CS477 Formal Software Development Methods

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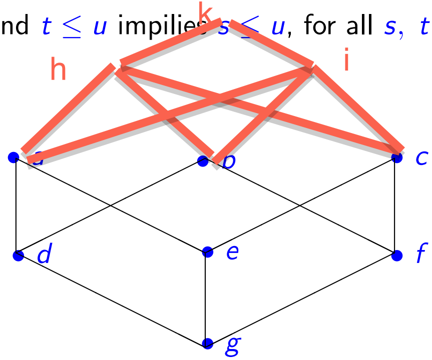
Partial orders and Complete Lattices

A partial order on a set S is a binary relation $\leq$ on S such that

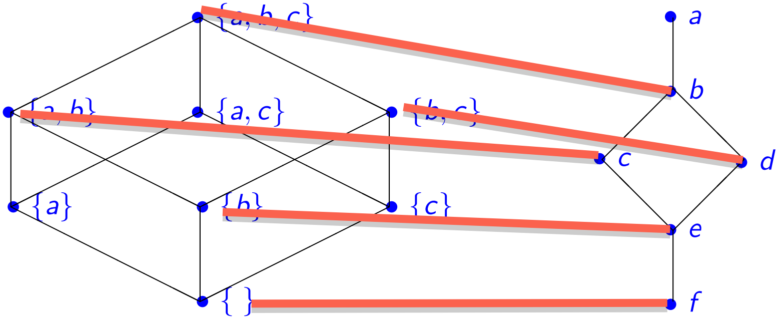
- **[Refl]** $s \leq s$ for all $s \in S$
- **[Antisym]** $s \leq t$ and $t \leq s$ impilies $s = t$, for all $s,\ t \in S$
- **[Trans]** $s \leq t$ and $t \leq u$ impilies $s \leq u$, for all $s,\ t,\ \in S$



Upper Bounds and Complete Latices

- In a partial order $(S, \leq)$, given $X \subseteq S$, y is an upper bound for X if for all $x \in X$ we have $x \leq y$.
- y is a least upper bound of X, y is an upper bound of X and whenever z is an upper bound of X, $y \leq z$.
- **Note:** Least upper bounds are unique.
- A complete lattice is a partial order $(L, \leq)$ such that for all $X \subseteq S$ there exists a (unique) least upper bound.
- Write $\mathrm{lub}(X)$ or $\bigvee X$ for the least upper bound of X.
- Write $x \vee y$ for $\bigvee\{x, y\}$
- **Note:** $x \vee y = x \iff y \leq x$
- **Note:** Given a set S, $(\mathcal{P}(S), \subseteq)$ is a complete lattice.
- Write $\bot = \bigvee\{\ \}$ and $\top = bigveeS$

Example Complete Lattices

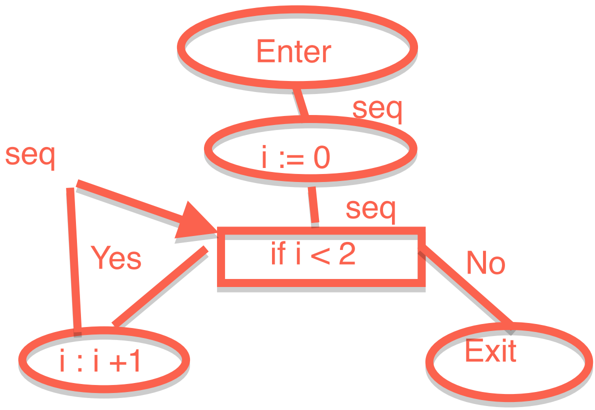


Control-Flow Graphs

A Control-Flow Graph is a tuple (N, E, l, k) where

- N is a finite set of nodes
- $l : N \to \{\mathrm{Entry}, \mathrm{Exit},$i:=e$, \mathrm{if} b, \}$
- $E \subseteq NtimesN$ such that
  - for all $m \in N$ we have $|\{n\,.\,(m, n) \in E\}| \leq 2$
  - if $m \in N$ and $l(m) = \mathrm{Exit}$ then $|\{n\,.\,(m, n) \in E\}| = 0$
  - if $m \in N$ and $l(m) = \mathrm{Entry}$ or $l(m) = i := e$ for some identifier i and expression e, then $|\{n\,.\,(m, n) \in E\}| = 1$
  - if $m \in N$ and $l(m) = \mathrm{if}\ b$ for some boolean expression b, then $|\{n\,.\,(m, n) \in E\}| = 2$
- $k : E \to \{\mathrm{seq}, \mathrm{yes}, \mathrm{no}\}$ such that
  - if $(m, n) \in E$ and $l(m) = \mathrm{Entry}$ or $l(m) = i := e$, then $k((m, n)) = \mathrm{seq}$
  - if $m, \in N$ and $l(m) = \mathrm{if}\ b$, then $\{k((m, n))\,.\,(m, n) \in E\} = \{\mathrm{yes}, \mathrm{no}\}$

Example



Abstract Interpretation

An abstract interpretation of control flow graphs is a pair $(A, \mathcal{I})$ where

- A is a complete latice and to be completed in class
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