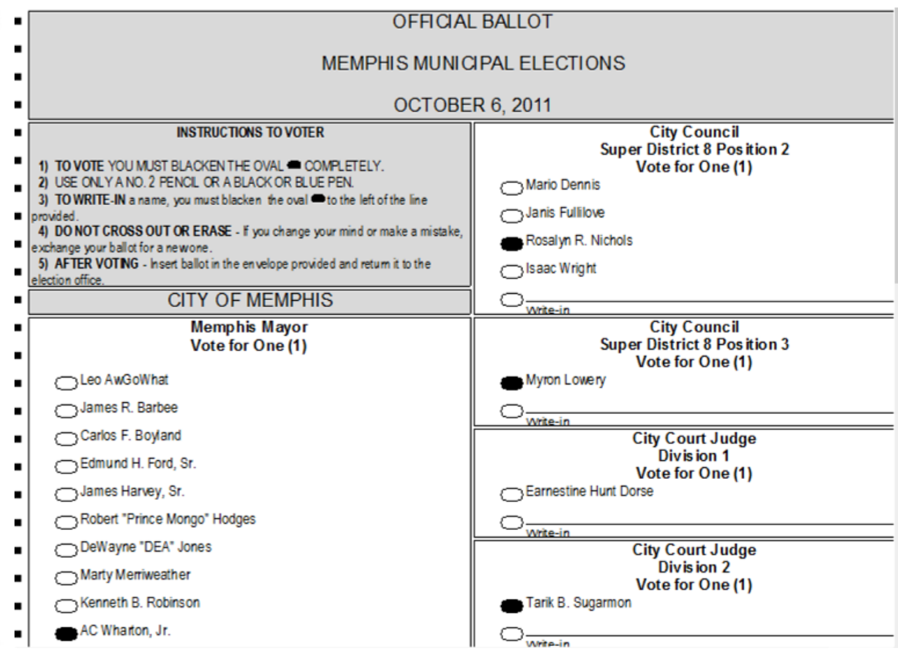
Continue until you have ranked as many or as few candidates as you like.

Fill in no more than one oval for each candidate or column.

To rank a write-in candidate, write the person's name in the write-in space and fill in the oval for the ranking of your choice.

Governor	1st Choice	2nd Choice	3rd Choice	4th Choice	5th Choice	6th Choice	7th Choice	8th Choice
Cote, Adam Roland Sanford	O	O	O	O	O	O	O	O
Dion, Donna J. Biddeford	O	O	O	O	O	O	O	O
Dion, Mark N. Portland	O	O	O	O	O	O	O	O
Eves, Mark W. North Berwick	O	O	O	O	O	O	O	O
Mills, Janet T. Farmington	O	O	O	O	O	O	O	O
Russell, Diane Marie Portland	O	O	O	O	O	O	O	O
Sweet, Elizabeth A. Hallowell	O	O	O	O	O	O	O	O
Write-in	O	O	O	O	O	O	O	O

SOURCE: Maine Secretary of State Office



OFFICIAL BALLOT

MEMPHIS MUNICIPAL ELECTIONS

OCTOBER 6, 2011

INSTRUCTIONS TO VOTER

1) **TO VOTE** YOU MUST BLACKEN THE OVAL ● COMPLETELY.
2) USE ONLY A NO. 2 PENCIL OR A BLACK OR BLUE PEN.
3) **TO WRITE-IN** a name, you must blacken the oval ● to the left of the line provided.
4) **DO NOT CROSS OUT OR ERASE** - If you change your mind or make a mistake, exchange your ballot for a new one.
5) **AFTER VOTING** - Insert ballot in the envelope provided and return it to the election office.

CITY OF MEMPHIS

Memphis Mayor
Vote for One (1)

- Leo AwGoWhat
- James R. Barbee
- Carlos F. Boyland
- Edmund H. Ford, Sr.
- James Harvey, Sr.
- Robert "Prince Mongo" Hodges
- DeWayne "DEA" Jones
- Marty Merriweather
- Kenneth B. Robinson
- AC Wharton, Jr.

City Council
Super District 8 Position 2
Vote for One (1)

- Mario Dennis
- Janis Fullilove
- Rosalyn R. Nichols
- Isaac Wright
- Write-in

City Council
Super District 8 Position 3
Vote for One (1)

- Myron Lowery
- Write-in

City Court Judge
Division 1
Vote for One (1)

- Earnestine Hunt Dorse
- Write-in

City Court Judge
Division 2
Vote for One (1)

- Tarik B. Sugarmon
- Write-in

Participatory Budgeting Problem

- The residents of a city are collectively the set of voters .
- They are voting on a set of proposals that they have identified to be worthwhile.
- The proposal has a cost .
- There is a fixed total budget of Dollars.
- The benefit a voter gets from proposal is .
- The set of winning or chosen proposals is .

$$\arg\max_{W \subseteq \mathcal{P}} \sum_{j \in W} \boxed{\frac{1}{|\mathcal{V}|} \sum_{i \in \mathcal{V}} v_{i,j}}, \quad \textbf{subject to} \quad \sum_{j \in W} c_j \leq B.$$

Average utility

Knapsack Voting

Goel, A., Krishnaswamy, A. K., Sakshuwong, S., and Aitamurto, T. (2015). Knapsack voting. Collective Intelligence.

Participatory Budgeting Voting Methods

- Current voting methods:
  - Approval voting (choosing all approved proposals)
  - -approval voting (choosing top- proposals)
- Issues with current voting methods: do not consider proposal costs
- Proposed approaches:
  - Knapsack voting (choosing while considering budget constraints)
  - Value-for-money comparisons (choose proposal that gives the better value among two given proposals)

Knapsack Voting

- Each voter submits a proposal that satisfies the budget constraint .
  - Set of voters
  - Set of proposals
  - Proposal has a cost
  - Fixed total budget of Dollars
- Each proposal receives a score equal to the number of voters that included it in their votes.
- Proposals are chosen in descending order.

Knapsack Voting

- Best response for voter is the vote that satisfies where
  - Proposal
  - is the cumulative votes of all voters except
  - is the set of winners when ’s vote is added to
  - Benefit a voter gets from proposal is
  - Set of proposals
  - Proposal has a cost
  - Fixed total budget of Dollars

Knapsack Voting

- Partial strategy-proofness
  - Partial strategy-proofness is new, relaxed notion of strategy-proofness
  - Refers to how a mechanism makes truthful reporting a dominant strategy for those agents whose preference intensities differ sufficiently between any 2 objects

Knapsack Voting

- Partial strategy-proofness theorem: Given a best response if , then there is another best response such that where
  - Best response for voter is the vote
  - is the cumulative votes of all voters except
  - the set of winners as determined by
  - Proposal
  - Set of proposals
  - Benefit a voter gets from proposal is
  - Proposal has a cost

Knapsack Voting

- Corollary 3.3: The partial strategy-proofness theorem fails to hold when each voter submits a -approval vote (i.e.), and the winning set is constrained by the budget B.
  - Each voter submits a proposal
- Knapsack voting is provably better than -approval voting, because knapsack voting can make truthful reporting a dominant strategy.

Value-for-money Comparisons

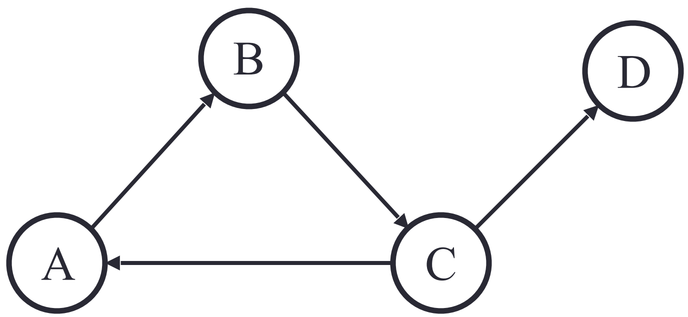
- For each pair of proposals presented to them, the voter is asked to choose a winner .
  - The benefit a voter gets from proposal is .
  - The proposal has a cost .
- Each voter has a fixed size, uniformly random subset of pairs to maintain uniformity.

Value-for-money Comparisons

- The resulting votes are used to calculate a strict rank ordering.
- is a complete directed graph on the set of proposals .
- The weight of each edge is the number of comparisons where j is favored to k.
- Find a strict rank order on that minimizes .
  - Weighted Minimum Feedback Arc Set problem

Weighted Minimum Feedback Arc Set Problem

- A directed graph may have directed cycles or a one-way loop of edges which we want to eliminate.



- What is the fewest number of edges to remove in order to eliminate these loops?
- NP-hard problem but can use LP-relaxation

Linear Programming Relaxation

- Removing the integrality constraint of each variable in a mixed integer linear program
  - A variable may initially be required to be an integer
  - The constraint is relaxed, so the variable can be a fraction instead.
- Transforms an NP-hard problem to a related problem solvable in P time
  - Requires less resources to solve

Preliminary Trial

- Digital voting system for participatory budgeting voting in Vallejo, California from September to October 2014
- Tested value-for-money comparisons voting method with voters
- Use LP-relaxation from Conitzer et al.
  - Changes Weighted Minimum Feedback Arc Set problem to minimizing which is subject to , , ,
    - is a complete directed graph on the set of proposals
    - The weight of each edge is the number of comparisons where j is favored to k
    - is set of all cycles in graph

Preliminary Trial

- They found integer-optimal results
- Indicates they may have found the optimal aggregate ranking
- Indicates value-for-money comparisons voting method could potentially be used for participatory budgeting

Budget Aggregation via Knapsack Voting: Welfare-maximization and Strategy-proofness

Goel, A., Krishnaswamy, A.K. and Sakshuwong, S., 2016. Budget aggregation via knapsack voting: welfare-maximization and strategy-proofness. Collective Intelligence, pp.783-809.

Knapsack Voting

- Each voter votes for an allocation such that where
  - Set of voters
  - Set of proposals
  - Fixed total budget of Dollars

Introduction

- Redefine Participatory Budgeting Problem and -approval voting
- Knapsack voting
  - Did not discuss how it can be welfare-maximizing
  - No empirical study

Knapsack Voting

- For each and any , define .
- The outcome is given by

$$\arg\max_{\sum_{p\in\mathcal{P}} w_p = B} \sum_{p\in\mathcal{P}} \sum_{0}^{w_p-1} \mathrm{score}(w_p)$$

Knapsack Voting

- They impose some assumptions on voter preferences to maintain strategyproofness
  - Assume natural model of voter utility
  - "Satisfaction" of voter is determined by overlap between preferred budget allocation and final outcome
- Voter utility for election outcome is
  - Voter has preferred allocation that satisfies the budget constraint
  - Outcome of the elections is
  - Voter utility for project is

Knapsack Voting

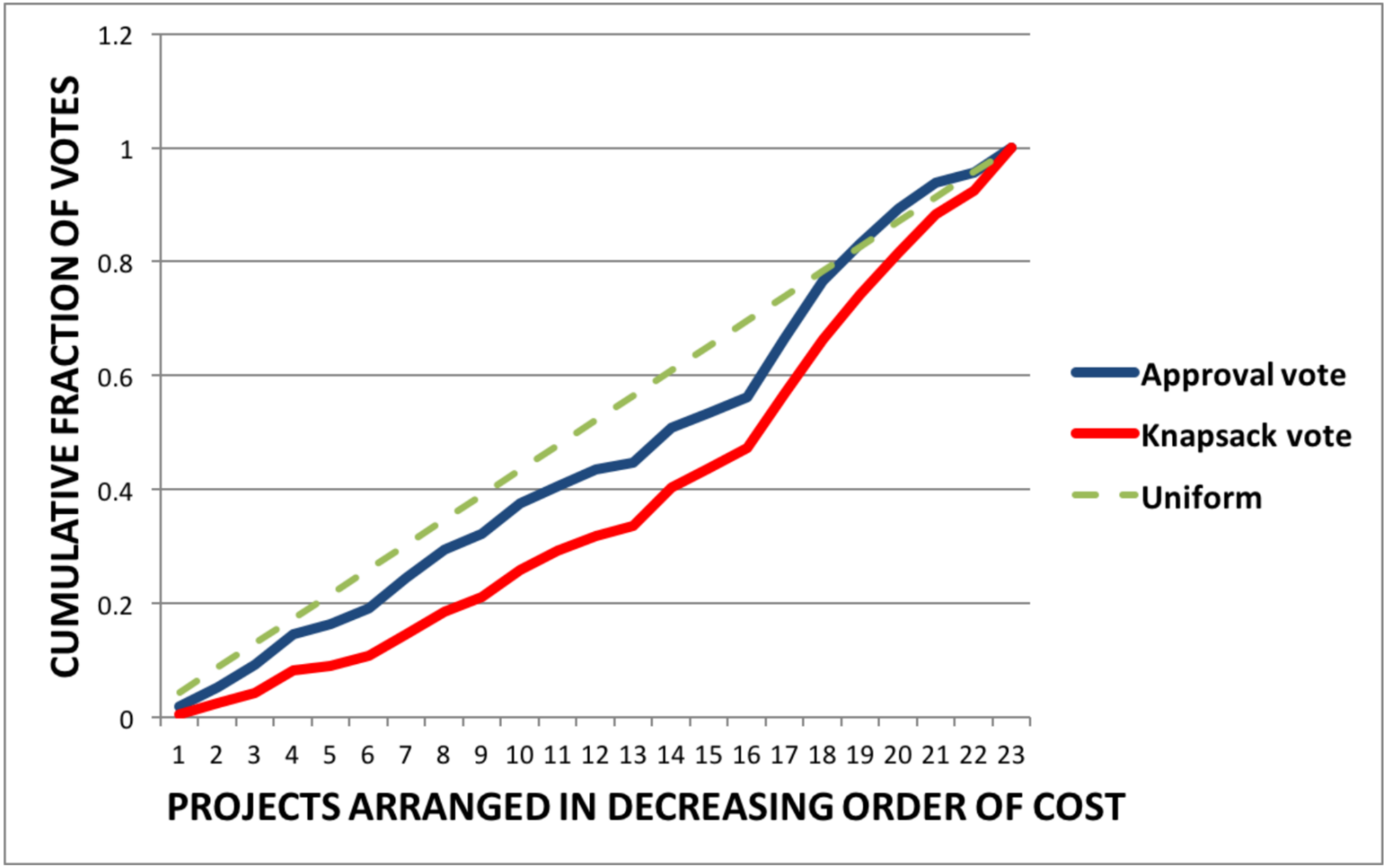
- Knapsack voting is strategy-proof, and its outcome is welfare-maximizing
  - Strategy-proof: the dominant strategy for a voter is voting for their true preferred budget allocation
  - Welfare-maximizing: maximizes the sum of utilities of all voters
- Neither property applies to -approval voting
- Knapsack voting is superior to -approval voting under these conditions and assumptions

Empirical Study

- Data from New York District 8 and Cambridge
- Similar trends across all elections
- Had experimental interface for knapsack voting in addition to -approval voting ballot
- -approval voting method biases the outcome towards projects of larger cost compared to knapsack voting
  - Bigger, costlier projects gain more support in -approval voting

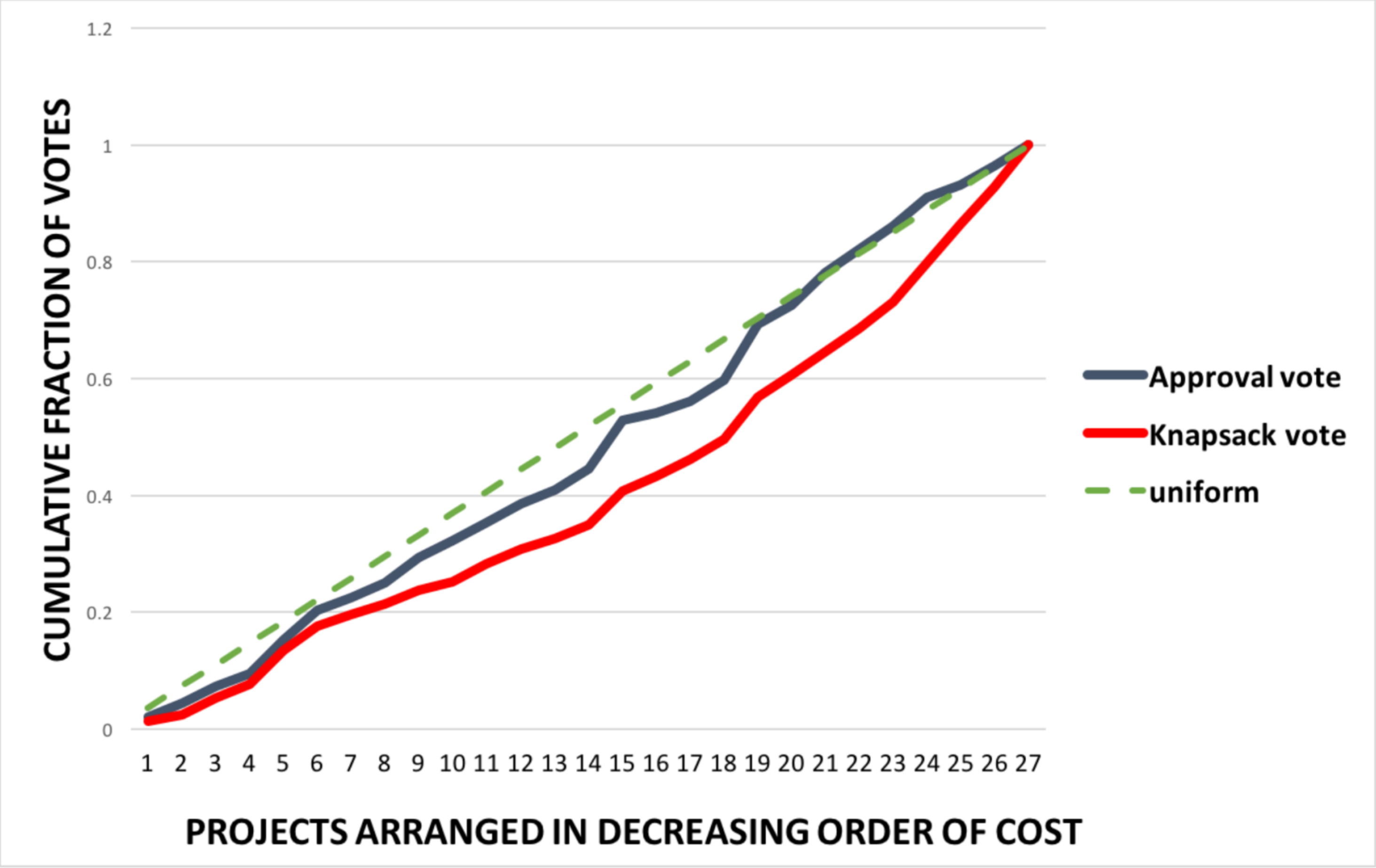
Empirical Study Hypothesis Data

Fig. 3. Cambridge



Empirical Study Hypothesis Data

Fig. 4. NYC District 8

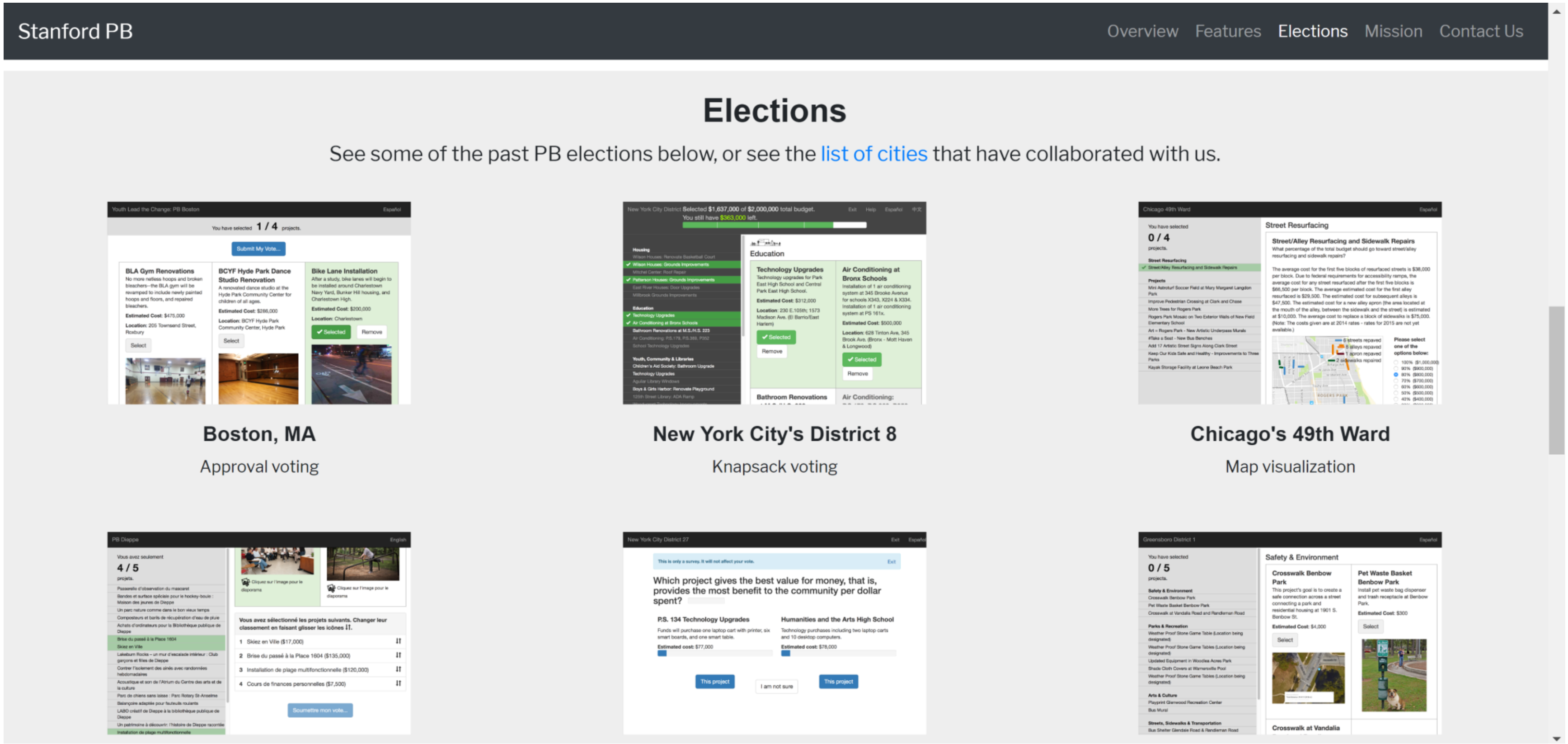


Empirical Study Hypothesis Data

Table I. Average cost of winning projects, as a fraction of the budget

	K-approval	*knapsack*
NYC District 8	0.20	0.12
Cambridge	0.15	0.10

https://pbstanford.org/



Conclusion

- Value-for-money comparisons is a possible participatory budgeting voting method
- Knapsack voting is strategy-proof, and its outcome is welfare-maximizing
- Knapsack voting is superior to -approval voting
  - But only with the paper's defined situation and assumptions:
    - In participatory budgeting
    - With natural model of user utility (voter satisfaction determined by overlap between preferred budget allocation and final outcome)

Questions?

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