HW 2 – Interpretation of First Order Logic Formulae

CS 477 – Spring 2013
Revision 1.1

Assigned February 8, 2013
Due February 15, 2013, 11:59 pm
Extension 48 hours (20% penalty)

1 Change Log

1.1 Changed $v \Rightarrow w$ in Problem 4 to $v < w$.

1.0 Initial Release.

2 Objectives and Background

The purpose of this HW is to test your understanding of

- modeling and interpretation of first order logic formulae

Another purpose of HWs is to provide you with experience answering non-programming written questions of the kind you may experience on the midterm and final.

3 Turn-In Procedure

The pdf for this assignment (`hw2.pdf`) should be found in the `mps/hw2/` subdirectory of your svn directory for this course. Your solution should be put in that same directory. Using your favorite tool(s), you should put your solution in a file named `hw2-sol.pdf`. If you have problems generating a pdf, please seek help from the course staff. Your answers to the following questions are to be submitted electronically from within `mps/hw2/` subdirectory by committing the file as follows:

```
svn add hw2-sol.pdf
svn commit -m "Turning in hw2"
```

4 Problems

Each of the following formulae is over the signature

$$\mathcal{G} = (V = \{u, v, w, x, y, z\}, F = \{+\}, af = \{+ \mapsto 2;\}, R = \{=, <\}, ar = \{= \mapsto 2, < \mapsto 2\})$$

. The operator $+$ and the relations $=$ and $<$ will be written as infixed. For each of the following formulae, give the following:

a. (2 pts) the list of free variables;

b. for the structure $\mathcal{S} = \{\mathcal{G}, \mathcal{D} = \mathbb{N}, \mathcal{F}, \phi, \mathcal{R}, \rho\}$ where $\phi(+)$ is normal addition, and where $\rho(=)$ is normal equality and $\rho(<)$ is normal less-than comparison

   (i) (3 pts) give an assignment for which the formula is valid and say why the assignment satisfies the formula, or say why none exists, and

   (ii) (3 pts) give an assignment for which the formula is invalid and say why the assignment fails to satisfy the formula, or say why none is possible;

c. for the structure $\mathcal{S} = \{\mathcal{G}, \mathcal{D} = \mathbb{R}, \mathcal{F}, \phi, \mathcal{R}, \rho\}$ where $\phi(+)$ is multiplication, and where $\rho(=)$ is normal equality but where $\rho(<)(x, y) = (x^2 < y^2)$.

   (i) (3 pts) give an assignment for which the formula is valid and say why the assignment satisfies the formula, or say why none exists, and

   (ii) (3 pts) give an assignment for which the formula is invalid and say why the assignment fails to satisfy the formula, or say why none is possible;

1. $\exists u.\ \forall v.\ (u < x) \wedge ((u < v) \Rightarrow ((x = v) \vee (x < v)))$

2. $x < y \wedge x < z \wedge \neg(y < (x + y))$

3. $((x < y) \wedge (y < z)) \Rightarrow (x < z)$

4. $(u < v) \wedge (v < w) \Rightarrow (\exists x.\ \exists y.((u < x) \wedge (x < v) \wedge (v < y) \wedge (y < w)))$

5. $\forall x.\ \forall y.(((x < y) \vee (x = y)) \wedge ((y < x) \vee (x = y))) \Rightarrow (x = y)$

5 Extra Credit

6. (5 pts) Give a structure that models the formula in Problem 1 (different for either structure I gave) and describe why the structure models the formula.