

---

# MP 1 – Basic Isabelle Proofs

CS 477 – Spring 2013

Revision 1.1

**Assigned** January 31, 2013

**Due** February 8, 2013, 11:59 PM

**Extension** 48 hours (penalty 20% of total points possible)

---

## 1 Change Log

**1.1** Removed all references to hw1.

**1.0** Initial Release.

## 2 Objectives and Background

The purpose of this MP is to test the student's ability to

- start up and interact with Isabelle;
- write proofs of propositional formulae using the basic proof methods.

Another purpose of MPs in general is to provide a framework to study for the exam. Several of the questions on the exam will appear similar to the MP problems. By the time of the exam, your goal is to be able to solve any of the following problems with pen and paper in less than 2 minutes.

## 3 Turn-In Procedure

The pdf for this assignment (`mp1.pdf`) should be found in the `mps/mp1/` subdirectory of your svn directory for this course, along with a skeleton version of the file `mp1.thy`. You should put code answering each of the problems below in the file `mp1.thy`. Your completed `mp1.thy` file should be put in the `mps/mp1/` subdirectory of your svn directory (where it was originally found) and committed as follows:

```
svn commit -m "Turning in mp1"
```

Please read the *Instructions for Submitting Assignments* in

<http://courses.engr.illinois.edu/cs477/mps/index.html>

## 4 Propositional Logic in Isabelle/HOL

Isabelle/HOL supports Natural Deduction proofs for propositional logic with the following introduction and elimination rules:

```
notI : (A  $\implies$  False)  $\implies$   $\neg$ A
notE :  $\llbracket \neg A; A \rrbracket \implies B$ 
conjI :  $\llbracket A; B \rrbracket \implies A \wedge B$ 
conjE :  $\llbracket A \wedge B; \llbracket A; B \rrbracket \implies C \rrbracket \implies C$ 
disjI1 : A  $\implies$  A  $\vee$  B
disjI2 : B  $\implies$  A  $\vee$  B
disjE :  $\llbracket A \vee B; A \implies C; B \implies C \rrbracket \implies C$ 
impI : (A  $\implies$  B)  $\implies$  A  $\longrightarrow$  B
impE :  $\llbracket A \longrightarrow B; A; B \implies C \rrbracket \implies C$ 
iffI :  $\llbracket A \implies B; B \implies A \rrbracket \implies A = B$ 
iffE :  $\llbracket A = B; \llbracket A \longrightarrow B; B \longrightarrow A \rrbracket \implies C \rrbracket \implies C$ 
```

The names of the variables are not the names used in the actual theorems as encoded in Isabelle, so you will need to use `thm` to

## 5 Problems

**Note:** For each of the formulae in each of the problems below, give a complete `apply` style proof. You must remove each occurrence of `sorry` and replace them with proofs. For this assignment, the proofs must be in `apply` style (as shown in class) and are restricted to proof methods that are the composition of one of `rule`, `rule_tac`, `erule`, or `erule_tac` with one of the (possibly specialized) rules given above.

**Note:** You are free to create your own lemmas in Isabelle, and use them in proofs, so long as they have complete, `apply` style proofs satisfying the above restrictions. (You may apply this extension recursively. ☺)

1. (2pts)  $A \longrightarrow A$
2. (4pts)  $(A \wedge B) \longrightarrow (A \vee B)$
3. (8pts)  $(A \vee A) \longrightarrow (A \wedge A)$
4. (4pts)  $A \longrightarrow \neg \neg A$
5. (8pts)  $(A \longrightarrow (B \longrightarrow C)) \longrightarrow ((A \wedge B) \longrightarrow C)$
6. (8pts)  $((A \wedge B) \wedge C) \longrightarrow (A \wedge (B \wedge C))$
7. (11pts)  $((A \vee B) \vee C) \longrightarrow (A \vee (B \vee C))$

8. (5pts)  $A \wedge B \longrightarrow B \wedge A$
9. (3pts)  $A \longrightarrow B \longrightarrow A$
10. (10pts)  $(A \longrightarrow B \longrightarrow C) \longrightarrow ((A \longrightarrow B) \longrightarrow (A \longrightarrow C))$

## 6 Extra Credit

The set of rules given above are sound for the standard model of propositional logic, but they are not complete. We require at least one more rule to admit what is called “classical” reasoning. (Reasoning done with only the rules given above is referred to as “intuitionistic” reasoning.) The one Isabelle rule we will add is the following:

$$\text{classical: } (\neg A \Longrightarrow A) \Longrightarrow A$$

In the problems below, you must follow the same rules as for the problems above except that you are also allowed to use `classical`.

11. (1 pt)  $\neg\neg A \longrightarrow A$
12. (2 pts)  $A \vee \neg A$
13. (2 pts)  $(\neg A \longrightarrow B) \longrightarrow (\neg B \longrightarrow A)$
14. (2 pts)  $((A \longrightarrow B) \longrightarrow A) \longrightarrow A$
15. (5 pts)  $(\neg(A \wedge B)) = (\neg A \vee \neg B)$
16. (3 pts)  $(\neg A \Longrightarrow \text{False}) \Longrightarrow A$

In this problem you will probably want to use `rule_tac`. If, for example, you wanted to use `rule_tac` with `impI`, you would first use

```
thm impI
```

to see the actual variable names. In this case, you would see:

```
(?P ==> ?Q) ==> ?P ==> ?Q
```

Then you could use

```
apply (rule_tac P = "A \ttand B" and Q = "A" in impI)
```

This will cause you to use the instance of the rule `impI` that says:

$$(A \wedge B \Longrightarrow A) \Longrightarrow A \wedge B \longrightarrow A$$