HW 1 – Truth and Proof in Propositional Logic

CS 477 – Spring 2013
Revision 1.0

Assigned January 23, 2013
Due January 30, 2013, 11:59 pm
Extension 48 hours (20% penalty)

1 Change Log

1.0 Initial Release.

2 Objectives and Background

The purpose of this HW is to test your understanding of

- validity of propositions in the standard model of propositional logic
- Natural Deduction proofs of propositions in propositional logic

Another purpose of HWs is to provide you with experience answering non-programming written questions of the kind you may experience on the midterm and final.

3 Turn-In Procedure

The pdf for this assignment (`hw1.pdf`) should be found in the `mps/hw1/` subdirectory of your svn directory for this course. Your solution should be put in that same directory. Using your favorite tool(s), you should put your solution in a file named `hw1-sol.pdf`. If you have problems generating a pdf, please seek help from the course staff. Your answers to the following questions are to be submitted electronically from within `mps/hw1/` subdirectory by commiting the file as follows:

```
svn add hw1-sol.pdf
svn commit -m "Turning in hw1"
```

4 Problem

For each of the following propositions, give both all possible valuations of every subformula of the proposition in the form of a truth table, and given a Natural Deduction proof of the proposition. For the Natural Deduction proof, you may use the pure style first indtruced in class, but it must be accompanied by a discription of how each assumption is discharged. Alternatively, you may use the sequent encoding of Natural Deduction proofs.

1. (5pts + 5pts) $(A \wedge B) \Rightarrow (A \vee B)$

2. (4pts + 6pts) $(A \vee A) \Rightarrow (A \wedge A)$

3. (3pts + 5pts) $A \Rightarrow \neg\neg A$

4. (16pts + 10pts) $(A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \wedge B) \Rightarrow C)$

5. (16pts + 8pts) $((A \wedge B) \wedge C) \Rightarrow (A \wedge (B \wedge C))$

5 Extra Credit

6. (10 pts) Given a detailed, rigorous proof that the sequent encoding of Natural Deduction proof is *sound* with respect to the standard model for propositional logic. That is, prove that every proposition having a closed Natural Deduction proof is valid in the standard model.

   Hint: What proposition corresponds to a sequent?